Infinity Learn by Sri Chaitanya Educational Institutions

**Chapter 13: Probability.**

## Exercise Miscellaneous

1. A and B are two events such that $P(A)\neq 0$. Find $P(B|A)$. If

i) A is a subset of B   ii) $A\cap B=\phi$

**Solution:**

Given, $P(A)\neq 0$

i) A is subset of B

$\Rightarrow A\cap B=A$

$\therefore P(A\cap B)=P(B\cap A)=P(A)$

$$\therefore P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{P(A)}{P(A)}=1$$

ii) $A\cap B=\phi$

$\Rightarrow P(A\cap B)=0$

$$\therefore P(B|A)=\frac{P(A\cap B)}{P(A)}=0$$

2. A couple has two children,

(i) Find the probability that both children are males, if it is known that at least one of the

children is male.

(ii) Find the probability that both children are females, if it is known that the elder child is

a female.

**Solution:**

If a couple has two children, then the sample space is $S=\{(B,B),(B,G),(G,B),(G,G)\}$

i) Let E and F respectively denote the events that both children are male and atleast one children is a male.

$$\therefore E \cap F = \{(G,G)\} \Rightarrow P(E \cap F) = \frac{1}{4}$$

$$P(E) = \frac{1}{4}$$

$$P(F) = \frac{3}{4}$$

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

ii) Let C and D respectively denote the events that both children are females and the elder child is a female.

$$C = \{(G,G)\} \Rightarrow P(C) = \frac{1}{4}$$

$$D = \{(G,B),(G,G)\} \Rightarrow P(D) = \frac{2}{4}$$

$$C \cap D = \{(G,G)\} \Rightarrow P(C \cap D) = \frac{1}{4}$$

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

3. Suppose that 5% of men and 0.25% of women have grey hair. A haired person is selected at random. What is the probability of this person being male? Assume that there are equal numbers of males and females.

**Solution:**

Give 5% of men and 0.25% of women have grey hair.

Thus, percentage of people with grey hair $= (5 + 0.25)\% = 5.25\%$

$\therefore$ Probability that the selected haired person is a male $= \dfrac{5}{5.25} = \dfrac{20}{21}$

4. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

**Solution:**

A person can be either right-handed or left-handed

Given, 90% of the people are right-handed

$\therefore\ p = $P(right-handed) $= \dfrac{9}{10}$

q = P (left-handed) $= 1 - \dfrac{9}{10} = \dfrac{1}{10}$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=1}^{10} {}^{10}C_r p^r q^{n-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

$= 1 - P$ (more than 6 are right-handed)

$$1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

5. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

(i) all will bear 'X' mark.

(ii) not more than 2 will bear 'Y' mark.

(iii) at least one ball will bear 'Y' mark.

(iv) the number of balls with 'X' mark and 'Y' mark will be equal.

**Solution:**

Given, total number of balls in the urn = 25

Balls bearing mark ‘X’ = 10

Balls bearing mark ‘Y’ = 15

p = P (ball bearing mark ‘X’) = $\frac{10}{25}=\frac{2}{5}$

q = P (ball bearing mark ‘Y’ = $\frac{15}{25}=\frac{3}{5}$

Let Z be the random variable that represents the number of balls with ‘Y’ mark on them in the trials.

Clearly, Z has a binomial distribution with n = 6 and p = $\frac{2}{5}$

$$\therefore P(Z=z)={}^{n}C_{z}\,p^{n-z}q^{z}$$

i) P (all will bear ‘X’ mark) $=P(Z=0)={}^{6}C_{0}\left(\frac{2}{5}\right)^{6}=\left(\frac{2}{5}\right)^{6}$

ii) P (not more than 2 bear ‘Y’ mark) $=P(Z\le 2)$

$$=P(Z=0)+P(Z=1)+P(Z=2)$$

$$={}^{6}C_{0}(p)^{6}(q)^{0}+{}^{6}C_{1}(p)^{5}(q)^{1}+{}^{6}C_{2}(p)^{4}(q)^{2}$$

$$=\left(\frac{2}{5}\right)^{6}+6\left(\frac{2}{5}\right)^{5}\left(\frac{3}{5}\right)+15\left(\frac{2}{5}\right)^{4}\left(\frac{3}{5}\right)^{2}$$

$$=\left(\frac{2}{5}\right)^{4}+\left[\left(\frac{2}{5}\right)^{5}+6\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)+15\left(\frac{3}{5}\right)^{2}\right]$$

$$=\left(\frac{2}{5}\right)^{4}+\left[\frac{4}{25}+\frac{36}{25}+\frac{135}{25}\right]$$

$$=\left(\frac{2}{5}\right)^{4}+\left[\frac{175}{25}\right]$$

$$=7\left(\frac{2}{5}\right)^{4}=0.1792$$

ii) P (at least one ball bear 'Y' mark) $= P(Z \geq 1) = 1 - P(Z = 0)$

$$= 1 - \left(\frac{2}{5}\right)^6$$

iv) P (equal number of balls with 'X' mark and 'Y' mark) $= P(Z = 3)$

$$= {}^6C_3\left(\frac{2}{54}\right)^3\left(\frac{3}{5}\right)^3$$

$$= \frac{20 \times 8 \times 27}{15625}$$

$$= \frac{864}{3125}$$

6. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

**Solution:**

Let p and q respectively be the probability that the player will clear and knock down the hurdle.

$$\therefore p = \frac{5}{6}$$

$$\Rightarrow q = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$$

Let X be the random variable that represents the number of times the player will knock down the hurdle.

Thus, by binomial distribution, we get

$$P(X = x) = {}^nC_x p^{n-x} q^x$$

P (Player knocking down less than 2 hurdles) $=P(x<2)$

$$=P(x=0)+P(X=1)$$

$$={}^{10}C_0(q)^0(p)^{10}+{}^{10}C_1(q)(p)^9$$

$$=\left(\frac{5}{6}\right)^{10}+10.\frac{1}{6}.\left(\frac{5}{6}\right)^9$$

$$=\left(\frac{5}{6}\right)^9\left[\frac{5}{6}+\frac{10}{6}\right]$$

$$=\frac{5}{2}\left(\frac{5}{6}\right)^9$$

$$=\frac{(5)^{10}}{2\times(6)^9}$$

7. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

**Solution:**

The probability of getting a six in a throw of die is $\frac{1}{6}$ and not getting a six is $\frac{5}{6}$

Let $p=\frac{1}{6}$ and $q=\frac{5}{6}$

The probability that the 2 sixes come in the first five throws of the die is

$${}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3=\frac{10\times(5)^3}{(6)^5}$$

Probability that third six comes in the sixth throw $=\frac{10\times(5)^3}{(6)^5}\times\frac{1}{6}$

$$=\frac{10\times125}{(6)^6}$$

$$=\frac{10\times 125}{46656}=\frac{625}{23328}$$

8. If a leap year is selected at random, what is the change that it will contain 53 Tuesdays?

**Solution:**

In a leap year, there are 366 days i..e, 52 weeks and 2 days.

In 52 weeks, there are 52 Tuesdays.

Therefore, the probability that the leap year will contain 53 Tuesday is equal to the probability that the remaining 2 days will be Tuesdays.

The remaining 2 days can be any of the following

Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday and Sunday and Monday

Total number of cases = 7

$\therefore$ Probability that a leap year will have 53 Tuesdays = $\frac{2}{7}$

9. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.

**Solution:**

Given, the probability of success is twice the probability of failure

Let the probability of failure be x

Probability of success = 2x

$x+2x=1$

$\Rightarrow 3x=1$

$\Rightarrow x = \frac{1}{3}$

$\therefore 2x = \frac{2}{3}$

Let $p = \frac{1}{3}$ and $q = \frac{2}{3}$

Let X be the random variable that represents the number of successes in six trials

By binomial distribution, we obtain

$P(X = x) = {}^{n}C_{x} p^{n-x} q^{x}$

Probability of at least 4 successes $= P(x \geq 4)$

$= P(X = 4) + P(X = 5) + P(X = 6)$

$= {}^{6}C_{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{2} + {}^{6}C_{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right) + {}^{6}C_{6}\left(\frac{2}{3}\right)^{6}$

$= \frac{15(2)^{4}}{3^{6}} + \frac{6(2)^{5}}{3^{6}} + \frac{(2)^{6}}{3^{6}}$

$= \frac{(2)^{4}}{(3)^{6}}[15 + 12 + 4]$

$= \frac{31 \times 2^{4}}{(3)^{6}} = \frac{31}{9}\left(\frac{2}{3}\right)^{4}$

10. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

**Solution:**

Let the man toss the coin n times. The n tosses are n Bernoulli trials

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$

$p=\frac{1}{2}, q=\frac{1}{2}$

$\therefore P(X=x)={}^{n}C_{x}p^{n-x}q^{x}={}^{n}C_{x}\left(\frac{1}{2}\right)^{n-x}\left(\frac{1}{2}\right)^{x}={}^{n}C_{x}\left(\frac{1}{2}\right)^{n}$

Given,

P (getting at least one head) $>\frac{90}{100}$

$P(x\geq 1)>0.9$

$1-P(x=1)>0.9$

$1-{}^{n}C_{0}.\frac{1}{2}>0.9$

${}^{n}C_{0}.\frac{1}{2}<0.1$

$\frac{1}{2^{n}}<0.1$

$2^{n}>\frac{1}{0.1}$

$2^{n}>10........(1)$

Thus, the minimum value of n is 4

Therefore, the man should toss the coin more than 4 times

11. In a game, a man wins a rupee for a six and loss a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

**Solution:**

Here, the probability of getting a six is $\frac{1}{6}$ and the probability of not getting a 6 is $\frac{5}{6}$

Three cases can occur.

(a) If he gets a six in the first throw, then the required probability is $\frac{1}{6}$

Amount he will receive = Re 1

b) If he does not get a six in the first in the first throw and gets a six in the second throw, then probability $\left(\frac{5}{6}\times\frac{1}{6}\right)=\frac{5}{36}$

Amount he will receive = – Re 1 + Re 1 = 0

(c) If he does not get a six in the first two throws and gets a six in the third throw, then probability $=\left(\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right)=\frac{25}{216}$

Amount he will receive = – Re 1 – Re 1 + Re 1 = -1

Expected value he can win = $\frac{1}{6}(1)+\left(\frac{5}{6}\times\frac{1}{6}\right)(0)+\left[\left(\frac{5}{6}\right)^2\times\frac{1}{6}\right](-1)$

$=\frac{1}{6}-\frac{25}{216}$

$=\frac{36-25}{216}=\frac{11}{216}$

12. Suppose we have four boxes A, B, C and D containing marbles as given below

| Box | Marble colour | | |
|---|---|---|---|
| | Red | White | Black |
| A | 1 | 6 | 3 |
| B | 6 | 2 | 2 |
| C | 8 | 1 | 1 |
| D | 0 | 6 | 4 |

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

**Solution:**

Let R be the event of drawing the red marble.

Let $E_A$, $E_B$, and $E_C$ respectively denote the events of selecting the box A, B, and C.

Total number of marbles = 40

Number of red marbles = 15

$$\therefore P(R) = \frac{15}{40} = \frac{3}{8}$$

Probability of drawing the red marble from box A is given by P ($E_A$|R).

$$\therefore P(E_A \mid R) = \frac{P(E_A \cap R)}{P(R)} = \frac{\frac{1}{40}}{\frac{3}{8}} = \frac{1}{15}$$

Probability that the red marble is from box B is P ($E_B$|R).

$$\Rightarrow P(E_B \mid R) = \frac{P(E_B \cap R)}{P(R)} = \frac{\frac{6}{40}}{\frac{3}{8}} = \frac{2}{15}$$

Probability that the red marble is from box C is P ($E_C$|R).

$$\Rightarrow P(E_C \mid R) = \frac{P(E_C \cap R)}{P(R)} = \frac{\frac{8}{40}}{\frac{3}{8}} = \frac{8}{15}$$

13. Assume that the changes of the patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its changes by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

**Solution:**

Let A, $E_1$ and $E_2$ respectively denote the events that a person has a heart attack, the Selected person followed the course of yoga and meditation, and the person adopted the Drugs prescription.

$\therefore P(A)=0.40$

$$P(E_1)=P(E_2)=\frac{1}{2}$$

$$P(A|E_1)=0.40\times 0.70=0.28$$

$$P(A|E_2)=0.40\times 0.75=0.30$$

Probability that the patient suffering a heart attack followed a course of mediation and yoga is given by $P(E_1|A)$

$$P(E_1|A)=\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28+\frac{1}{2}\times 0.30}$$

$$=\frac{14}{29}$$

14. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).

**Solution:**

The total number of determinants of second order with each element being 0 or 1 is

$(2)^4 = 16$.

The value of determinant is positive in the following cases $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$\therefore$ Required probability $= \dfrac{3}{16}$

15. An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

P (A fails) = 0.2

P (B fails alone) = 0.15

P (A and B fails) = 0.15

Evaluate the following probabilities

(i) P (A fails | B has failed) (ii) P (A fails alone)

**Solution:**

Let the event in which A fails and B fails be denote by $E_A$ and $E_B$

$P(E_A) = 0.2$

$P(E_A \; and \; E_B) = 0.15$

P (B fails alone) = $P(E_B) - P(E_A \; and \; E_B)$

$\therefore 0.15 = P(E_B) - 0.15$

$\therefore P(E_B) = 0.3$

i) $P(A_A \mid E_B) = \dfrac{P(E_A \cap E_B)}{P(E_B)} = \dfrac{0.15}{0.3} = 0.5$

ii) P (A fails alone) $= P(E_A) - P(E_A \; and \; E_B)$

$= 0.2 - 0.15 = 0.05$

16. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The

ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

**Solution:**

Let $E_1$ and $E_2$ respectively denote the event that a red ball is transferred from bag I to II and a black ball is transferred from bag I to II.

$$P(E_1)=\frac{3}{7} \text{ and } P(E_2)=\frac{4}{7}$$

Let A be the event that the ball drawn is red.

When a red ball is transferred from bag I to II,

$$P(A|E_1)=\frac{5}{10}=\frac{1}{2}$$

When a black ball is transferred from bag I to II,

$$P(A|E_2)=\frac{4}{10}=\frac{2}{5}$$

$$P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{\frac{4}{7}\times\frac{2}{5}}{\frac{3}{7}\times\frac{1}{2}+\frac{4}{7}\times\frac{2}{5}}=\frac{16}{31}$$

17. If A and B are two events such that $P(A)\neq 0$ and $P(B|A)=1$, then

A) $A\subset B$ B) $B\subset A$ C) $B\neq\phi$ D) $A\neq\phi$

**Solution:**

Given, $P(A)\neq 0$ and $P(B|A)=1$

$$P(B|A)=\frac{P(B\cap A)}{P(A)}$$

$$1=\frac{P(B\cap A)}{P(A)}$$

$$P(A)=P(B\cap A)$$

$$\Rightarrow A\subset B$$

Thus, (A) is the correct answer

18. If $P(A|B)>P(A)$, the which of the following is correct

A) $P(B|A)<P(B)$ B) $P(A\cap B)<P(A).P(B)$

C) $P(B|A)>P(B)$ D) $P(B|A)=P(B)$

**Solution:**

Given, $P(A|B)>P(A)$

$$\Rightarrow\frac{P(A\cap B)}{P(B)}>P(A)$$

$$\Rightarrow P(A\cap B)>P(A).P(B)$$

$$\Rightarrow\frac{P(A\cap B)}{P(A)}\geq P(B)$$

$$\Rightarrow P(B|A)\geq P(B)$$

Thus, (C) is the correct answer

19. If A and B are any two events such that $P(A)+P(B)-P(A\,and\,B)=P(A)$, then

A) $P(B|A)=1$ B) $P(A|B)=1$ C) $P(B|A)=0$ D) $P(A|B)=0$

**Solution:**

Given,

$$P(A)+P(B)-P(A\,and\,B)=P(A)$$

$$\Rightarrow P(A)+P(B)-P(A\cap B)=P(A)$$

$$\Rightarrow P(B)-P(A\cap B)=P(B)$$

$$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B)}{P(B)}=1$$

Thus, (B) is the correct answer

## Exercise 13.1

1. Given that $E$ and $F$ are events such that $P(E)=0.6, P(F)=0.3$ and $P(E\cap F)=0.2$ then find the values of $P(E|F)$ and $P(F|E)$

**Solution:** Given $P(E)=0.6, P(F)=0.3$ and $P(E\cap F)=0.2$

The conditional probability states that

$$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{0.2}{0.3}=\frac{2}{3}$$

Therefore, $P(E\mid F)=\frac{2}{3}$

The conditional probability states that

$$P(F|E)=\frac{P(E\cap F)}{P(E)}=\frac{0.2}{0.6}=\frac{1}{3}$$

Therefore, $P(F\mid E)=\frac{1}{3}$

2. Compute $P(A|B)$ if $P(B)=0.5$ and $P(A\cap B)=0.32$

**Solution:** Given, $P(B)=0.5$ and $P(A\cap B)=0.32$

The conditional probability states that

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0.32}{0.5}=\frac{16}{25}$$

Therefore, $P(A|B)=\frac{16}{25}$

3. If $P(A)=0.8, P(B)=0.5$ and $P(B|A)=0.4$ find

i) $P(A\cap B)$ ii) $P(A|B)$ iii) $P(A\cup B)$

**Solution:** Given, $P(A)=0.8, P(B)=0.5$ and $P(B|A)=0.4$

The conditional probability states that

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$

(i) Hence, $0.4=\frac{P(A\cap B)}{0.8} \Rightarrow P(A\cap B)=0.32$

(ii) Use conditional probability

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$$
$$=\frac{0.32}{0.5}$$
$$=0.64$$

(iii) Use addition theorem on probability

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=0.8+0.5-0.32$$
$$=0.98$$

Hence, $P(A\cup B)=0.98$

4. Evaluate $P(A\cup B)$, if $2P(A)=P(B)=\frac{5}{13}$ and $P(A|B)=\frac{2}{5}$

**Solution:** Given, $2P(A)=P(B)=\frac{5}{13}$

It implies that $P(A)=\frac{5}{26}$ and

Use the conditional probability $P(A|B)=\frac{P(A\cap B)}{P(B)}$

Substitute the appropriate values in the above equation.

$$\frac{2}{5}=\frac{P(A\cap B)}{\frac{5}{13}}$$

$$P(A\cap B)=\frac{5}{13}\times\frac{2}{5}$$
$$=\frac{2}{13}$$

Use the Addition theorem on probability

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}$$
$$=\frac{5+10-4}{26}$$
$$=\frac{11}{26}$$

Therefore, $P(A\cup B)=\frac{11}{26}$

5. If $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}$ and $P(A\cup B)=\frac{7}{11}$, find

i. $P(A\cap B)$

ii. $P(A|B)$

iii. $P(B|A)$

**Solution:** Given, $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}$ and $P(A\cup B)=\frac{7}{11}$

i. Addition theorem of probability states that

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

Substitute the appropriate values

$$\frac{7}{11}=\frac{6}{11}+\frac{5}{11}-P(A\cap B)$$

$$P(A\cap B)=\frac{4}{11}$$

Therefore, $P(A\cap B)=\frac{4}{11}$

ii. The conditional probability states that

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$=\frac{\frac{4}{11}}{\frac{5}{11}}$$

$$=\frac{4}{5}$$

Therefore, $P(A|B)=\frac{4}{5}$

iii. The conditional probability states that

$$P(B|A)=\frac{P(A\cap B)}{P(A)}$$

$$=\frac{\frac{4}{11}}{\frac{6}{11}}$$

$$=\frac{2}{3}$$

Therefore, $P(B|A)=\frac{2}{3}$

6. A coin is tossed three times, where

i. $E$ : head on third toss, $F$ : heads on first two tosses

ii. $E$ : At least two heads, $F$ : At most two heads

iii. $E$ : At most two heads, $F$ : At least one tail.

Find the value of $P(E|F)$ in each part.

**Solution**: The sample space $S$ is $S=\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

i. Given that $E$ : head on third toss, $F$ : heads on first two tosses

$$E=\{HHH, HTH, THH, TTH\} \text{ and } F=\{HHH, HHT\}$$

Hence, $E\cap F=\{HHH\}$

Conditional probability states that

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$=\frac{n(E\cap F)}{n(F)}$$

$$=\frac{1}{4}$$

Therefore, $P(E|F)=\frac{1}{4}$

ii. Given that $E$ : At least two heads, $F$ : At most two heads

$E=\{HHH,HTH,THH,HHT\}$ and

$F=\{TTT,TTH,THT,HTT,HTH,THH,HHT\}$

Hence, $E\cap F=\{HHH,HTH,THH,HHT\}$

Conditional probability states that

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$=\frac{n(E\cap F)}{n(F)}$$

$$=\frac{4}{7}$$

Therefore, $P(E|F)=\frac{4}{7}$

iii. Given that $E$ : At most two tails, $F$ : At least one tail.

$E=\{HHH,TTH,THT,HTT,THH,HTH,HHT\}$ and

$F=\{TTT,HHT,THH,HTH,TTH,THT,HTT\}$

Hence, $E\cap F=\{HHT,THH,HTH,TTH,THT,HTT\}$

Conditional probability states that

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$=\frac{n(E\cap F)}{n(F)}$$

$$=\frac{6}{7}$$

Therefore, $P(E|F)=\frac{6}{7}$

7. Two coins are tossed once, where

i. $E$ : tail appears on one coin, $F$ : one coin shows head

ii. $E$ : no tail appears, $F$ : no head appears.

Find the value of $P(E|F)$ in each part.

**Solution:** The sample space $S$ is $S=\{HH,HT,TH,TT\}$

i. Given that $E$ : tail appears on one coin, $F$ : one coin shows head

$$E=\{HT,TH\} \text{ And } F=\{HT,TH\}$$

Hence, $E\cap F=\{HT,TH\}$

Conditional probability states that

$$\begin{aligned} P(E|F) &= \frac{P(E\cap F)}{P(F)} \\ &= \frac{n(E\cap F)}{n(F)} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

Therefore, $P(E|F)=1$

ii. Given that $E$ : no tail appears, $F$ : no head appears.

$$E=\{HH\} \text{ And } F=\{TT\}$$

Hence, $E\cap F=\{\ \}$

Conditional probability states that

$$\begin{aligned} P(E|F) &= \frac{P(E\cap F)}{P(F)} \\ &= \frac{n(E\cap F)}{n(F)} \\ &= \frac{0}{1} \\ &= 0 \end{aligned}$$

Therefore, $P(E|F)=0$

8. A die is thrown three times, suppose that $E:$ Appear 4 on third toss, $F:6,5$ appear on first two tosses. Then find $P(E|F)$

**Solution:** The number of possible cases for the event $E:$ Appear 4 on third toss:

First two tosses any one of first six numbers, so that $n(E)=6\times6=36$

For the event $F:6,5$ appear on first two tosses

$$F=\{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

Hence, $E\cap F=\{(6,5,4)\}$

Conditional probability states that

$$\begin{aligned} P(E|F) &= \frac{P(E\cap F)}{P(F)} \\ &= \frac{n(E\cap F)}{n(F)} \\ &= \frac{1}{6} \end{aligned}$$

Therefore, $P(E|F)=\frac{1}{6}$

9. Mother, Father and son line up at random for a family picture

$E:$ Son on one end, $F:$ Father in middle

Then find $P(E|F)$

**Solution:** Suppose that $A$:Mother, $B:$ Father, $C:$Son

Given that $E:$ Son on one end, $F:$ Father in middle

It implies that $E=\{ABC,BAC,CAB.CBA\}$ and $F=\{ABC,CBA\}$

Hence, $E\cap F=\{ABC,CBA\}$

Conditional probability states that

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$
$$=\frac{n(E\cap F)}{n(F)}$$
$$=\frac{2}{2}$$
$$=1$$

Therefore, $P(E|F)=1$

10. A black and a red dice are rolled.

a. Find the conditional probability of obtaining a sum greater than 9, given that the black die results 5

b. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

**Solution:** Given that one black and one red dice thrown.

a. Suppose that $E$ be the event of getting total 9 and $F$ be the event of getting 5 on the black die

The set of favourable cases to the event $E$ is

$$E=\{(B_4,R_6),(B_5,R_5),(B_5,R_6),(B_6,R_4),(B_6,R_5),(B_6,R_6)\}$$

The set of favourable cases to the event $F$ is

$$F=\{(B_5,R_1),(B_5,R_2),(B_5,R_3),(B_5,R_4),(B_5,R_5),(B_5,R_6)\}$$

Hence, $E\cap F=\{(B_5,R_5),(B_5,R_6)\}$

Conditional probability states that

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$
$$=\frac{n(E\cap F)}{n(F)}$$
$$=\frac{2}{6}$$
$$=\frac{1}{3}$$

Therefore, $P(E|F)=\frac{1}{3}$

b. Suppose that $E$ be the event of getting sum 8 and $F$ be the event of getting number less than 4 on the red dice

The set of favourable cases to the event $E$ is

$$E=\{(B_2,R_6),(B_3,R_5),(B_4,R_4),(B_5,R_3),(B_6,R_2)\}$$

The set of favourable cases to the event $F$ is

$$F=\left\{\begin{matrix}(B_1,R_1),(B_2,R_1),(B_3,R_1),(B_4,R_1),(B_5,R_1),(B_6,R_1),\\(B_1,R_1),(B_2,R_1),(B_3,R_1),(B_4,R_2),(B_5,R_2),(B_6,R_2),\\(B_1,R_3),(B_2,R_3),(B_3,R_3),(B_4,R_3),(B_5,R_3),(B_6,R_3)\end{matrix}\right\}$$

Hence, $E\cap F=\{(B_5,R_3),(B_6,R_2)\}$

Conditional probability states that

$$\begin{aligned}P(E|F)&=\frac{P(E\cap F)}{P(F)}\\&=\frac{n(E\cap F)}{n(F)}\\&=\frac{2}{18}\\&=\frac{1}{9}\end{aligned}$$

Therefore, $P(E|F)=\frac{1}{9}$

11. A fair die is rolled. Consider the events $E=\{1,3,5\},F=\{2,3\},G=\{2,3,4,5\}$ then find the following

i) $P(E|F)$ and $P(F|E)$

ii) $P(E|G)$ and $P(G|E)$

iii) $P(E\cup F|G)$ and $P(E\cap F|G)$

**Solution:** Given that $E=\{1,3,5\},F=\{2,3\},G=\{2,3,4,5\}$

i) Conditional probability states that $P(E|F)=\frac{P(E\cap F)}{P(F)}$

Here $E\cap F=\{3\}$.

It implies that

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$=\frac{n(E\cap F)}{n(F)}$$

$$=\frac{1}{2}$$

Therefore, $P(E|F)=\frac{1}{2}$

Conditional probability states that $P(F|E)=\frac{P(E\cap F)}{P(E)}$

Here $E\cap F=\{3\}$.

It implies that

$$P(F|E)=\frac{P(E\cap F)}{P(E)}$$

$$=\frac{n(E\cap F)}{n(E)}$$

$$=\frac{1}{3}$$

Therefore, $P(F|E)=\frac{1}{3}$

ii) Conditional probability states that $P(E|G)=\frac{P(E\cap G)}{P(G)}$

Here $E\cap G=\{3,5\}$.

It implies that

$$P(E|\mathrm{G})=\frac{P(E\cap G)}{P(G)}$$

$$=\frac{n(E\cap G)}{n(G)}$$

$$=\frac{2}{4}$$

Therefore, $P(E|G)=\frac{1}{2}$

Conditional probability states that $P(G|E)=\dfrac{P(E\cap G)}{P(E)}$

Here $E\cap G=\{3,5\}$.

It implies that

$$\begin{aligned}P(G|E)&=\frac{P(E\cap G)}{P(E)}\\&=\frac{n(E\cap G)}{n(E)}\\&=\frac{2}{3}\end{aligned}$$

Therefore, $P(G|E)=\dfrac{2}{3}$

iii) Conditional probability states that $P(E\cup F|G)=\dfrac{P(E\cup F\cap G)}{P(G)}$

Here $E\cup F=\{1,2,3,5\}E\cup F\cap G=\{2,3,5\}$.

It implies that

$$P(E\cup F|G)=\frac{P(E\cup F\cap G)}{P(G)}=\frac{n(E\cup F\cap G)}{n(G)}=\frac{3}{4}$$

Therefore, $P(E\cup F|G)=\dfrac{3}{4}$

Conditional probability states that $P(E\cap F|G)=\dfrac{P(E\cap F\cap G)}{P(G)}$

Here $E\cap F\cap G=\{3\}$

It implies that

$$P(E\cap F|G)=\frac{P(E\cap F\cap G)}{P(G)}=\frac{n(E\cap F\cap G)}{n(G)}=\frac{1}{4}$$

Therefore, $P(E\cap F|G)=\dfrac{1}{4}$

12. Assume that each born child is equally like to be a boy or a girl. If a family had two children. What is the conditional probability that both are girls given that

i) The youngest is a girl

ii) At least one is girl.

**Solution:** Suppose that $B$: boy and $G$: girl.

Sample space for the experiment is $S=\{(BB),(BG),(GB),(GG)\}$

i) Let $E$ be the event of getting family having both are girl children

$$E=\{GG\}$$

Let $F$ be the event of getting a family having youngest is a girl

$$F=\{(GG),(GB)\}$$

Hence, $E\cap F=\{(GG)\}$

The conditional probability states that $P(E|F)=\dfrac{P(E\cap F)}{P(F)}$

$$P(E|F)=\frac{P(E\cap F)}{P(F)}$$

$$=\frac{n(E\cap F)}{n(F)}$$

$$=\frac{1}{2}$$

Therefore, $P(E|F)=\dfrac{1}{2}$

ii) Let $E$ be the event of getting family having both are girl children

$$E=\{GG\}$$

Let $F$ be the event of getting at least one girl

$$F=\{(GG),(GB),(BG)\}$$

Hence, $E \cap F = \{(GG)\}$

The conditional probability states that $P(E|F) = \frac{P(E \cap F)}{P(F)}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$
$$= \frac{n(E \cap F)}{n(F)}$$
$$= \frac{1}{3}$$

Therefore, $P(E|F) = \frac{1}{3}$

13. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions If a question is selected at random from the question bank, what is the probability that it will be an easy question that it is a multiple choice questions?

**Solution:** The given data can be tabulated as below

| | True/False $(T)$ | Multiple $(M)$ | Total |
|---|---|---|---|
| Easy $(E)$ | 300 | 500 | 800 |
| Difficult $(D)$ | 200 | 400 | 600 |
| Total | 500 | 900 | 1400 |

We want to find the probability getting an easy question that it is a multiple choice question.

Conditional probability states that $P(E|M) = \frac{P(E \cap M)}{P(M)}$

$$P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{n(E \cap M)}{n(M)} = \frac{500}{900} = \frac{5}{9}$$

Therefore, $P(E|M) = \frac{5}{9}$

14. Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event " The sum of the numbers on the dice is 4"

**Solution:** Let $E$ be the event of getting different numbers on two dice

The number of favourable cases to the event $E$ is $n(E)=36-6(\text{doublet})=30$

Let $F$ be the event of getting the sum of the numbers on the dice is 4.

$$F=\{(1,3),(2,2),(3,1)\}$$

Hence, $E\cap F=\{(1,3),(3,1)\}$

Conditional probability states that $P(F\mid E)=\dfrac{P(E\cap F)}{P(E)}$

$$P(F\mid E)=\frac{P(E\cap F)}{P(E)}$$
$$=\frac{2}{30}$$
$$=\frac{1}{15}$$

Therefore, $P(E\mid F)=\dfrac{1}{15}$

15. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event the coin shows a tail, given that at least one die shows as 3

**Solution:** The sample space of the experiment is

$$S=\begin{Bmatrix}(1,H),(1,T),(2,H),(2,T),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,H),(4,T),(5,H),(5,T),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

Here $n(S)=20$

Let $E$ be the event of showing tail on the coin

$$E=\{(1,T),(2,T),(4,T),(5,T)\}$$

Here, $n(E)=4$

Let $F$ be the event of getting 3 at least on one dice

$$F=\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(6,3)\}$$

Here, $n(F)=7$

Consider $E\cap F=\varphi$

Hence, $n(E\cap F)=0$

Conditional probability states that $P(E|F)=\dfrac{P(E\cap F)}{P(F)}$

Hence,

$$\begin{aligned}P(E|F)&=\frac{P(E\cap F)}{P(F)}\\&=\frac{n(E\cap F)}{n(F)}\\&=\frac{0}{7}\\&=0\end{aligned}$$

Therefore, the probability of the event the coin shows a tail, given that at least one die shows as 3 is zero.

16. If $P(A)=\frac{1}{2}, P(B)=0$ then the value of $P(A|B)$

A) 0

B) $\frac{1}{2}$

C) Not defined

D) 1

**Solution:** Given $P(A)=\frac{1}{2}, P(B)=0$

Conditional probability states that $P(A|B)=\frac{P(A\cap B)}{P(B)}$

$$\begin{aligned}P(A|B)&=\frac{P(A\cap B)}{P(B)}\\&=\frac{n(A\cap B)}{n(B)}\\&=\text{Not defined}\end{aligned}$$

This is matching with the option $(C)$

17. If $A$ and $B$ are the events such that $P(A|B)=P(B|A)$ then

A) $A\neq B$

B) $A=B$

C) $A\cap B$ is an empty set

D) $P(A)=P(B)$

Solution: Given $P(A|B)=P(B|A)$

Conditional probability states that $P(A|B)=\frac{P(A\cap B)}{P(B)}$

It implies that $P(A)=P(B)$

This is matching with the option $(D)$

## Exercise 13.2

1. If $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$, find $P(A\cap B)$ if $A$ and $B$ are independent events

**Solution:** Given, $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$

Given $A$ and $B$ are independent events. It implies that $P(A\cap B)=P(A)\cdot P(B)$

Hence,

$$\begin{aligned}P(A\cap B)&=P(A)\cdot P(B)\\&=\frac{3}{5}\cdot\frac{1}{5}\\&=\frac{3}{25}\end{aligned}$$

Therefore, $P(A\cap B)=\frac{3}{25}$

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black

**Solution:** Since there are 26 black cards in the deck of 52 cards

Let $A$ be the event of the getting two black cards without replacement

It implies that

$$\begin{aligned}P(A)&=\frac{C(26,2)}{C(52,2)}\\&=\frac{26\cdot 25}{52\cdot 51}\\&=\frac{25}{102}\end{aligned}$$

Therefore, the probability that two cards drawn are to be black is $\frac{25}{102}$

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale

**Solution:** Given that there are 15 oranges in box out of 12 are good and 3 are bad ones.

Suppose that three randomly selected oranges drawn without replacement

Suppose that $A$ be the event of getting all three oranges are good

$$\begin{aligned} P(A) &= \frac{C(12,3)}{C(15,3)} \\ &= \frac{12\cdot 11\cdot 10}{15\cdot 14\cdot 13} \\ &= \frac{44}{91} \end{aligned}$$

Therefore, the probability that the box approved for sale is $\frac{44}{91}$

4. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not

**Solution:** The Sample space S is given by

$$S = \begin{Bmatrix} (H,1),(H,2),(H,3),(H,4),(H,5),(H,6) \\ (T,1),(T,2),(T,3),(T,4),(T,5),(T,6) \end{Bmatrix}$$

Let $A$ be the event which shows Head on the coin

$$A = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$$

It implies that $P(A) = \frac{6}{12} = \frac{1}{2}$

Let $B$ be the event which shows 3 on the die

$$B = \{(H,3),(T,3)\}$$

It implies that $P(B)=\frac{2}{12}=\frac{1}{6}$

$\therefore A\cap B=\{(H,3)\}$

$P(A\cap B)=\frac{1}{12}$

$P(A).P(B)=\frac{1}{2}\times\frac{1}{6}=P(A\cap B)$

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the events, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

**Solution:**

The sample space (S) is $S=\{1,2,3,4,5,6\}$

Let A: the number is even $=\{2,4,6\}$

$\Rightarrow P(A)=\frac{3}{6}=\frac{1}{2}$

B: the number is red $=\{1,2,3\}$

$\Rightarrow P(B)=\frac{3}{6}=\frac{1}{2}$

$\therefore A\cap B=\{2\}$

$P(AB)=P(A\cap B)=\frac{1}{6}$

$P(A)P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}\neq\frac{1}{6}$

$\Rightarrow P(A).P(B)\neq P(AB)$

Therefore, A and B are not independent

6. Let E and F be the events with $P(E)=\frac{3}{5}, P(F)=\frac{3}{10}$ and $P(E\cap F)=\frac{1}{5}$. Are E and F independent?

**Solution:**

Given, $P(E)=\frac{3}{5}, P(F)=\frac{3}{10}$ and $P(EF)=P(E\cap F)=\frac{1}{5}$

$P(E).P(F)=\frac{3}{5}\times\frac{3}{10}=\frac{9}{50}\neq\frac{1}{5}$

$\Rightarrow P(E).P(F)\neq P(EF)$

Thus, E and F are not independent

7. Given that the events A and B are such that $P(A)=\frac{1}{2}, P(A\cap B)=\frac{3}{5}$ and $P(B)=p$

Find p if they are (i) mutually exclusive (ii) independent.

**Solution:**

Given, $P(A)=\frac{1}{2}, P(A\cap B)=\frac{3}{5}$ and $P(B)=p$

(i) When A and B are mutually exclusive, $A\cap B=\phi$

$\therefore P(A\cap B)=0$

Since, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-0$

$\Rightarrow p=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$

(ii) when A and B are independent, $P(A\cap B)=P(A).P(B)=\frac{1}{2}p$

Since, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-\frac{1}{2}p$$

$$\Rightarrow \frac{3}{5}=\frac{1}{2}+\frac{p}{2}$$

$$\Rightarrow \frac{p}{2}=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$$

$$\Rightarrow p=\frac{2}{10}=\frac{1}{5}$$

8. Let A and B be independent events with $P(A)=0.3$ and $P(B)=0.4$. Find

i) $P(A\cap B)$ ii) $P(A\cup B)$ iii) $P(A/B)$ iv) $P(B/A)$

**Solution:**

Given, $P(A)=0.3$ and $P(B)=0.4$

i) If A and B are independent events, then

$$P(A\cap B)=P(A).P(B)=0.3\times 0.4=0.12$$

ii) Since, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow P(A\cup B)=0.3+0.4-0.12=0.58$$

iii) Since, $\Rightarrow P(A/B)=\frac{P(A\cap B)}{P(B)}$

$$\Rightarrow P(A/B)=\frac{0.12}{0.4}=0.3$$

iv) Since, $P(B/A)=\frac{P(A\cap B)}{P(A)}$

$$\Rightarrow P(B/A)=\frac{0.12}{0.3}=0.4$$

9. If A and B are two events such that $P(A)=\frac{1}{4}, P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{8}$, find P (not A and not B)

**Solution:**

Given, $P(A)=\frac{1}{4}$ and $P(A\cap B)=\frac{1}{8}$

P (not on A and not on B) $=P(A'\cap B')$

P (not on A and not on B) $=P((A\cup B))'$

$=1-P(A\cup B)$

$=1-[P(A)+P(B)-P(A\cap B)]$

$=1-\frac{5}{8}$

$=\frac{3}{8}$

10. Events A and B are such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and P (not A or not B) $=\frac{1}{4}$.

State whether A and B are independent?

**Solution:**

Given, $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and P (not A or not B) $=\frac{1}{4}$

$\Rightarrow P(A'\cup B')=\frac{1}{4}$

$\Rightarrow P((A\cap B))'=\frac{1}{4}$

$\Rightarrow 1-P(A\cap B)=\frac{1}{4}$

$\Rightarrow P(A\cap B)=\frac{3}{4}$..........(1)

But, $P(A).P(B)=\frac{1}{2}.\frac{7}{12}=\frac{7}{24}$.........(2)

Here, $\frac{3}{4} \neq \frac{7}{24}$

$\therefore P(A \cap B) = P(A).P(B)$

Thus, A and B are independent events

11. Given two independent events A and B such P(A) = 0.3, P(B) = 0.6. Find (i) P (A and B) (ii) P (A and not B) (iii) P (A or B) (iv) P (neither A or B)

**Solution:**

Given, $P(A) = 0.3, P(B) = 0.6$

i) $\therefore P(A and\ B) = P(A).P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.6 = 0.18$

ii) P (A and not B) = $P(A \cap B')$

$= P(A) - P(A \cap B)$

$= 0.3 - 0.18$

$= 0.12$

iii) P (A or B) = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18$

$= 0.72$

iv) P (neither A nor B) $= P(A' \cap B')$

$= P((A \cup B))'$

$= 1 - P(A \cup B)$

$=1-0.72$

$=0.28$

12. A die tossed thrice. Find the probability of getting an odd number at least once

**Solution:**

Probability of getting an odd number in a single throw of a die $=\frac{3}{6}=\frac{1}{2}$

Similarly, probability of getting an even number $=\frac{3}{6}=\frac{1}{2}$

Probability of getting an even number three times $=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

Therefore, probability of getting an odd number at least once

=1-probability of getting an odd number in none of the throws

=1-probability of getting an even number thrice

$=1-\frac{1}{8}$

$=\frac{7}{8}$

13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red

(ii) first ball is black and second is red.

(iii) one of them is black and other is red

**Solution:**

Given,

Total number of balls = 18

Number of red balls = 8

Number of black balls = 10

(i) Probability of getting a red ball in the first draw = $\frac{8}{18}=\frac{4}{9}$

The ball is replaced after the first draw.

$\therefore$ Probability of getting a red ball in the second draw = $\frac{8}{18}=\frac{4}{9}$

Thus, probability of getting both the balls red = $\frac{4}{9}\times\frac{4}{9}=\frac{16}{81}$

(ii) Probability of getting first ball black = $\frac{10}{18}=\frac{5}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as red = $\frac{8}{18}=\frac{4}{9}$

Thus, probability of getting first ball as black and second ball as red = $\frac{5}{9}\times\frac{4}{9}=\frac{20}{81}$

(iii) Probability of getting first ball as red = $\frac{8}{18}=\frac{4}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as black = $\frac{10}{18}=\frac{5}{9}$

Thus, probability of getting first ball as black and second ball as red = $\frac{4}{9}\times\frac{5}{9}=\frac{20}{81}$

Therefore, probability that one of them is black and other is red

= Probability of getting first ball black and second as red + Probability of getting first ball red and second ball black = $\frac{20}{81}+\frac{20}{81}$

$$=\frac{40}{81}$$

14. Probability of solving specific problem independently by A and B are respectively. $\frac{1}{2}$ and $\frac{1}{3}$ If both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one of them solves the problem.

**Solution:**

Probability of solving the problem by A, $P(A)=\frac{1}{2}$

Probability of solving the problem by B, $P(B)=\frac{1}{3}$

Since the problem is solved independently by A and B

$\therefore P(A\cap B)=P(A).P(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

$P(A')=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$

$P(B')=1-P(B)=1-\frac{1}{3}=\frac{2}{3}$

i) Probability that the problem is solved $=P(A\cup B)$

$=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$

$=\frac{4}{6}$

$=\frac{2}{3}$

ii) Probability that exactly one of them solves the problem is given by

$P(A).P(B')+P(B).P(A')$

$=\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}$

$=\frac{1}{3}+\frac{1}{6}$

$=\frac{1}{2}$

15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E: 'the card drawn is a spade'

F: 'the card drawn is an ace'

(ii) E: 'the card drawn is black'

F: 'the card drawn is a king'

(iii) E: 'the card drawn is a king and queen'

F: 'the card drawn is a queen or jack'

**Solution:**

i) Since, in a deck of 52 cards, 13 cards are spades and 4 cards are aces.

$\therefore P(E)=P$ (the card drawn is a spade) $=\frac{13}{52}=\frac{1}{4}$

$\therefore P(F)=P$ (the cards drawn in an ace) $=\frac{4}{52}=\frac{1}{13}$

In the deck of cards, only 1 card is an ace of spades.

P(EF) = P (the card drawn is spade and an ace) = $\frac{1}{52}$

$P(E)\times P(F)=\frac{1}{4}.\frac{1}{13}=\frac{1}{52}=P(EF)$

$\Rightarrow P(E)\times P(F)=P(EF)$

Thus, the events E and F are independent

ii) Since, in a deck of 52 cards, 26 cards are black and 4 cards are kings

$\therefore P(E)=$ P(the card drawn is a black) $=\frac{26}{52}=\frac{1}{2}$

$\therefore P(F) =$ P(the card drawn in an king) $= \frac{4}{52} = \frac{1}{13}$

In the pack of 52 cards, 2 cards are black as well as kings

$\therefore P(EF) =$P (the card drawn is black king) $= \frac{2}{52} = \frac{1}{26}$

$$P(E) \times P(F) = \frac{1}{2} . \frac{1}{13} = \frac{1}{26} = P(EF)$$

Thus, the given events E and F are independent

iii) Since, in a deck of 52 cards, 4 cards are kings, 4 cards are queens and 4 cards are jacks

$\therefore P(E) =$P (the card drawn is a king or a queen) $= \frac{8}{52} = \frac{2}{13}$

$\therefore P(F) =$P (the card drawn in a queen or a jack) $= \frac{8}{52} = \frac{2}{13}$

There are 4 cards which are king and queen or jack

$\therefore P(EF) =$P(the card drawn is king or a queen, or queen or a jack)

$$= \frac{4}{52} = \frac{1}{13}$$

$$P(E) \times P(F) = \frac{2}{13} . \frac{2}{13} = \frac{4}{169} \neq \frac{1}{3}$$

$$\Rightarrow P(E).P(F) \neq P(EF)$$

Thus, the given events E and F are not independent

16. In a hotel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English news papers. A student is selected at random. (a) Find the probability that she reads neither Hindi and English news papers.

(b) If she reads Hindi news paper, find the probability that she reads English news paper.

(c) If she reads English news paper, find the probability that she reads Hindi news paper.

**Solution:**

Let H denote the students who read Hindi newspaper and E denote the students who read English newspaper.

Given,

$$P(H) = 60\% = \frac{60}{100} = \frac{3}{5}$$

$$P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(H \cap E) = 20\% = \frac{20}{100} = \frac{1}{5}$$

i) Probability that a student reads Hindi and English newspaper is,

$$P(H \cup E') = 1 - P(H \cup E)$$

$$= 1 - \{P(H) + P(E) - P(H \cap E)\}$$

$$= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right)$$

$$= 1 - \frac{4}{5} = \frac{1}{5}$$

ii) Probability that a randomly chosen student reads English newspapers, if she reads Hindi news paper, is given by P (E/H).

$$P(E/H) = \frac{P(E \cap H)}{P(H)} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$$

iii) Probability that a random chosen student reads Hindi newspaper, if she reads English newspaper, is given by $P(H/E)$

$$P(H/E)=\frac{P(H\cap E)}{P(E)}=\frac{\frac{1}{5}}{\frac{2}{5}}=\frac{1}{2}$$

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{12}$ D) $\frac{1}{36}$

**Solution:**

The only even prime number is 2

Let E be the event of getting an even prime number on each die

$\therefore E=\{(2,2)\}$

$\Rightarrow P(E)=\frac{1}{36}$

18. Two events A and B will be independently, if

A) A and B are mutually exclusive

B) $P(A'B')=[1-P(A)][1-P(B)]$

C) $P(A)=P(B)$

D) $P(A)+P(B)=1$

**Solution:**

Two events A and B are said to be independent, if $P(AB)=P(A)\times P(B)$

Let take option B

$$P(A'B')=[1-P(A)][1-P(B)]$$

$\Rightarrow P(A' \cap B') = 1 - P(A) + P(A).P(B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A).P(B)$

$\Rightarrow P(A) + P(B) - P(AB) = P(A) + P(B) - P(A).P(B)$

$\Rightarrow P(AB) = P(A).P(B)$

This implies that A and B are independent, if $P(A'B') = [1 - P(A)][1 - P(B)]$

A. Let $P(A) = m, P(B) = n, 0 < m, n < 1$

A and B are mutually exclusive

$\therefore A \cap B = \phi$

$\Rightarrow P(AB) = 0$

However, $P(A).P(B) = mn \neq 0$

$\therefore P(A).P(B) \neq P(AB)$

C. Let A: Event of getting an odd number on throw of a die $= \{1, 3, 5\}$

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B: Event of getting an even number on throw of a die $= \{2, 4, 6\}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

Here, $A \cap B = \phi$

$\therefore P(AB) = 0$

$P(A).P(B) = \frac{1}{4} \neq 0$

$P(A).P(B) \neq P(AB)$

D: $P(A)+P(B)=\frac{1}{2}+\frac{1}{2}=1$

But, it cannot be inferred that A and B are independently

Thus, the correct answer is B.

## Exercise 13.3

1. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

**Solution:** Given there are 5 red and 5 black balls.

Suppose that in the first attempt a red ball drawn

Hence, the probability of drawing red ball is $P(R_1)=\frac{1}{2}$

If two red balls are added to the urn, then the urn contains 7 red balls and 5 black balls

Hence, the probability of getting red ball in the second attempt is $P(R_2 \mid R_1)=\frac{7}{12}$

Let a black ball be drawn in the first attempt

$\therefore$ P (drawing a black ball in the first attempt) $=\frac{5}{10}=\frac{1}{2}$

If two black balls are added to the urn, then the urn contains 5 red and 7 black balls.

P (drawing a red ball) = $\frac{5}{12}$

Thus, probability of drawing second ball as red is

$$=\frac{1}{2}\times\frac{7}{12}+\frac{1}{2}\times\frac{5}{12}$$

$$=\frac{1}{2}\left(\frac{7}{12}+\frac{5}{12}\right)$$

$$=\frac{1}{2}\times 1$$

$=\frac{1}{2}$

2. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

**Solution:**

Let $E_1$ and $E_2$ be the events of selecting first bag and second bag respectively.

$$P(E_1)=P(E_2)=\frac{1}{2}$$

Let A be the event of getting a red ball.

$\Rightarrow P(A|E_1)$ =P (drawing a red ball from first bag) = $\frac{4}{8}=\frac{1}{2}$

$\Rightarrow P(A|E_2)$ = P (drawing a red ball from second bag) $=\frac{2}{8}=\frac{1}{4}$

The probability of drawing a ball from the first bag, given that it is red, is given by

$P(E_2|A)$

$$P(E_1|A)=\frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)}$$

$$=\frac{\frac{1}{2}.\frac{1}{2}}{\frac{1}{2}.\frac{1}{2}+\frac{1}{2}.\frac{1}{4}}$$

$$=\frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{8}}=\frac{\frac{1}{4}}{\frac{3}{8}}=\frac{2}{3}$$

3. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is hostler?

**Solution:**

Let $E_1$ and $E_2$ be the events that the students is a hostler and a day scholar respectively and A be the event that the chosen student gets grade A

$$\therefore P(E_1) = 60\% = \frac{60}{100} = 0.6$$

$$P(E_2) = 40\% = \frac{40}{100} = 0.4$$

$P(A|E_1) =$ P(student getting an A grade is a hostler) = 30% = 0.3

$P(A|E_2) =$ P(student getting an A grade is a day scholar) = 20% = 0.2

The probability that a randomly chosen student is a hostler, given that he has an A grade, is given by $P(E_1|A)$

By using Baye's theorem, we obtain

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.2}$$

$$= \frac{0.18}{0.26} = \frac{18}{26} = \frac{9}{13}$$

4. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct

with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

**Solution:**

Let $E_1$ and $E_2$ be the respective events that the student knows the answer and he guesses the answer.

Let A be the event that the answer is correct

$$\therefore P(E_1)=\frac{3}{4}$$

$$P(E_2)=\frac{1}{4}$$

The probability that the students answered correctly, given that he known the answer, is 1

$$\therefore P(A|E_1)=1$$

Probability that the student answered correctly, given that he guessed, is $\frac{1}{4}$.

$$\therefore P(A|E_2)=\frac{1}{4}$$

The probability that the student knows the answer, given that the answered it correctly, is given by $P(E_1|A)$

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)}$$

$$=\frac{\frac{3}{4}.1}{\frac{3}{4}.1+\frac{1}{4}.\frac{1}{4}}$$

$$=\frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}}=\frac{\frac{3}{4}}{\frac{13}{16}}=\frac{12}{13}$$

5. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yield a false positive result for 0.5% of the healthy person tested (that is, if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

**Solution:**

Let $E_1$ and $E_2$ be the respective events that a person has a disease and a person has no disease.

Since $E_1$ and $E_2$ are events complimentary to each other,

$$\therefore P(E_1)+P(E_2)=1$$

$$\Rightarrow P(E_2)=1-P(E_1)=1-0.001=0.999$$

Let A be the event that the blood test result is positive.

$$P(E_1)=0.1\%=\frac{0.1}{100}=0.001$$

$P(A|E_1)$ = P (result is positive given the person has disease) = 99% = 0.99

$P(A|E_2)$ =P (result is positive given that the person has no disease) = 0.5% = 0.005

Probability that a person has a disease has a disease, given that his test result is positive, is given by $P(E_1|A)$

By using Baye's theorem, we get

$$P(E_1|A)=\frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)}$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$$

$$= \frac{0.00099}{0.00099 + 0.004995}$$

$$= \frac{0.00099}{0.005985}$$

$$= \frac{990}{5985} = \frac{110}{665} = \frac{22}{133}$$

6. There are three coins. One is two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

**Solution:**

Let $E_1, E_2$ and $E_3$ be the respective events of choosing a two headed coin, a biased coin, and an unbiased coin

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads.

$\therefore P(A/E_1) = P$(coin showing heads, given that it is two-headed coin) = 1

Probability of heads coming up, given that it is a biased coin = 75%

$\therefore P(A/E_2) = P$ (coin showing heads, given that it is a biased coin) = $\frac{75}{100} = \frac{3}{4}$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$

$\therefore P(A|E_3) = P$(coin showing heads, given that it is an unbiased coin) = $\frac{1}{2}$

The probability that the coin is two – headed, given that it shows heads, is given by $P(E_1|A)$

By using Baye's theorem, we get

$$P(E_1|A)=\frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A/E_2)+P(E_3).P(A/E_3)}$$

$$=\frac{\frac{1}{3}.1}{\frac{1}{3}.1+\frac{1}{3}.\frac{3}{4}+\frac{1}{3}.\frac{1}{2}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)}$$

$$=\frac{1}{\frac{9}{4}}=\frac{4}{9}$$

7. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

**Solution:**

Let $E_1, E_2$ and $E_3$ be the respective events that the driver is a scooter driver, a car driver, and a truck driver

Let A be the event that the person meets with an accident

There are 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers.

Total number of drivers = 2000 + 4000 + 6000 = 12000

$P(E_1)=P$ (driver is a scooter driver) = $\frac{2000}{12000}=\frac{1}{6}$

$P(E_2)=P$ (driver is a car driver) = $\frac{4000}{12000}=\frac{1}{3}$

$P(E_3)=P$ (driver is a truck driver) = $\frac{6000}{12000}=\frac{1}{2}$

$P(A|E_1)=P$(scooter driver met with an accident) $=0.01=\frac{1}{100}$

$P\left(A|E_2\right)=P$ (car driver met with an accident) $=0.03=\frac{3}{100}$

$P\left(A|E_3\right)=P$ (truck driver met with an accident) $=0.15=\frac{15}{100}$

The probability that the driver is a scooter driver, given that he meet with an accident, is given by $P\left(E_1\,|\,A\right)$

By using Baye's theorem, we get

$$P\left(E_1\,|\,A\right)=\frac{P\left(E_1\right).P\left(A|E_1\right)}{P\left(E_1\right).P\left(A|E_1\right)+P\left(E_2\right).P\left(A|E_2\right)+P\left(E_3\right).P\left(A/E_3\right)}$$

$$\frac{\frac{1}{6}.\frac{1}{100}}{\frac{1}{6}.\frac{1}{100}+\frac{1}{3}.\frac{3}{100}+\frac{1}{2}.\frac{15}{100}}$$

$$=\frac{\frac{1}{6}.\frac{1}{100}}{\frac{1}{100}\left(\frac{1}{6}+1+\frac{15}{2}\right)}$$

$$=\frac{\frac{1}{6}}{\frac{104}{12}}=\frac{1}{6}\times\frac{12}{104}=\frac{1}{52}$$

8. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Future, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen random from this and is found to be defective. What is the probability that was produced by machine B?

**Solution:**

Let $E_1$ and $E_2$ be the respective events of items produced by machines A and B.

Let X be the event that the produced items was found to be defective.

$\therefore$ Probability of items produced by machine A, $P(E_1) = 60\% = \frac{3}{5}$

Probability of items produced by machine B, $P(E_2) = 40\% = \frac{2}{5}$

Probability that machine A produced defective items, $P(X \mid E_1) = 2\% = \frac{2}{100}$

Probability that machine B produced defective items, $P(X \mid E_2) = 1\% = \frac{1}{100}$

The probability that the randomly selected items was from machine B, gives that it is

Defective, is given by $P(E_2 \mid \text{X})$

By using Baye's theorem, we get

$$P(E_2 \mid \text{X}) = \frac{P(E_2).P(X \mid E_2)}{P(E_1).P(X / E_1) + P(E_2).P(X / E_2)}$$

$$\frac{\frac{2}{5}.\frac{1}{100}}{\frac{3}{5}.\frac{2}{100} + \frac{2}{5}.\frac{1}{100}}$$

$$\frac{\frac{2}{500}}{\frac{6}{500} + \frac{2}{500}} = \frac{2}{8} = \frac{1}{4}$$

9. Two groups are competing for the position on board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

**Solution:**

Let $E_1$ and $E_2$ be the respective events that the first group and the second group win the competition. Let A be the event of introducing a new product.

$P(E_1) =$ Probability that the first group wins the competition = 0.6

$P(E_2) =$ Probability that the second group wins the competition = 0.4

$P(A|E_1) =$ Probability of introducing a new product if the first group wins = 0.7

$P(A/E_2) =$ Probability of introducing a new product if the second group wins = 0.3

The probability that the new product is introduced by the second group is given by

$P(E_2|A)$

By using Baye's theorem, we get

$$P(E_2|A) = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)}$$

$$= \frac{0.4\times 0.3}{0.6\times 0.7+0.4\times 0.3}$$

$$= \frac{0.12}{0.42+0.12}$$

$$= \frac{0.12}{0.54} = \frac{12}{54} = \frac{2}{9}$$

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

**Solution:**

Let $E_1$ be the event that the outcome on the die is 5 or 6 and $E_2$ be the event that the outcome on the dies is 1,2,3 or 4.

$P(E_1)=\frac{2}{6}=\frac{1}{3}$ and $P(E_2)=\frac{4}{6}=\frac{2}{3}$

Let A be the event of getting exactly one head.

$P(A|E_1)=$ Probability of getting exactly one head by tossing the coin three times if she gets 5 or 6 $=\frac{3}{8}$

$P(A/E_2)=$ Probability of getting exactly one head in a single throw of coin if she gets 1,2,3 or $4=\frac{1}{2}$

The probability that the girl threw 1,2,3 or 4 with die, if she obtained exactly on head, is given by $P(E_2|A)$

By using Baye's theorem, we get

$$P(E_2|A)=\frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)}$$

$$=\frac{\frac{2}{3}.\frac{1}{2}}{\frac{1}{3}.\frac{3}{8}+\frac{2}{3}.\frac{1}{2}}$$

$$=\frac{\frac{2}{3}.\frac{1}{2}}{\frac{1}{3}.\frac{3}{8}+\frac{2}{3}.\frac{1}{2}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}\left(\frac{3}{8}+1\right)}=\frac{1}{\frac{11}{8}}=\frac{8}{11}$$

11. A manufacture has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that was produced by A?

**Solution:**

Let $E_1$ and $E_2$ be the respective events that the time consumed by machine A, B and C for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X \mid E_1) = 1\% = \frac{1}{100}$$

$$P(X \mid E_2) = 5\% = \frac{5}{100}$$

$$P(X \mid E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by $P(E_1 \mid A)$

By using Baye's theorem, we get

$$P(E_1 \mid X) = \frac{P(E_1).P(X / E_1)}{P(E_1).P(X \mid E_1) + P(E_2).P(X \mid E_2) + P(E_3).P(X \mid E_3)}$$

$$\frac{\frac{1}{2}.\frac{1}{100}}{\frac{1}{2}.\frac{1}{100} + \frac{3}{10}.\frac{5}{100} + \frac{1}{5}.\frac{7}{100}}$$

$$= \frac{\frac{1}{100}.\frac{1}{2}}{\frac{1}{100}\left(\frac{1}{2} + \frac{3}{2} + \frac{7}{5}\right)}$$

$$=\frac{\frac{1}{2}}{\frac{17}{5}}=\frac{5}{34}$$

12. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

**Solution:**

Let $E_1$ and $E_2$ be the respective events of choosing a diamond cards and a card which is not diamond.

Let A denote the lost card.

Out of 52 cards, 13 cards are diamonds and 39 cards are not diamonds

$$\therefore P(E_1)=\frac{13}{52}=\frac{1}{4}$$

$$P(E_2)=\frac{39}{52}=\frac{3}{4}$$

When one diamond card is lost, there are 12 diamonds cards out of 51 cards.

Two cards can be drawn out of 12 diamonds cards in ${}^{12}C_2$ ways.

Similarly, 2 diamonds cards can be drawn out of 51 cards in ${}^{51}C_2$ ways.

The probability of getting two cards, when one diamond card is lost, is given by

$P(A|E_1)$

$$P(A|E_1)=\frac{{}^{12}C_2}{{}^{51}C_2}=\frac{12!}{2!\times 10!}\times\frac{2!\times 49!}{5!}=\frac{11\times 12}{50\times 51}=\frac{22}{425}$$

When the lost card is not a diamond, there are 13 diamonds cards out of 51 cards

Two cards can be drawn out of 13 diamonds cards in ${}^{13}C_2$ ways whereas 2 cards can be drawn out of 51 cards in ${}^{51}C_2$ ways

The probability of getting two cards, when one card is lost which is not diamond, is given by $P(A|E_2)$

$$P(A|E_2)=\frac{{}^{13}C_2}{{}^{51}C_2}=\frac{13!}{2!\times 11!}\times\frac{2!\times 49!}{5!}=\frac{12\times 13}{50\times 51}=\frac{26}{425}$$

The probability that the lost card is diamond is given by $P(E_1|A)$

By using Baye's theorem, we get

$$P(E_1|A)=\frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)}$$

$$=\frac{\frac{1}{4}.\frac{22}{425}}{\frac{1}{4}.\frac{22}{425}+\frac{3}{4}.\frac{26}{425}}$$

$$=\frac{\frac{11}{2}}{25}=\frac{11}{50}$$

13. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

A) $\frac{4}{5}$ B) $\frac{1}{2}$ C) $\frac{1}{5}$ D) $\frac{2}{5}$

**Solution:**

Let $E_1$ and $E_2$ be the events such that

$E_1$ : A speaks truth

$E_2$ : A speak false

Let X be the event that a head appears.

$$P(E_1)=\frac{4}{5}$$

$\therefore P(E_2)=1-P(E_1)=1-\frac{4}{5}=\frac{1}{15}$

If a coin is tossed, then it may result in either head (H) or tail (T).

The probability of getting a head is $\frac{1}{2}$ whether A speaks truth or not

$\therefore P(X \mid E_1)=P(X \mid E_2)=\frac{1}{2}$

The probability that there is actually a head is given by $P(E_1 \mid X)$

$$P(E_1 \mid X)=\frac{P(E_1).P(X/E_2)}{P(E_1).P(X/E_1)+P(E_2).P(X \mid E_2)}$$

$$=\frac{\frac{4}{5}.\frac{1}{2}}{\frac{4}{5}.\frac{1}{2}+\frac{1}{5}.\frac{1}{2}}$$

$$=\frac{\frac{1}{2}.\frac{4}{5}}{\frac{1}{2}\left(\frac{4}{5}+\frac{1}{5}\right)}$$

$$=\frac{\frac{4}{5}}{1}=\frac{4}{5}$$

14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

A) $P(A|B)=\frac{P(B)}{P(A)}$

B) $P(A|B)<P(A)$

C) $P(A|B)\geq P(A)$

D) None of these

**Solution:**

If $A \subset B$, then $A \cap B = A$

$\Rightarrow A \cap B = P(A)$

Also, $\Rightarrow P(A) < P(B)$

Consider $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)} \neq \dfrac{P(B)}{P(A)}$..........(1)

Consider $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)}$..........(2)

Since, $P(B) \leq 1$

$\Rightarrow \dfrac{1}{P(B)} \geq 1$

$\Rightarrow \dfrac{P(A)}{P(B)} \geq P(A)$

Form (2), we get

$\Rightarrow P(A|B) \geq P(A)$..........(3)

$\therefore P(A|B)$ is not less than P (A)

Thus c is correct

## Exercise 13.4

1. State which of the following are not the probability distribution of a random variable. Give reasons for your answer.

(i)

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | 0.4 | 0.4 | 0.2 |

(ii)

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| P(X) | 0.1 | 0.5 | 0.2 | - 0.1 | 0.3 |

(iii)

| Y | - 1 | 0 | 1 |
|---|---|---|---|
| P(Y) | 0.6 | 0.1 | 0.2 |

(iv)

| Z | 3 | 2 | 1 | 0 | -1 |
|---|---|---|---|---|---|
| P(Z) | 0.3 | 0.2 | 0.4 | 0.1 | 0.05 |

**Solution:**

Since the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities = 0.4 + 0.4 +0.2 =1

Thus, the given table is a probability distribution of random variable.

(ii) For X = 3, P (X) = - 0.1

Since probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities = $0.6 + 0.1 + 0.2 = 0.9 \neq 1$

Thus, the given table is not a probability distribution of random variables

(iv) Sum of the probabilities = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = $1.05 \neq 1$

Thus, the given table is not a probability distribution of random variable.

2. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X? Is X a random variable?

**Solution:**

Let B represents a black ball and R represents a red ball.

The two balls selected can be represented as BB, BR, RB, RR

X represents the number of black balls.

Thus, the possible values of X are 0, 1 and 2.

Yes, X is a random variable

3. Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

**Solution:**

A coin is tossed six times and X represents the difference between the number of heads and the number of tails.

$\therefore X(6H, 0T) = |6-0| = 6$

$X(5H, 1T) = |5-1| = 4$

$X(4H, 2T) = |4-2| = 2$

$X(3H, 3T) = |3-3| = 0$

$X(2H, 4T) = |2-4| = 2$

$X(1H, 5T) = |1-5| = 4$

$X(0H, 6T) = |0-6| = 6$

Therefore, the possible values of X are 6, 4, 2 and 0

4. Find the probability distribution of

(i) Number of heads in two tosses of a coin

(ii) Number of tails in the simultaneous tosses of three coins

(iii) Number of heads in four tosses of a coin

**Solution:**

i) The sample space is $\{HH, HT, TH, TT\}$

Let X represent the number of heads.

$\therefore X(HH)=2, X(HT)=1, X(TH)=1, X(TT)=0$

Thus, X can take the value of 0, 1 or 2

Since,

$$P(HH)=P(HT)=P(TH)=P(TT)=\frac{1}{4}$$

$$P(X=0)=P(TT)=\frac{1}{4}$$

$$P(X=1)=P(HT)+P(TH)=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

$$P(X=2)=P(HH)=\frac{1}{4}$$

Therefore, the required probability distribution is as follows

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

(ii) The sample space is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Let X represents the number of tails.

Since, X can take the value of 0, 1, 2 or 3

$$P(X=0)=P(HHH)=\frac{1}{8}$$

$$P(X=1)=P(HHT)+P(HTH)+P(THH)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

$$P(X=2)=P(HTT)+P(THT)+P(TTH)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

$$P(X=3)=P(TTT)=\frac{1}{8}$$

Therefore, the probability distribution is as follows..

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

(iii) The sample space is

$$S=\left\{\begin{matrix} HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT, \\ THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT \end{matrix}\right\}$$

Let X be the random variable, which represents the number of heads.

Since, X can take the value of 0, 1, 2, 3 or 4

$$P(X=0)=P(TTTT)=\frac{1}{16}$$

$$P(X=1)=P(TTTH)+P(TTHT)+P(THTT)+P(HTTT)=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$$

$$P(X=2)=P(HHTT)+P(THHT)+P(TTHH)+P(HTTH)+P(HTHT)+P(THTH)$$

$$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{6}{16}=\frac{3}{8}$$

$$P(X=4)=P(HHHH)=\frac{1}{16}$$

Therefore, the probability distribution is as follows.

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| P(X) | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{16}$ |

5. Find the probability distribution of the number of success in two tosses of die, where a success is defined as

(i) Number greater than 4

(ii) Six appears on at least one die

**Solution:**

Let X be the random variable, which represents the number of success

(i) Here, success refers to the number greater than 4

P(X=0) = P (number less than or equal to 4 on both the tosses) = $\frac{4}{6}\times\frac{4}{6}=\frac{4}{9}$

P (X= 1) = P (number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$$=\frac{4}{6}\times\frac{2}{6}+\frac{4}{6}\times\frac{2}{6}=\frac{4}{9}$$

P(X=2) = P (number greater than 4 on both the tosses)

$$=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$$

Therefore, the probability distribution is as follows.

| X | 1 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{4}{9}$ | $\frac{4}{9}$ | $\frac{1}{9}$ |

(ii) Here, success means six appears on at least one die

P (Y=0) = P (six does not appear on any of the dice) = $\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$

P(Y=1) = P (six appears on at least one of the dice) $=\frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}=\frac{5}{36}+\frac{5}{36}=\frac{10}{36}$

Therefore, the required probability is as follows.

| Y | 0 | 1 |
|---|---|---|
| P(Y) | $\frac{25}{36}$ | $\frac{10}{36}$ |

6. From a lot of 30 bulbs which includes 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

**Solution:**

Given, out of 30 bulbs, 6 are defective.

$\Rightarrow$ Number of non – defective bulbs = 30 -6 =24

4 bulbs are drawn from the lot with replacement

Let X be the random variable that denotes the number of defective bulbs in the selected Bulbs

$\therefore P(X=0) = P$(4 non – defective and 0 defective) $= {}^4C_0 . \frac{4}{5} . \frac{4}{5} . \frac{4}{5} . \frac{4}{5} = \frac{256}{625}$

P (X = 1) = P (3 non – defective and 1 defective) $= {}^4C_1 . \left(\frac{1}{5}\right) . \left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P (X = 2) = P (2 non – defective and 2 defective) $= {}^4C_2 . \left(\frac{1}{5}\right)^2 . \left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P (X = 3) = P (1 non – defective and 3 defective) $= {}^4C_3 . \left(\frac{1}{5}\right)^3 . \left(\frac{4}{5}\right) = \frac{16}{625}$

P (X = 4) = P (0 non – defective and 4 defective) $= {}^4C_4 . \left(\frac{1}{5}\right)^4 . \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Thus, the required probability distribution is as follows

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|

| P(X) | $\frac{256}{625}$ | $\frac{256}{625}$ | $\frac{96}{625}$ | $\frac{16}{625}$ | $\frac{1}{625}$ |
|---|---|---|---|---|---|

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

**Solution:**

Let the probability of getting a tail in the biased coin be x.

$\therefore P(T) = x$

$\Rightarrow P(H) = 3x$

For a biased coin, $P(T) + P(H) = 1$

$\Rightarrow x + 3x = 1$

$\Rightarrow 4x = 1$

$\Rightarrow x = \frac{1}{4}$

$\therefore P(T) = \frac{1}{4}$ *and* $P(H) = \frac{3}{4}$

When the coin is tossed twice, the sample space is $\{HH, TT, HT, TH\}$.

Let X be the random variable representing the number of tails.

$\therefore P(X = 0) = P$(no tail) $= P(H) \times P(H) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

P (X=1) = P (one tail) = P(HT) + P(TH)

$= \frac{3}{4} . \frac{1}{4} + \frac{1}{4} . \frac{3}{4}$

$= \frac{3}{16} + \frac{3}{16}$

$= \frac{3}{8}$

P (X=2) = P (two tails) = $P(TT) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Thus, the required probability distribution is as follows

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{9}{16}$ | $\frac{3}{8}$ | $\frac{1}{16}$ |

8. A random variable X has the following probability distribution

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(X) | 0 | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2+k$ |

Determine

(i) k

(ii) $P(X<3)$

(iii) $P(X>6)$

(iv) $P(0<X<3)$

**Solution:**

(i) Since, the sum of probabilities of a probability distribution of random variable is one

$\therefore 0+k+2k+3k+k^2+2k^2+(7k^2+k)=1$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow (10k-1)(k+1)=0$

$\Rightarrow k=-1, \frac{1}{10}$

(ii) P (X< 3) = P (X = 0) + P (X = 1) + P (X = 2)

= 0 + k + 2k

= 3k

$= 3 \times \frac{1}{10}$

$$= \frac{3}{10}$$

(iii) P (X > 6) = P (X = 7)

$$= 7k^2 + k$$

$$= 7 \times \left(\frac{1}{10}\right)^2 + \frac{1}{10}$$

$$= \frac{7}{100} + \frac{1}{10}$$

$$= \frac{17}{100}$$

(iv) $P(0 < x < 3) = P(x = 1) + P(x = 2)$

$$= k + 2k$$

$$= 3k$$

$$= 3 \times \frac{1}{10}$$

$$= \frac{3}{10}$$

9. The random variable X has probability P(X) of the following form, where k is some

number $P(X) = \begin{cases} k, & if \;\; x = 0 \\ 2k, & if \;\; x = 1 \\ 3k, & if \;\; x = 2 \\ 0 & otherwise \end{cases}$

(a) Determine the value of k

(b) Find $P(X < 2), P(X \geq 2), P(X \geq 2)$

**Solution:**

(a) Since, the sum of probabilities of a probability distribution of random variable is one.

$$\therefore k + 2k + 3k + 0 = 1$$

$$\Rightarrow 6k = 1$$

$$\Rightarrow k = \frac{1}{6}$$

$(b)\ P(X < 2) = P(X = 0) + P(X = 1)$

$$\therefore k + 2k$$

$$=3k=\frac{3}{6}=\frac{1}{2}$$

$$P(X\le 2)=P(X=0)+P(X=1)+P(X=2)$$

$$=k+2k+3k$$

$$=6k=\frac{6}{6}=1$$

$$P(X\ge 2)=P(X=2)+P(X>2)$$

$$=3k+0$$

$$=3k=\frac{3}{6}=\frac{1}{2}$$

10. Find the mean number of heads in three tosses of a fair coin

**Solution:**

Let X denote the success of getting heads.

Thus, the sample space is $S=\{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT\}$

Here, X can take the value of 0, 1, 2 or 3

$$\therefore P(X=0)=P(TTT)$$

$$=P(T).P(T).P(T)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$=\frac{1}{8}$$

$$\therefore P(X=1)=P(HHT)+P(HTH)+P(THH)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$$

$$\therefore P(X=2)=P(HHT)+P(HTH)+P(THH)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$$

$$\therefore P(X=2)=P(HHT)+P(HTH)+P(THH)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$$

$$\therefore P(X=3)=P(HHH)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$$

Thus, the required probability is as follows

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

Mean $E(X)=\sum X_i P(X_i)$

$$=0\times\frac{1}{8}+1\times\frac{3}{8}+2\times\frac{3}{8}+3\times\frac{1}{8}$$

$$=\frac{3}{8}+\frac{3}{4}+\frac{3}{8}$$

$$=\frac{3}{2}=1.5$$

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

**Solution:**

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Thus, X can be take the value of 0, 1 or 2

$\therefore P(X=0)=P$ (not getting six on any of the dice) $=\frac{25}{26}$

$P(X=1)=P$ (six on first die and no six on second die) + P (no six on first die and six on second die)

$$=2\times\left(\frac{1}{6}\times\frac{5}{6}\right)=\frac{10}{36}$$

$P(X=2)=P$ (six on both the dice) = $\frac{1}{36}$

Thus, the required probability distribution is as follows

| X | 0 | 1 | 2 |
|---|---|---|---|

| P(X) | $\frac{25}{36}$ | $\frac{10}{36}$ | $\frac{1}{36}$ |
|---|---|---|---|

Then, expectation of $X = E(X) = \sum X_i P(X_i)$

$$= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$$

$$= \frac{1}{3}$$

12. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of two numbers obtained. Find E(X)

**Solution:**

The two positive integers can be selected from the first six positive integers without replacement in $6 \times 5 = 30$ *ways*

X represents the larger of the two numbers obtained

Thus, X can take the value of 2, 3, 4, 5 or 6

For X = 2, the possible observations are (1, 2) and (2, 1)

$$\therefore P(x = 2) = \frac{2}{30} = \frac{1}{15}$$

For X = 3, the possible observations are $(1,3),(2,3),(3,1)$ *and* $(3,2)$

$$\therefore P(X = 3) = \frac{4}{30} = \frac{2}{15}$$

For X = 4, the possible observations are $(1,4),(2,4),(3,4),(4,3),(4,2)$ *and* $(4,1)$

$$\therefore P(X = 4) = \frac{6}{36} = \frac{1}{5}$$

For X = 5, the possible observations are

$(1,5),(2,5),(3,5),(4,5),(5,4),(5,3),(5,2)$ *and* $(5,1)$

For X = 6, the possible observations are

$(1,6),(2,6),(3,6),(4,6),(5,6),(6,4),(6,3),(6,2)$ *and* $(6,1)$

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Thus, the required probability distribution is as follows

| X | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| P(X) | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{4}{15}$ | $\frac{1}{3}$ |

Then, $E(x)=\sum X_i P(X_i)$

$$=2.\frac{1}{15}+3.\frac{2}{15}+4.\frac{1}{5}+5.\frac{4}{15}+6.\frac{1}{3}$$

$$=\frac{2}{15}+\frac{2}{5}+\frac{4}{5}+\frac{4}{3}+2$$

$$=\frac{70}{15}=\frac{14}{3}$$

13. Let X denotes the sum of the number obtained when two fair dice are rolled. Find the variance and standard deviation of X.

**Solution:**

Here, X can take values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12

$$P(X=2)=P(1,1)=\frac{1}{36}$$

$$P(X=3)=P(1,2)+P(2,1)=\frac{2}{36}=\frac{1}{18}$$

$$P(X=4)=P(1,3)+P(2,2)+P(3,1)=\frac{3}{36}=\frac{1}{12}$$

$$P(X=5)=P(1,4)+P(2,3)+P(3,2)+P(4,1)=\frac{4}{36}=\frac{1}{9}$$

$$P(X=6)=P(1,5)+P(2,4)+P(3,3)+P(4,2)+P(5,1)=\frac{5}{36}$$

$$P(X=7)=P(1,6)+P(2,5)+P(3,4)+P(4,3)+P(5,2)+P(6,1)=\frac{6}{36}=\frac{1}{6}$$

$$P(X=8)=P(2,6)+P(3,5)+P(4,4)+P(5,3)+P(6,2)=\frac{5}{36}$$

$$P(X=9)=P(3,6)+P(4,5)+P(5,4)+P(6,3)=\frac{4}{36}=\frac{1}{9}$$

$$P(X=10)=P(4,6)+P(5,5)+P(6,4)=\frac{3}{36}=\frac{1}{12}$$

$$P(X=11)=P(5,6)+P(6,5)=\frac{2}{36}=\frac{1}{18}$$

$$P(X=12)=P(6,6)=\frac{1}{36}$$

Thus, the required probability distribution is as follows.

| X | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P(X) | $\frac{1}{36}$ | $\frac{1}{18}$ | $\frac{1}{12}$ | $\frac{1}{9}$ | $\frac{5}{36}$ | $\frac{1}{6}$ | $\frac{5}{36}$ | $\frac{1}{9}$ | $\frac{1}{12}$ | $\frac{1}{18}$ | $\frac{1}{36}$ |

Then, $E(X)=\sum X_i.P(X_i)$

$$=2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}$$

$$=\frac{1}{18}+\frac{1}{6}+\frac{1}{3}+\frac{5}{9}+\frac{7}{6}+\frac{10}{9}+1+\frac{5}{6}+\frac{11}{18}+\frac{1}{3}$$

= 7

$$E(X^2)=\sum X_i^2.P(X_i)$$

$$= 4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{36}$$

$$=\frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{9}+9+\frac{25}{3}+\frac{121}{18}+4$$

$$=\frac{987}{18}=\frac{329}{6}=54.833$$

Then, Var $(X)=E(X)^2-[E(X)]^2$

$$=54.833-(7)^2$$

$$=54.833-49$$

$$=5.833$$

$\therefore$ Standard deviation = $\sqrt{Var(X)}$

$$=\sqrt{5.833}$$

$$=2.415$$

14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One students is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X

**Solution:**

There are 15 students in the class. Each student has the same chance to be chosen

Thus, the probability of each student to be selected is $\frac{1}{15}$

The given information can be shown in the frequency table as follows

| x | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|
| f | 2 | 1 | 2 | 3 | 1 | 2 | 3 | 1 |

$$P(X=14)=\frac{2}{15}, P(X=15)=\frac{1}{15}, P(X=16)=\frac{2}{15}, P(X=16)=\frac{3}{15},$$

$$P(X=18)=\frac{1}{15}, P(X=19)=\frac{2}{15}, P(X=20)=\frac{3}{15}, P(X=21)=\frac{1}{15}$$

Thus, the probability distribution of random variable X is as follows.

| x | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|
| f | $\frac{2}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{1}{15}$ |

Then, mean X= E(X)

$$\sum X_i P(X_i)$$

$$=14\times\frac{2}{15}+15\times\frac{1}{15}+16\times\frac{2}{15}+17\times\frac{3}{15}+18\times\frac{1}{15}+19\times\frac{2}{15}+20\times\frac{3}{15}+21\times\frac{1}{15}$$

$$=\frac{1}{15}(28+15+32+51+18+38+60+21)$$

$$=\frac{263}{15}=17.53$$

$$E(X^2)=\sum X_i^2 P(X_i)$$

$$=(14)^2.\frac{2}{15}+(15)^2.\frac{1}{15}+(16)^2.\frac{2}{15}+(17)^2.\frac{3}{15}+(18)^2.\frac{1}{15}+(19)^2.\frac{2}{15}+(20)^2.\frac{3}{15}+(21)^2\frac{1}{15}$$

$$=\frac{1}{15}(392+225+512+867+324+722+1200+441)$$

$$=\frac{4683}{15}=312.2$$

$$\therefore Variance(X)=E(X)^2-[E(X)]^2$$

$$=312.2-\left(\frac{263}{15}\right)^2$$

$$=312.2-307.4177$$

$= 4.7823 = 4.78$

Standard deviation = $\sqrt{Variance(X)}$

$= \sqrt{4.78}$

$= 2.816 \approx 2.19$

15. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and var (X)

**Solution:**

Given, $P(X = 0) = 30\% = \frac{30}{100} = 0.3$

$P(X = 1) = 70\% = \frac{70}{100} = 0.7$

Thus, the probability distribution is as follows.

| X | 0 | 1 |
|---|---|---|
| P(X) | 0.3 | 0.7 |

Then, $E(X) = \sum X_i P(X_i)$

$= 0 \times 0.3 + 1 \times 0.7$

$= 0.7$

$E(X^2) = \sum X_i^2 P(X_i)$

$= 0^2 \times 0.3 + (1)^2 \times 0.7 = 0.7$

Since, $Var(X) = E(X^2) - [E(X)]^2$

$= 0.7 - (0.7)^2$

$= 0.7 - 0.49 = 0.21$

16. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

A) 1 B) 2 C) 5 D) $\frac{8}{3}$

**Solution:**

Let X be the random variable representing a number on the die.

The total number of observations is six.

$$\therefore P(X=1)=\frac{3}{6}=\frac{1}{2}$$

$$P(X=2)=\frac{2}{6}=\frac{1}{3}$$

$$P(X=5)=\frac{1}{6}$$

Thus, the probability distribution is as follows.

| X | 1 | 2 | 5 |
|---|---|---|---|
| P(X) | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{6}$ |

Mean $=E(X)=\sum X_i P(X_i)$

$$=\frac{1}{2}\times 1+\frac{1}{3}\times 2+\frac{1}{6}.5$$

$$=\frac{1}{2}+\frac{2}{3}+\frac{5}{6}$$

$$=\frac{3+4+5}{6}=\frac{12}{6}=2$$

17. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

A) $\frac{37}{221}$ B) $\frac{5}{13}$ C) $\frac{1}{13}$ D) $\frac{2}{13}$

**Solution:**

Let X denote the number of aces obtained.

Thus, X can be take any of the value of 0, 1 or 2.

Since, in a deck of 52 cards, 4 cards are aces. Thus, there are 48 non – ace cards.

$\therefore P(X=0)=P$ (0 ace and 2 non – ace cards) = $\frac{{}^{4}C_{0}\times {}^{48}C_{2}}{{}^{52}C_{2}}=\frac{1128}{1326}$

P(X= 1) = P (1 ace and 1 non – ace cards) = $\frac{{}^{4}C_{1}\times {}^{48}C_{1}}{{}^{52}C_{2}}=\frac{192}{1326}$

P (X = 2) = P (2 ace and 0 non – ace cards) = $\frac{{}^{4}C_{2}\times {}^{48}C_{0}}{{}^{52}C_{2}}=\frac{6}{1326}$

Thus, the probability distribution is as follows

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{1128}{1326}$ | $\frac{192}{1326}$ | $\frac{6}{1326}$ |

Then, $E(X)=\sum X_i P(X_i)$

$$=0\times\frac{1128}{1326}+1\times\frac{192}{1326}+2\times\frac{6}{1326}$$

$$=\frac{204}{1326}=\frac{2}{13}$$

## Exercise 13.5

1. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

(i) 5 successes? (ii) at least 5 success? (iii) at most 5 successes?

**Solution:**

Let X denote the number of success of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is, $p=\frac{3}{6}=\frac{1}{2}$

$\therefore q=1-p=\frac{1}{2}$

X has a binominal distribution

Thus, $P(X=x)={}^{n}C_{n-1}q^{n-x}p^{x}$, where n = 0, 1, 2 ……. N

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6-x}\left(\frac{1}{2}\right)^{x}$

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6}$

(i) P (5 success) = P (X = 5)

$={}^{6}C_{5}\left(\frac{1}{2}\right)^{6}$

$=6.\frac{1}{64}=\frac{3}{32}$

(ii) P (at least 5 success) = $P(X\geq 5)$

$$= P(X=5)+P(X=6)$$

$$= {}^6C_5\left(\frac{1}{2}\right)^6 + {}^6C_6\left(\frac{1}{2}\right)^6$$

$$= 6.\frac{1}{64}+1.\frac{1}{64}$$

$$= \frac{7}{64}$$

(iii) P (at most 5 success) = $P(X \le 5)$

$$=1-P(X>5)$$

$$=1-P(X=6)$$

$$=1-{}^6C_6\left(\frac{1}{2}\right)^6$$

$$=1-\frac{1}{64}=\frac{63}{64}$$

2. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two success

**Solution:**

Let X denote the number of times of getting doublets in an experiment of throwing two 3 dice simultaneously four times.

Probability of getting doublets in a single throw of the pair of dice is

$$p=\frac{6}{36}=\frac{1}{6}$$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

Clearly, X has the binomial distribution with $n=4, p=\frac{1}{6} \text{ and } q=\frac{5}{6}$

$\therefore P(X=r)={}^{n}C_{x}q^{n-x}p^{x}$, where x = 0, 1, 2, 3 ……n

$$={}^{4}C_{x}\left(\frac{5}{6}\right)^{4-x}\left(\frac{1}{6}\right)^{x}$$

$$={}^{6}C_{x}\frac{5^{4-x}}{6^{4}}$$

$\therefore$ P (2 successes) = $P(X=2)$

$$={}^{4}C_{2}\frac{5^{4-2}}{6^{4}}$$

$$=6\frac{25}{1296}=\frac{25}{216}$$

3. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

**Solution:**

Let X denote the number of defective items in a sample of 10 items drawn successively

$$\Rightarrow p=\frac{5}{100}=\frac{1}{20}$$

$$\therefore q=1-\frac{1}{20}=\frac{19}{20}$$

X has a binomial distribution with n = 10 and $p=\frac{1}{20}$

$P(X=x)={}^{n}C_{x}q^{n-x}p^{x}$, where x = 0, 1, 2 …… n

$$={}^{10}C_{x}\left(\frac{19}{20}\right)^{10-x}\left(\frac{1}{20}\right)^{x}$$

P (not more than 1 defective item) = $P(X\le 1)$

$$=P(X=0)+P(X=1)$$

$$= {}^{10}C_0\left(\frac{19}{20}\right)^{10}\left(\frac{1}{20}\right)^{0} + {}^{10}C_1\left(\frac{19}{20}\right)^{9}\left(\frac{1}{20}\right)^{1}$$

$$= \left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^{9}\left(\frac{1}{20}\right)$$

$$= \left(\frac{19}{20}\right)^{9}\left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \left(\frac{29}{20}\right)\left(\frac{19}{20}\right)^{9}$$

4. Five cards are drawn successively with replacement from a well – shuffled deck of 52 cards. What is the probability that

(i) All the five cards are spades?

(ii) Only 3 cards are spades?

(iii) None is a spade?

**Solution:**

Let X represent the number of spade cards among the five cards drawn.

In a well shuffled deck of 52 cards, there are 13 spades cards.

$$\Rightarrow p = \frac{13}{52} = \frac{1}{4}$$

$$\therefore q = 1 - \frac{1}{4} = \frac{3}{4}$$

X has a binomial distribution with n = 5 and $p = \frac{1}{4}$

$P(X = x) = {}^{n}C_x q^{n-x} p^{x}$, where x = 0, 1, 2 ……. N

$$= {}^{5}C_x\left(\frac{3}{4}\right)^{5-x}\left(\frac{1}{4}\right)^{x}$$

(i) P (all five cards are spades) = P (X = 5)

$$= {}^5C_5\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^5$$

$$=1.\frac{1}{1024}=\frac{1}{1024}$$

(ii) P (only 3 card are spades) = P (X = 3)

$$= {}^5C_3\left(\frac{3}{4}\right)^2.\left(\frac{1}{4}\right)^3$$

$$=10.\frac{9}{6}.\frac{1}{64}$$

$$=\frac{45}{512}$$

(iii) P (none is a spade) = P (X = 0)

$$= {}^5C_0\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^0$$

$$=1.\frac{243}{1024}=\frac{243}{1024}$$

5. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. What is the probability that out of 5 such bulbs

(i) None

(ii) Not more than one

(iii) More than one

(iv) At least one

Will fuse after 150 days of use

**Solution:**

Let X represents the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials

Given, p = 0.05

$\therefore q=1-p=1-0.05=0.95$

X has a binomial distribution with n = 5 and p = 0.05

$\therefore P(X=x) = {}^{n}C_{x}q^{n-x}p^{x}$, where x = 0, 1, 2 ….. n

$= {}^{5}C_{x}(0.95)^{5-x}.(0.05)^{x}$

(i) P (none) = P (X = 0)

$= {}^{5}C_{0}(0.95)^{5}.(0.05)^{0}$

$=1\times(0.95)^{5}$

$=(0.95)^{5}=0.7737$

(ii) P (not more than one) = $P(X\le 1)$

$=P(X=0)+P(X=1)$

$= {}^{5}C_{0}\times(0.95)^{5}.(0.05)^{0} + {}^{5}C_{1}(0.95)^{4}.(0.05)^{1}$

$=1\times(0.95)^{5}+5\times(0.95)^{4}.(0.05)$

$=(0.95)^{5}+(0.25)(0.95)^{4}$

$=(0.95)^{4}+[0.95+0.25]$

$=(0.95)^{4}\times 1.2$

= 0.977

(iii) P (more than 1) = P (X>1)

$=1-P(X\le 1)$

= 1 – P (not more than 1)

= $1-(0.95)^{4}\times 1.2$

= 0.02

(iv) P (at least one) = $P(X \geq 1)$

$$=1-P(X<1)$$

$$=1-P(X=0)$$

$$=1-{}^{5}C_{0}(0.95)^{5}\times(0.05)^{0}$$

$$=1-1\times(0.95)^{5}$$

$$=1-(0.95)^{5}$$

$$=0.2263$$

6. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

**Solution:**

Let X denote the number of balls marked with the digit 0 among the 4 balls drawn.

X has a binominal distribution with n = 4 and $p=\frac{1}{10}$

$$\therefore q=1-p=1-\frac{1}{10}=\frac{9}{10}$$

$$P(X=x)={}^{n}C_{x}q^{n-x}p^{x}, x=1,2.....n$$

$$={}^{n}C_{x}\left(\frac{9}{10}\right)^{4-x}\left(\frac{1}{10}\right)^{x}$$

P (none marked with 0) = P(X= 0)

$$={}^{4}C_{x}\left(\frac{9}{10}\right)^{4}\left(\frac{1}{10}\right)^{0}$$

$$=1.\left(\frac{9}{10}\right)^{4}$$

$$=\left(\frac{9}{10}\right)^{4}$$

7. In an examination, 20 questions of true – false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tail, he answers 'false'. Find the probability that he answers at least 12 questions correctly

**Solution:**

Let X represent the number of correctly answered questions out of 20 questions.

Since "head" on a coin represents the true answer and "tail" represents the false answer, the correctly answered questions are Bernoulli trials.

$$\therefore p=\frac{1}{2}$$

$$\therefore q=1-p=1-\frac{1}{2}=\frac{1}{2}$$

X has a binominal distribution with n = 20 and $p=\frac{1}{2}$

$$\therefore P(X=x)={}^{n}C_{x}q^{n-x}p^{x}, where\ x=0,1,2\ldots\ldots n$$

$$={}^{20}C_{x}\left(\frac{9}{10}\right)^{20-x}\left(\frac{1}{2}\right)^{x}$$

$$={}^{20}C_{x}\left(\frac{1}{2}\right)^{20}$$

P (at least 12 questions answered correctly) = $P(X\geq 12)$

$$=P(X=12)+P(X=13)+\ldots.+P(X=20)$$

$$={}^{20}C_{12}\left(\frac{1}{2}\right)^{20}+{}^{20}C_{13}\left(\frac{1}{2}\right)^{20}+\ldots.+{}^{20}C_{20}\left(\frac{1}{2}\right)^{20}$$

$$=\left(\frac{1}{2}\right)^{20}.\left[{}^{20}C_{12}+{}^{20}C_{13}+\ldots\ldots+{}^{20}C_{20}\right]$$

8. Suppose X has a binominal distribution $B\left(6,\frac{1}{2}\right)$. Show that X = 3 is the most likely outcome. (Hint: P(X=3) is the maximum among all $P(x_i), x_i = 0,1,2,3,4,5,6$)

**Solution:**

X is the random variable whose binomial distribution is $B\left(6,\frac{1}{2}\right)$

Thus, n = 6 and $p=\frac{1}{2}$

$\therefore q = 1-p = 1-\frac{1}{2}=\frac{1}{2}$

Then, $P(X=x) = {}^nC_x q^{n-x} p^x$

$= {}^6C_x \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^x$

$= {}^6C_x \left(\frac{1}{2}\right)^6$

Here, $P(X=x)$ will be maximum, if ${}^6C_x$ will be maximum

Then, ${}^6C_0 = {}^6C_6 = \frac{6!}{0!.6!} = 1$

${}^6C_1 = {}^6C_5 = \frac{6!}{1!.5!} = 6$

${}^6C_2 = {}^6C_4 = \frac{6!}{2!.4!} = 15$

${}^6C_3 = \frac{6!}{3!.3!} = 20$

The value of ${}^6C_3$ is maximum

Thus, for X = 3, P (X = x) is maximum

Therefore, X = 3 is the most likely outcome

9. On a multiple choice examination with three possible answer for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

**Solution:**

Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is $p=\frac{1}{3}$

$\therefore q=1-p=1-\frac{1}{3}=\frac{2}{3}$

Clearly, X has a binomial distribution with n = 5 and $p=\frac{1}{3}$

$\therefore P(X=x)={}^{n}C_{x}q^{n-x}p^{x}$

$={}^{5}C_{x}\left(\frac{2}{3}\right)^{5-x}.\left(\frac{1}{3}\right)^{x}$

P (guessing more than 4 correct answers) = $P(X\geq 4)$

$=P(X=4)+P(X=5)$

$={}^{5}C_{4}\left(\frac{2}{3}\right).\left(\frac{1}{3}\right)^{4}+{}^{5}C_{5}\left(\frac{1}{3}\right)^{5}$

$=5.\frac{2}{3}.\frac{1}{81}+1.\frac{1}{243}$

$=\frac{10}{243}+\frac{1}{243}=\frac{11}{243}$

10. A person buys a lottery ticket in 50 lotteries, in each of which his change of winning a prize is $\frac{1}{100}$. What is the probability that he will in a prize (a) at least once (b) exactly once (c) at least twice?

**Solution:**

Let X represents the number of winning prizes in 50 lotteries

Clearly, X has a binominal distribution with n = 50 and $p=\frac{1}{100}$

$\therefore q=1-p=1-\frac{1}{100}=\frac{99}{100}$

$\therefore P(X=x)={}^{n}C_{x}q^{n-x}p^{x}={}^{50}C_{x}\left(\frac{99}{100}\right)^{50-x}\left(\frac{1}{100}\right)^{x}$

(a) P (winning at least once) = $P(X\geq 1)$

$=1-P(X<1)$

$=1-P(X=0)$

$=1-{}^{50}C_0\left(\frac{99}{100}\right)^{50}$

$=1-1.\left(\frac{99}{100}\right)^{50}$

$=1-\left(\frac{99}{100}\right)^{50}$

(b) P (Winning exactly once) = P (X =1)

$={}^{50}C_1\left(\frac{99}{100}\right)^{49}\left(\frac{1}{100}\right)^{1}$

$=50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$

$=\frac{1}{2}\left(\frac{99}{100}\right)^{49}$

(c) P (at least twice) = $P(X\geq 2)$

$=1-P(X<2)$

$=1-P(X\leq 1)$

$=1-\left[P(X=0)+P(X=1)\right]$

$=\left[1-P(X=0)\right]-P(X=1)$

$=1-\left(\frac{99}{100}\right)^{50}-\frac{1}{2}\left(\frac{99}{100}\right)^{49}$

$=1-\left(\frac{99}{100}\right)^{49}\left[\frac{99}{100}+\frac{1}{2}\right]$

$=1-\left(\frac{99}{100}\right)^{49}\left[\frac{149}{100}\right]$

$=1-\left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

11. Find the probability of getting 5 exactly twice in 7 thrown of a die

**Solution:**

Let X represents the number of times of getting 5 in 7 throws of the die

Probability of getting 5 in a single throw of the die, $p=\frac{1}{6}$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

Clearly, X has the probability distribution with n = 7 and $p=\frac{1}{6}$

$$\therefore P(X=x)={}^{n}C_{x}q^{n-x}p^{x}={}^{7}C_{x}\left(\frac{5}{6}\right)^{7-x}\left(\frac{1}{6}\right)^{x}$$

P (getting 5 exactly twice) = P (X =2)

$$={}^{7}C_{2}\left(\frac{5}{6}\right)^{5}\left(\frac{1}{6}\right)^{2}$$

$$=21\left(\frac{5}{6}\right)^{5}.\frac{1}{36}$$

$$=\left(\frac{7}{12}\right)\left(\frac{5}{6}\right)^{5}$$

12. Find the probability of throwing at most 2 sixes in 6 throws of a single die

**Solution:**

Let X represent the number of times of getting sixes in 6 throws of the die.

Probability of getting six in a single throw of die, $p=\frac{1}{6}$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

Clearly, X has a binomial distribution with n = 6

$$\therefore P\left(X=x\right)={}^{n}C_{x}q^{n-x}p^{x}={}^{6}C_{x}\left(\frac{5}{6}\right)^{6-x}\left(\frac{1}{6}\right)^{x}$$

P (at most 2 sixes) $= P(X \le 2)$

$$= P(X=0)+P(X=1)+P(X=2)$$

$$= {}^6C_0\left(\frac{5}{6}\right)^6 + {}^6C_1\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + {}^6C_2\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)^2$$

$$=1.\left(\frac{5}{6}\right)^6 + 6.\frac{1}{6}.\left(\frac{5}{6}\right)^5 + 15.\frac{1}{36}\left(\frac{5}{6}\right)^4$$

$$=\left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + \frac{5}{12}.\left(\frac{5}{6}\right)^4$$

$$=\left(\frac{5}{6}\right)^4\left[\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right) + \left(\frac{5}{12}\right)\right]$$

$$=\left(\frac{5}{6}\right)^4\left[\frac{25}{36} + \frac{5}{6} + \frac{5}{12}\right]$$

$$=\left(\frac{5}{6}\right)^4\left[\frac{25+30+15}{36}\right]$$

$$=\frac{70}{36}.\left(\frac{5}{6}\right)^4 = \frac{35}{18}.\left(\frac{5}{6}\right)^4$$

$$=0.9377$$

13. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

**Solution:**

Let X denote the number of times of selecting defective articles in a random sample space of 12 articles

Clearly, X has a binominal distribution with n = 12 and p = 10% = $\frac{10}{100} = \frac{1}{10}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\therefore P(X=x)={}^{n}C_{x}q^{n-x}p^{x}={}^{12}C_{x}\left(\frac{9}{10}\right)^{12-x}\left(\frac{1}{10}\right)^{x}$$

P (Selecting 9 defective articles) $={}^{12}C_{9}\left(\frac{9}{10}\right)^{3}\left(\frac{1}{10}\right)^{9}$

$$=220.\frac{9^3}{10^3}.\frac{1}{10^9}$$

$$=\frac{22\times 9^3}{10^{11}}$$

14. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs none is defective is

(A) $10^{-1}$ (B) $\left(\frac{1}{2}\right)^{5}$ (C) $\left(\frac{9}{10}\right)^{5}$ (D) $\frac{9}{10}$

**Solution:**

Let X denote the number of defective bulbs out of a sample of 5 bulbs

Probability of getting a defective bulb, $p=\frac{10}{100}=\frac{1}{10}$

$$\therefore q=1-p=1-\frac{1}{10}=\frac{9}{10}$$

Clearly, X has binomial distribution with n = 5 and $p=\frac{1}{10}$

$$\therefore P(X=x)={}^{n}C_{x}q^{n-x}p^{x}={}^{5}C_{x}\left(\frac{9}{10}\right)^{5-x}\left(\frac{1}{10}\right)^{x}$$

P (None of the bulbs is defective) = P (X = 0)

$$={}^{5}C_{0}\left(\frac{9}{10}\right)^{5}$$

$$=1.\left(\frac{9}{10}\right)^{5}=\left(\frac{9}{10}\right)^{5}$$

15. The probability that a student is not a swimming is $\frac{1}{5}$. The probability that out of five students, four are swimmers is

(A) ${}^{5}C_{4}\left(\frac{4}{5}\right)^{4}\frac{1}{5}$ (B) $\left(\frac{4}{5}\right)^{4}\frac{1}{5}$ (C) ${}^{5}C_{1}\frac{1}{5}\left(\frac{4}{5}\right)^{4}$ (D) None of these

**Solution:**

Let X denote the number of students, out of 5 students, who are swimmers.

Probability of students who are not swimmers, $q=\frac{1}{5}$

$\therefore p=1-q=1-\frac{1}{5}=\frac{4}{5}$

Clearly, X has a binomial distribution with n = 5 and $p=\frac{4}{5}$

$\therefore P(X=x)={}^{n}C_{x}q^{n-x}p^{x}={}^{5}C_{x}\left(\frac{1}{5}\right)^{5-x}\left(\frac{4}{5}\right)^{x}$

P (four students are swimmers) $=P(X=4)={}^{5}C_{4}\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{4}$