Examples

Example 1: Two coins (a one-rupee coin and a two-rupee coin) are tossed once. Find a sample space.

Answer: When we tossed the coin

Then the either coin can turn up Head (H) or Tail(T),

When two coins are tossed simultaneously

Then the required sample space is $S = \{HH, HT, TH, TT\}$

Example 2: Find the sample space associated with the experiment of rolling a pair of dice (one is blue and the other red) once. Also, find the number of elements of this sample space.

Answer: when a pair of dice is thrown

Then possible outcome is

$1, 2, 3, 4, 5, 6$ only

Hence required sample space is

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

So, the number of elements in sample space $= 6 \times 6 = 36$

Example 3: In each of the following experiments specify appropriate sample space

(i) A boy has a 1 rupee coin, a 2 rupee coin and a 5-rupee **coin in his pocket. He takes out two coins out of his pocket, one after the other.**

(ii): A person is noting down the number of accidents along a busy highway during a year

Answer: (i) Let Q denote a 1rupee coin,

Let H denotes a 2 rupee coin

Let R denotes a 5-rupee coin.

He takes out two coins, one after the other

All possible outcomes are

$S=\{QH, QR, HQ, HR, RQ, RH\}$

(ii) These can be occurred either 0 accidents or $1,2,3,\ldots\infty$ accidents

We know that accidents can't be in fraction or negative. It can only be whole numbers

So, sample space is

$S=\{0,1,2,\ldots\}$

Example 4: A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 blue and 4 white balls; if it shows tail we throw a die. Describe the sample space of this experiment.

Answer: when a coin is tossed it can be either are Head (H) or Tail(T),

Also, 3 blue $\&$ 4 white balls

Can be denoted by B_1, B_2, B_3 and W_1, W_2, W_3, W_4

Possible outcomefor dice is

$1,2,3,4,5,6$

So, the required sample space for this

$S=\{HB_1, HB_2, HB_3, HW_1, HW_2, HW_3, HW_4, T1, T2, T3, T4, T5, T6\}$

Example 5: Consider the experiment in which a coin is tossed repeatedly until a head comes up. Describe the sample space.

Answer: when we tossed a coin then head may come up on the first toss, or the

2^{nd} toss, or the 3^{rd}toss and so on till head is obtained.

Hence, the required sample space is

$S=\{H, TH, TTH, TTTH, TTTTH, \ldots\}$

Example 6: Consider the experiment of rolling a die. Let A be the event 'getting a prime number', B be the event 'getting an odd number'. Write the sets representing the events (i) A or B

(ii) A and B

(iii) A but not B

(iv) 'not A'.

Answer: Here $S=\{1,2,3,4,5,6\}$, $A=\{2,3,5\}$ and $B=\{1,3,5\}$

Obviously

(i) A or B = $A \cup B = \{1,2,3,5\}$
(ii) A and B= $A \cap B = \{3,5\}$
(iii) A but not B = A - B = {2}
(iv) not A =A' = {1,4,6}

Example 7: Two dice are thrown and the sum of the numbers which come up on the dice is noted. Let us consider the following events associated with this experiment

A: 'the sum is even'.

B: 'the sum is a multiple of 3.

C: "the sum is less than 4 ".

D: 'the sum is greater than 11.

Which pairs of these events are mutually exclusive?

Answer: when dice is thrown twice $6 \times 6 = 36$ element

So, the sample space $S = \{(x,y): x,y = 1,2,3,4,5,6\}$.

Then

$A = \{(1,1),(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),(6,2),(6,4),(6,6)\}$
$B = \{(1,2),(2,1),(1,5),(5,1),(3,3),(2,4),(4,2),(3,6),(6,3),(4,5),(5,4) \ \& \ (6,6)\}$
$C = \{(1,1),(2,1),(1,2)\}$ and $D = \{(6,6)\}$

We find that

$A \cap B = \{(1,5),(2,4),(3,3),(4,2),(5,1),(6,6)\} \neq \varphi$

Therefore, A and B are not mutually exclusive events.

Similarly, $A \cap C \neq \phi, A \cap D \neq \phi, B \cap C \neq \phi$ and $B \cap D \neq \phi$

Thus, those events, $(A,C),(A,D),(B,C),(B,D)$ are not mutually exclusive events.

Also $C \cap D = \phi$ and so C and D are mutually exclusive events.

Example 8: A coin is tossed three times, consider the following events.

A: 'No head appears',

B: 'Exactly one head appears' and

C: 'Atleast two heads appear'.

Do they form a set of mutually exclusive and exhaustive events?

Answer: The sample space of the coin when it is tossed three time

$S=\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

and $A=\{TTT\}, B=\{HTT, THT, TTH\}, C=\{HHT, HTH, THH, HHH\}$

Now

$A\cup B\cup C=\{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\} = S$

So A, B and C are exhaustive events.

Also, $A\cap B=\varphi$, $A\cap C=\varphi$ and $B\cap C=\varphi$

Therefore, the events are pair-wise disjoint, i.e. we can say they are mutually exclusive.

Hence, A, B and C form a set of mutually exclusive and exhaustive events.

Example 9: Let a sample space be $S=\{\omega_1,\omega_2,\ldots,\omega_6\}$. Which of the following assignments of probabilities to each outcome are valid?

Outcome	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
(a)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
(b)	1	0	0	0	0	0
(c)	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{3}$
(d)	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{2}$
(e)	0.1	0.2	0.3	0.4	0.5	0.6

Answer: Condition (i): Each of the number $p(\omega_i)$ is positive and less than one.

Condition (ii): Sum of probabilities

$$=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=1$$

Therefore, the assignment is valid

(b) Condition (i): Each of the number $p(\omega)$ is either 0 or 1.

Condition (ii) Sum of the probabilities $=1+0+0+0+0+0=1$

Therefore, the assignment is valid

(c) Condition (i) Two of the probabilities $p(\omega_5)$ and $p(\omega_6)$ are negative, the assignment is not valid

(d) Since $p(\omega_6)=\frac{3}{2}>1$, the assignment given in question is not valid

(e) Since sum of probabilities $=0.1+0.2+0.3+0.4+0.5+0.6=2.1$, the assignment is not valid.

Example 10: One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be

(i) a diamond

(ii) not an ace

(iii) a black card (i.e., a club or, a spade)

(iv) not a diamond

(v) not a black card.

Answer: When a card is chosen from a well shuffled deck of 52 cards, so the number of possible outcomes is 52.

(i) Let us assume A be the event 'the card drawn is a diamond'

Clearly the number of outcomes in set A is 13.

Therefore, $P(A)=\frac{13}{52}=\frac{1}{4}$

i.e., Probability of a diamond card $=\frac{1}{4}$

(ii) Let us assume that the event 'Card drawn is an ace' is B

Therefore 'Card drawn is not an ace' should be B'.

We know that $P\left(B'\right)=1-P(B)=1-\frac{4}{52}=1-\frac{1}{13}=\frac{12}{13}$

(iii) Let us assume C denote the event 'card drawn is black card'

Therefore, number of outcomes i.e.

$P(C)=\frac{26}{52}=\frac{1}{2}$

Thus, the required probability of a black card $=\frac{1}{2}$

(iv) We assumed in (i) above that A is the event 'card drawn is a diamond', so the event 'card drawn is not a diamond' can be denoted as A' or not A

Now $P(\text{not } A)=1-P(A)=1-\frac{1}{4}=\frac{3}{4}$

(v) The event 'card drawn is not a black card' assume can be denoted as C ' or 'not C '. We know that

$$P(\text{not } C) = 1 - P(C) = 1 - \frac{1}{2} = \frac{1}{2}$$

So, the required probability of not a black card $= \frac{1}{2}$

Example 11: A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Calculate the probability that it will be

(i) red,

(ii) yellow,

(iii) blue,

(iv) not blue,

(v) either red or blue.

Answer: Total number of disc is 9

So, the total number of possible outcomes is 9.

We can define the events A, B, C be as

A: If 'the disc drawn is red'

B: If 'the disc drawn is yellow'

C: 'the disc drawn is blue'.

(i) Sample space of red discs $= 4$, i.e., $n(A) = 4$

$$P(A) = \frac{4}{9}$$

(ii) Sample space of yellow discs $= 2$, i.e., $n(B) = 2$

Therefore, $P(B) = \frac{2}{9}$

(iii) Sample space of blue discs $= 3$, i.e., $n(C) = 3$

Therefore, $P(C) = \frac{3}{9} = \frac{1}{3}$

(iv) Clearly the event 'not blue' is 'not C '. We know that $P(\text{not } C) = 1 - P(C)$

Therefore $P(\text{not } C) = 1 - \frac{1}{3} = \frac{2}{3}$

(v) Probability of That event 'either red or blue' may be define by the set 'A or C' Since, A and Cboth are mutually exclusive events, we have

$$\mathrm{P(A\ or\ C)} = \mathrm{P}(\mathrm{A} \cup \mathrm{C}) = \mathrm{P(A)} + \mathrm{P(C)} = \frac{4}{9} + \frac{1}{3} = \frac{7}{9}$$

Example 12: Two students Anil and Ashima appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that

(a) Both Anil and Ashima will not qualify the examination.

(b) Atleast one of them will not qualify the examination and

(c) Only one of them will qualify the examination.

Answer: Let us assume E and F denote the events that Anil and Ashima will qualify the examination, respectively.

We have to Given that

$$\mathrm{P(E)} = 0.05, \mathrm{P(F)} = 0.10 \text{ and } \mathrm{P}(\mathrm{E} \cap \mathrm{F}) = 0.02$$

Then

(a) The event 'both Anil and Ashima will not qualify the examination' can be written as $\mathrm{E}' \cap \mathrm{F}'$.

Since, E' is 'not E ', i.e., if Anil will not qualify the examination

and F' = 'not F ', i.e., if Ashima will not qualify the examination.

Also $\mathrm{E}' \cap \mathrm{F}' = (\mathrm{E} \cup \mathrm{F})'$ (by Demorgan's Law)

Now $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

or $\mathrm{P}(\mathrm{E} \cup \mathrm{F}) = 0.05 + 0.10 - 0.02 = 0.13$

Therefore $\mathrm{P}\left(\mathrm{E}' \cap \mathrm{F}'\right) = \mathrm{P}(\mathrm{E} \cup \mathrm{F})' = 1 - \mathrm{P}(\mathrm{E} \cup \mathrm{F}) = 1 - 0.13 = 0.87$

(b) P (atleast one of these will not qualify)

= 1 - P(both of them will qualify)

= 1 - 0.02 = 0.98

(c) Probability of those event only one of them will qualify the examination is same as the event either (Anil will qualify, and Ashima will not qualify) or (Anil will not qualify and Ashima will qualify)

i.e., $\mathrm{E} \cap \mathrm{F}'$ or $\mathrm{E}' \cap \mathrm{F}$, where $\mathrm{E} \cap \mathrm{F}'$ and $\mathrm{E}' \cap \mathrm{F}$ are mutually exclusive.

Therefore, P (only one will be qualify) $= \mathrm{P}\left(\mathrm{E} \cap F' \text{ or } E' \cap \mathrm{F}\right)$

$$= P\left(E \cap F'\right) + P\left(E' \cap F\right) = P(E) - P(E \cap F) + P(F) - P(E \cap F)$$

$=0.05-0.02+0.10-0.02=0.11$

Example 13: A committee of two persons is selected from two men and two women. What is the probability that the committee will have

(a) no man?

(b) one man?

(c) two men?

Answer: The total number of persons in the committee $=2+2=4$.

From the four persons, two can be selected in ${}^4C_2$ ways.

(a) No men in the committee of can be say that there will be two women in the committee. Out of two women, two can be selected in ${}^2C_2=1$ way.

Therefore $P(\text{no man})=\frac{{}^2C_2}{{}^4C_2}=\frac{1\times2\times1}{4\times3}=\frac{1}{6}$

(b) One man in the committee implies that there is one woman.

Let us assume One man out of 2 can be selected in ${}^2C_1$ ways and one woman out of 2 can be selected in ${}^2C_1$ ways.

Both of them can be selected in ${}^2C_1\times{}^2C_1$ ways.

Therefore $P(\text{One man})=\frac{{}^2C_1\times{}^2C_1}{{}^4C_2}=\frac{2\times2}{2\times3}=\frac{2}{3}$

(c) Two men selected as ${}^2C_2$ way.

$P(\text{Two men})=\frac{{}^2C_2}{{}^4C_2}=\frac{1}{{}^4C_2}=\frac{1}{6}$

Example 14: On her vacations Veena visits four cities A, B, C and D in a random order. What is the probability that she visits

(i) A before B?

(ii) A before B and B before C?

(iii) A first and B last?

(iv) A either first or second?

(v) A just before B?

Answer: The number of arrangements (orders) in which Veena can visit four cities A, B, C, or D is 4! i.e.,24 .Therefore, $n(S) = 24$

Since the number of elements in the sample space of the experiment is 24 all of these outcomes are considered to be equally likely. A sample space for the experiment is

$$\begin{aligned} S = \{&ABCD, ABDC, ACBD, ACDB, ADBC, ADCB \\ &BACD, BADC, BDAC, BDCA, BCAD, BCDA \\ &CABD, CADB, CBDA, CBAD, CDAB, CDBA \\ &DABC, DACB, DBCA, DBAC, DCAB, DCBA\} \end{aligned}$$

(i) Let us assume E be the event 'she visits A before B' be denoted by E

Therefore,

$$\begin{aligned} E = \{&ABCD, CABD, DABC, ABDC, CADB, DACB \\ &ACBD, ACDB, ADBC, CDAB, DCAB, ADCB\} \end{aligned}$$

Thus $P(E) = \frac{n(E)}{n(S)} = \frac{12}{24} = \frac{1}{2}$

(ii) Let us assume F the event 'Veena visits A before B and B before C'

Here $F = \{ABCD, DABC, ABDC, ADBC\}$

Therefore, $P(F) = \frac{n(F)}{n(S)} = \frac{4}{24} = \frac{1}{6}$

(iii) A first and B last

Letus assume G be the event "she visits A first and B last"

$G = \{ACDB, ADCB\}$

So, $n(G) = 2$

$$P(G) = \frac{n(G)}{n(S)}$$

$$= \frac{2}{24}$$

$$= \frac{1}{12}$$

(iv) A either first or second

Let us assume H be the event “she visits A either first or second”

$$H = \begin{Bmatrix} ABCD, ABDC, ADBC, ACDB, ADBC, ADCB, \\ BACD, BADC, CABD, CADB, DABC, DACB, \end{Bmatrix}$$

So, n(H) = 12

$$P(H) = \frac{n(H)}{n(S)}$$

$$= \frac{12}{24} = \frac{1}{2}$$

(v) A just before B

Let us assume I be the event “she visit A just before B”

I = {ABCD,ABDC,CABD,CDAB,DABC,DCAB,}

So, n(I) = 6

$$P(I) = \frac{n(I)}{n(S)}$$

$$= \frac{6}{24} = \frac{1}{4}$$

Example 15: Find the probability that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains (i) all Kings

(ii) 3 Kings

(iii) atleast 3 Kings.

Answer: Total number of hands in a shuffled pack $= {}^{52}C_7$

(i) Number of hands 4 Kings $= {}^{4}C_4 \times {}^{48}C_3$ (3 cards will be chosen from the remaining 48 cards)

Hence P (a hand of 4 Kings) $= \dfrac{{}^{4}C_4 \times {}^{48}C_3}{{}^{52}C_7} = \dfrac{1}{7735}$

(ii) Number of hands of possible ways with 3 Kings and 4 non-King cards $= {}^{4}C_3 \times {}^{48}C_4$

So, $P(3\text{Kings}) = \dfrac{{}^{4}C_3 \times {}^{48}C_4}{{}^{52}C_7} = \dfrac{9}{1547}$

(iii) P(atleast 3 King) = P(3 Kings or 4 Kings)

$= P(3 \text{ Kings }) + P(4 \text{ Kings })$

$$= \frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$$

Example 16: If A, B, C are three events associated with a random experiment, prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$$
$$- P(A \cap C) - P(B \cap C) + P(A \cap E = B \cap C)$$

Answer: consider $E = B \cap C$ so that

$$P(A \cup B \cup C) = P(A \cup E) = P(A) + P(E) - P(A \cap E)............(i)$$

Now,

$$P(E) = P(B \cup C) = P(B) + P(C) - P(B \cap C)............(2)$$

Also, $A \cap E = A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (By distribution property of intersection are union)

So, $P(A \cap E) = P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$

$$= P(A \cap B) + P(A \cap C) - P[A \cap B \cap C]...............(3)$$

Use equation (2) and (3) in (1), we get

$$P[A \cup B \cup C] = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

Example 17: In a relay race there are five teams A, B, C, D and E.

(a) What is the probability that A, B, C finish first, second and third, respectively.

(b) What is the probability that A, B, C are first three to finish (in any order) (Assume that all finishing orders are equally likely)

Answer: let us assume the sample space consisting of all finishing orders in the first three places, we will have ${}^5P_3$, *i.e.*, $\frac{5!}{(5-3)!} = 5 \times 4 \times 3 = 60$ sample points, each with a probability of $\frac{1}{60}$

(a) Probability of A, B and C finish first, second and third, respectively.

Probability of finding is only one finishing order for this, i.e., ABC.

Thus P (A, B and C finish first, second and third respectively) $= \frac{1}{60}$

(b) Finish first three of A, B and C the number of way $3!$ arrangements for A, B and C. Therefore, the sample space to this event will be $3!$ in number.

So P(A, B and C are first three to finish) = $\frac{3!}{60} = \frac{6}{60} = \frac{1}{10}$

Exercise – 16.1

1: Describe the sample space for the indicated experiment: A coin is tossed three times.

Answer: Since either coin can tum up Head (H) or Tail (T).

The possible outcomes may be So, when 1 coin is tossed once the sample space $= 2$

Coin is tossed 3 times the sample space $2^3 = 8.$

Thus, the sample space is

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

2: Describe the sample space for the indicated experiment: die is thrown two times.

Answer: Let us assume that $1,2,3,4,5$ and 6 are the possible outcomes when the die is thrown. Then, the total number of sample space $= (6 \times 6).= 36$

Thus, the sample space is

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),[2,4),(2,5),(2,6),(3,1),(3,2),(3,3)(3,4),(3,5)$
$,(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),$
$(6,5),(6,6)\}$

3: Describe the sample space for the indicated experiment: A coin is tossed four times.

Answer: Since either coin can turn up Head (H) or Tail (T). are the possible outcomes.

So, when 1 coin is tossed then the sample space $= 2$

Then,

When Coin is tossed 3 times the possible cases $= 2^4 = 16$

Thus, the sample space is

$$S = \left\{\begin{matrix} HHHH, THHH, HTHH, HHTH, HHHT, TTT, HTTT, THTT, TTHT, TTH, TTHH, \\ HHTT, THTH, HTHT, THHT, HTTH \end{matrix}\right\}$$

4: Describe the sample for the indicated experiment: A coin is tossed and a die is thrown.

Answer: Since either coin can turn up Head (H) or Tail (T). are the possible outcomes.

Let us assume that $1,2,3,4,5$ and 6 are the possible numbers comes when the die is thrown.

Then, total number of spaces $= \{2 \times 6) = 12$

Thus, the sample space is,

$S = \{(H, 1),(H, 2),(H, 3),(H, 4),(H, 5),(H, 6),(T, 1),(T, 2),(T, 3),(T, 4),(T, 5),(T, 6)\}$

5: Describe the sample space for the indicated experiment: A coin is tossed and then a die is rolled only in case a head is shown on the coin.

Answer: Since either coin can turn up Head (H) or Tail (T). are the possible outcomes.

Let us assume that $1,2,3,4,5$ and 6 are the possible numbers comes when the die is thrown.

When head in encountered,

Then, number of spaces $=(1 \times 6)=6$

Sample Space $\mathrm{S}_{\mathrm{H}}=\{\mathrm{H}1, \mathrm{H}2, \mathrm{H}3, \mathrm{H}4, \mathrm{H}5, \mathrm{H}6\}$

Now, tail is encountered, Sample space $\mathrm{S}_{\mathrm{T}}=(\mathrm{T})$

Therefore, the total sample space $\mathrm{S}=[\mathrm{H}1,\mathrm{H}2,\mathrm{H}3,\mathrm{H}4,\mathrm{HS},\mathrm{H}6, \mathrm{T}]$

6: 2 boys and 2 girls are in Room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Answer: From the question it is given that,

2 boys and 2 girls are in Room X,

1 boy and 3 girls in Room Y.

Let us assume $\mathrm{b}1, \mathrm{b}2$ and $\mathrm{g}1, \mathrm{g}2$ be 2 boys and 2 girls are in Room X,

And also assume $\mathrm{b}3$ and $\mathrm{g}3, \mathrm{g}4, \mathrm{g}5$ be 1 boy and 3 girls in Room Y.

Now we will take two cases

Case 1: Room X is selected

Sample Space $\mathrm{S}_{\mathrm{X}}=\{(\mathrm{X},\mathrm{b}1),(\mathrm{X},\mathrm{b}2),(\mathrm{X},\mathrm{g}1),(\mathrm{X},\mathrm{g}2)\}$

Case 2: Room Y is selected

Sample Space $\mathrm{S}_{\mathrm{y}}=((\mathrm{Y}, \mathrm{b}3),(\mathrm{Y}, \mathrm{g}3),(\mathrm{Y}, \mathrm{g}4),(\mathrm{Y}_{,\mathrm{g}} 5))$

The overall sample spaces

$\mathrm{S}=\{(\mathrm{X},\mathrm{b}1),(\mathrm{X},\mathrm{b}2),(\mathrm{X},\mathrm{g}1),(\mathrm{X},\mathrm{g}2),(\mathrm{Y},\mathrm{b}3),(\mathrm{Y},\mathrm{g}3),(\mathrm{Y},\mathrm{g}4),(\mathrm{Y},\mathrm{g}5)\}$

7: One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Answer: Let us assume that $1,2,3,4,5$ and 6 are the possible numbers comes when the die is thrown.

And also assume die of red colour be $'R'$, die of white colour be $'W'$, die of blue colour be $'B'$

Then, total number of spaces $= \{6 \times 3) = 18$

Thus, the sample space is,

$S = \{(R,1),(R,2),(R,3),(R,4),(R,5),(R,6),(W,1),(W,2),(W,3),(W,4),(W,5),(W,6),$
$(B,1),(B,2),(B,3),(B,4),(B,5),(B,6)\}$

8: An experiment consists of recording boy-girl composition of families with 2 children.

(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?

(ii) What is the sample space if we are interested in the number of girls in the family?

Answer: Let us assume boy be $'B'$ and girl be $'G'$

(i) The sample space for it is a boy or girl in the order of their births. $S = \{GG,BB,GB,BG\}$

(ii) The sample space for the number of girls in the family when there are two child in the family then,

Sample space $5 = \{2,1,0\}$

9: A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Answer: From the question it is given that, a box contains 1 red and 3 identical white balls. Let us assume $'R'$ be the event of red ball is drawn and "W" be the event of white ball is drawn.

In the question we have given that white balls are identical, so the event of drawing any one of the three white ball is same.

Then, total number of sample space $=\left(2^2-1\right)=3$

So, Sample space $S=\{WW,WR,RW\}$

10: An experiment consists of tossing a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

Answer: Let us assume that $1,2,3,4,5$ and 6 are the possible outcomes when the die is thrown.

Since either coin can turn up Head (H) or Tail (T). are the possible outcomes.

Let us take,

Case 1: Head is encountered

Sample space $S_1 = \{HT, HH\}$

Case 2: Tail is encountered

Sample Space $S_2 = ([T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$ Then,

The Overall Sample spaces

$S = \{(HT),(HH),(T1),(T2),(T3),(T4),(TS),(T6)\}$

11: Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment?

Answer: From the question,

' D ' denotes the event of bulb is defective and ' N ' denotes event of non-defective bulbs. Then,

Total number of Sample space $= 2 \times 2 \times 2 = 8.$

Thus, Sample space $S = \{$ DDD, DDN, DND, NDD, DNN, NDN, NND, NNN\ $\}$

12: A coin is tossed. If the outcome is the head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?

Answer:

Since either coin can turn up Head (H) or Tail (T). are the possible outcomes.

Let us assume that 1,2,3,4,5 and 6 are the possible outcomes when the die is thrown. The problem can be solved by dividing it into 3 cases

Case 1: When coin show Head and then the corresponding number on the die shows Odd number

Total number of sample space $= (1 \times 3) = 3$

Sample space $S_{HO} = \{(H,1),(H,3),(H,5)\}$

Case 2: Now, when the coin show Head and then the corresponding number on the die shows Even number

Total number of sample space $= (1 \times 3 \times 6) = 18$

$S_{HE} = \{(H,2,1),(H,2,2),(H,2,3),(H,2,4),(H,2,5),(H,2,6),(H,4,1),(H,4,2),(H,4,3),(H,2,4),(H,4,5),$
$(H,4,6),(H,6,1),(H,6,2),(H,6,3),(H,6,4),(H,6,5),(H,6,6)\}$

Case 3: The outcome is Tail

Total number of sample space = 1

Sample space $S_T = \{(T)\}$

The overall sample spaces

$S = \{(H,1),(H,3),(H,5),(H,2,1),(H,2,2),(H,2,3),(H,2,4),(H,2,5),(H,2,6),(H,4,1),(H,4,2),(H,4,3),$
$(H,2,4),(H,4,5),(H, 4,6),(H, 6,1),(H, 6,2),[H, 6,3),(H, 6,4),(H, 6,5),(H, 6,6),(T)\}$

13: The number $1,2,3,4$ are written separately on four slips of paper. The slips are put in a box and mixed thoroughly, A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

Answer: From the question it is given that, $1,2,3,4$ are the numbers written on the four slips. When two slips are drawn without replacement the first event has 4 possible outcomes and the second event has 3 possible outcomes because 1 slip is already picked.

Therefore, the total number of possible outcomes $= (4 \times 3) = 12.$

Thus, sample space,

$S=((1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1),(4,2),(4,3))$

14: An experiment consists of rolling a die and then tossing a coin if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Answer: Since either coin can turn up Head (H) or Tail (T). are the possible outcomes.

Let us assume that $1,2,3,4,5$ and 6 are the possible outcomes when the die is thrown. The following sample space can be written in ways

(i) The number on the die is even.

The sample space $S_E = \{(2,H),(4,H),(6,H),(2,T),(4,T),(6,T)\}$

(ii) Sample for those when the die is odd and the coin is tossed twice.

The sample spaces

$S_O = \{(1,H,H),(3,H,H),(5,H,H),(1,H,T),(3,H,T),(5,H,T),(1,T,H),(3,T,H),(5,T,H),$
$(1, T, T),(3, T, T),(5, T, T)\}$

Hence, the total sample space $= S_E + S_O$

$S=\{(2,H),(4,H),(6,H),(2,T),(4,T),(6,T),(1,H,H),(3,H,H),(5,H,H),(1,H,T),(3,H,T),(5,H,T),$
$(1,T,H),(3,T,H),(5,T,H),(1,T,T),(3,T,T),(5,T,T)\}$

15: A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.

Answer: Since either coin can turn up Head (H) or Tail (T). are the possible outcomes.

Let us assume $R_{1,}R_2$ denote the event the red balls are drawn and $B_{1,}B_{2,}B_3$ denote the event the black balls are drawn

Let us assume that 1,2,3,4,5 and 6 are the possible outcomes when the die is thrown.

(i) Coin shows Tail.

So, the sample space $S_T = \{(TR_1),(TR_2),(TB_1),(TB_2),(TB_3)\}$

(ii) Coin shows head.

So, the sample space $S_H = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$

Hence, the total sample space for the given condition $= S_T + S_H$

$$S = \{(T,R_1),(T,R_2),(T,B_1),(T,B_2),(T,B_3),(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$$

16: A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

Answer: Let us assume that 1,2,3,4,5 and 6 are the possible outcomes when the die is thrown.

According to the condition given in the question, a die is thrown again and again until a six comes up.

when six may come up for the first time or six may come up on second time this process will repeat until the six comes.

The sample space when 6 comes on very first throw $S_1 = \{6\}$

The sample space when 6 comes on second throw $S_2 = ((1,6),(2,6),(3,6),(4,6),(5,6))$ This event can go for infinite times.

So, the sample space is infinitely defined

$$S = \{(6),(1,6),(2,6),(3,6),(4,6),(5,6),(1,1,6),(1,2,6)\ldots\ldots\}$$

Exercise 16.2

1: A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Answer: Let us assume that 1,2,3,4,5, and 6 are the possible outcomes when the die is thrown.

So, $S = 1,2,3,4,5,6$

As per the conditions given the question]

E be the event "die shows 4"

$E = (4)$

F be the event "die shows even number".

$F = (2,4,6)$

$E \cap F = (4) \cap (2,4,6)$

$= 4$

$4 \neq \varphi$

[because there is a common element in E and F]

Therefore E and F are not mutually exclusive event.

2: A die is thrown. Describe the following events:

(i) A: a **number less than** 7

(ii) B: a **number greater than** 7

(iii) C: a **multiple of** 3

(iv) D: a **number less than** 4

(v) E: a **n even number greater than** 4

(vi) F: a **number not less than** 3

Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $D - E$, $A - C$, $E \cap F'$, F'

Answer: Let us assume that $1,2,3,4,5,$ and 6 are the possible outcomes when the die is thrown.

So, $S = 1,2,3,4,5,6$

As per the conditions given in the question,

(i) A: a number less than 7

All the numbers in the die are less than 7

$A = (1,2,3,4,5,6)$

(ii) B: a number greater than 7

There is no number greater than 7 on the die.

Then,

$B = (\phi)$

(iii) C: a multiple of 3

In this have only two numbers which are multiple of 3

Then,

$C = (3,6)$

(iv) D: a number less than 4

$D = (1,2,3)$

(v) E: a n even number greater than 4

$E = (6)$

(vi) F: a number not less than 3

$F = (3,4,5,6)$

Also, we have to find, $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $D - E$, $A - C$, $E \cap F'$, F'

So,

$$
\begin{aligned}
&A \cap B = (1,2,3,4,5,6) \cap (\varphi) = (\varphi) \\
&B \cup C = (\varphi) \cup (3,6) = (3,6) \\
&E \cap F = (6) \cap (3,4,5,6) = (6) \\
&D \cap E = (1,2,3) \cap (6) = (\varphi) \\
&D - E = (1,2,3) - (6) = (1,2,3) \\
&A - C = (1,2,3,4,5,6) - (3,6) = (1,2,4,5) \\
&F' = S - F = (1,2,3,4,5,6) - (3,4,5,6) = (1,2) \\
&E \cap F' = (6) \cap (1,2) = (\varphi)
\end{aligned}
$$

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events: A: the sum is greater than 8, B: 2 occurs on either die C: the sum is at least 7 and a multiple of 3. Which pairs of these events are mutually exclusive?

Answer: Let us assume that $1,2,3,4,5,$ and 6 are the possible outcomes when the die is thrown.

When a pair of die is thrown, so sample space will be formed as,

$$
S = \begin{Bmatrix}
(1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)
\end{Bmatrix}
$$

A: the sum is greater than 8

$$\therefore A=\left\{\begin{matrix}(3,6),(4,5),(5,4),(6,3),(4,6),\\(5,5),(6,4),(5,6),(6,5),(6,6)\end{matrix}\right\}$$

Possible sum greater than 8 are $9,10,11,12$

B: 2 occurs on either die

$$B=\left\{\begin{matrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(1,2),(3,2),(4,2),(5,2),(6,2)\end{matrix}\right\}$$

In these conditions possibilities are there that the number 2 will come on either first die or second die or both the die simultaneously

C: The sum is at least 7 and multiple of 3

$C=\{(3,6),(4,5),(5,4),(6,3),(6,6)\}$

So, the sum of this will be only 9 or 12

Let we will find pairs of those events are mutually exclusive or not.

(i) $A\cap B=\varphi$

Since there is no common element in A and B

Therefore A and B are mutually exclusive

(ii) $B\cap C=\varphi$

Since there is no common element between

Therefore B and C are mutually exclusive.

(iii)

$A\cap C\{(3,6),(4,5),(5,4),(6,3),(6,6)\}$
$\Rightarrow\{(3,6),(4,5),(5,4),(6,3),(6,6)\}\neq\varphi$

Since $A and$ C has common elements.

Therefore $A and$ C are mutually exclusive.

4: Three coins are tossed once. Let A denote the event 'three heads show", B denote the event "two heads and one tail show", C denote the event" three tails show and D denote the event 'a head shows on the first coin". Which events are

(i) Mutually exclusive?

(ii) Simple?

(iii) Compound?

Answer: Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

When three coins are tossed once then possible sample space contains,

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTH\}$

Now,

A: 'three heads'

$A = (HHH)$

B: "two heads and one tail"

$B = (HHT, THH, HTH)$

C: 'three tails'

$C = (TTT)$

D: be the sample space has a head shows on the 1^{st}coin

$D = (HHH, HHT, HTH, HTT)$

(i) Mutually exclusive

$A \cap B = (HHH) \cap (HHT, THH, HTH) = \varphi$

Therefore, A and C are mutually exclusive.

$A \cap C = (HHH) \cap (TTT) = \varphi$

There, A and C are mutually exclusive.

$A \cap D = (HHH) \cap (HHH, HHT, HTH, HTT) = (HHH)$
$A \cap D \neq \varphi$

So, they are not mutually exclusive

$B \cap C = (HHT, HTH, THH) \cap (TTT) = \varphi$

Since there is no common element in B & C, so they are mutually exclusive.

$B \cap D = (HHT, THH, HTH) \cap (HHH, HHT, HTH, HTT) = (HHT, HTH)$
$B \cap D \neq \varphi$

Since there are common elements in B & D,

So, they are not mutually exclusive.

$C \cap D = (TTT) \cap (HHH, HHT, HTH, HTT) = \varphi$

Since there is no common element in C & D, So they are not mutually exclusive.

(ii) Simple event

If an event has occurred one sample point of a sample space, it is called a simple (or elementary) event.

$A = (HHH)$

$C = (TTT)$

Both $A \& C$ have only one element,

So, they are simple events.

(iii) Compound events

If an event has greater than one sample point, it is called a Compound event

$B = (HHT, HTH, THH)$

$D = (HHH, HHT, HTH, HTT)$

Both $B \& D$ have more than one element,

So, they are compound events.

5. Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii) Two events, which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Answer: Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

When 3 coins are tossed once then the possible sample space,

$S = (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)$

(i) Two events which are mutually exclusive.

Let us assume A be the event of getting only head

$A = (HHH)$

And also let us suppose B be the event of getting only Tail

$B=(TTT)$

So, $A\cap B=\varphi$

Since there is no common element in $A \,\&\, B$ so these two are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive

Now,

Let us assume P be the event of getting exactly two tails

$P=(HTT, TTH, THT)$

Let us assume Q be the event of getting at least two heads

$Q=(HHT, HTH, THH, HHH)$

Let us assume R be the event of getting only one tail

Since there is no common element in P and Q,

Therefore, they are mutually exclusive

$C=(TTT)$

$P\cap Q=(HTT,TTH,THT)\cap(HHT,HTH,THH,HHH)=\varphi$

$Q\cap R=(HHT,HTH,THH,HHH)\cap(TTT)=\varphi$

Since there is no common element in Q and R

Hence, they are mutually exclusive.

Since there is no common element in P and R,

$P\cap R=(HTT,TTH,THT)\cap(TTT)=\varphi$ So, they are mutually exclusive.

Now, P and Q,, Q and R, P and R, are mutually exclusive

$\therefore$ P,Q, and R are mutually exclusive.

And also,

$P\cup Q\cup R=(HTT, TTH, THT, HHT, HTH, THH, HHH, TTT)=S$

Hence P,Q and R are exhaustive events.

(iii) Two events, which are not mutually exclusive

Let us assume 'A' be the event of getting at least two heads,

$A=(HHH,HHT,THH,HTH)$

Let us assume ' B ' be the event of getting only head

$B=(HHH)$

Now $A\cap B=(HHH,HHT,THH,HTH)\cap(HHH)$

$=(HHH)$

$A\cap B\neq\varphi$

Since there is a common element in A and B,

So, they are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive

Let us assume ' P ' be the event of getting only Head

$P=(HHH)$

Let us assume ' Q ' be the event of getting only tail

$Q=(TTT)$

$P\cap Q=(HHH)\cap(TTT)$

$=\varphi$

Since there is no common element in P and Q, These are mutually exclusive events.

But,

$P\cup Q=(HHH)\cup(TTT)$

$=\{HHH,TTT\}$

$P\cup Q\neq S$

Since $P\cup Q\neq S$ these are not exhaustive events.

(v) Now, we will check the Three events which are mutually exclusive but not exhaustive

Let us assume ' X ' be the event of getting only head

$X=(HHH)$

Let us assume ' Y ' be the event of getting only tail

$Y=(TTT)$

Let us assume ' Z ' be the event of getting exactly two heads

$Z=(HHT,THH,HTH)$

Now,

$$X \cap Y = (HHH) \cap (TTT) = \varphi$$
$$X \cap Z = (HHH) \cap (HHT, THH, HTH)$$
$$= \varphi$$
$$Y \cap Z = (TTT) \cap (HHT, THH, HTH)$$
$$= \varphi$$

Therefore, they are mutually exclusive

Also

$$X \cup Y \cup Z = (HHHTTT, HHT, THH, HTH)$$
$$X \cup Y \cup Z \neq S$$

So, X, Y and Z are not exhaustive.

Hence it is proved that X, Y and Z are mutually exclusive but not exhaustive,

6: Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5.

Describe the events

(i) A'

(ii) not B

(iii) A or B

(iv) A and B

(v) A **but not** C

(vi) B **or** C

(vii) B **and** C

(viii) $A \cap B' \cap C'$

Answer: Let us assume that $1,2,3,4,5$ and 6 are the possible outcomes when the die is thrown. In the question is given that pair of die is thrown, so sample space will be,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

Let us assume that $1,2,3,4,5$ and 6 are the possible outcomes when the die is thrown.

when we throw a pair of die, then sample space will be,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5)\times(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

A be the sample space getting an even number on the 1stdie.

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5)\times(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

B be the sample space getting an odd number on the 1st die.

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

C be the sample space getting the sum of the numbers is on the dice ≤ 5

$$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$$

Then,

(i) $A' = \begin{Bmatrix} ((1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix} = B$

(ii) $B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = A$

(iii) $A \cup B(A \text{ or } B) = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = S$

(iv) $A \text{ and } B(A \cap B) = \varphi$

(v) $A \text{ but not } C = A - C = \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5)\times(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$

(vi) $B \text{ or } C = B \cup C = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$

(vii) $B \text{ and } C = B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii)

$C' = \begin{Bmatrix} (1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2) \\ (4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$

$\therefore \; A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$

$= \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$

7: Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5.

State true or false: (give reason for your answer)

(i) A and B **are mutually exclusive**

(ii) A and B **are mutually exclusive and exhaustive**

(iii) $A = B'$

(iv) A and C **are mutually exclusive**

(v) A and B **are mutually exclusive.**

(vi) A', B', C **are mutually exclusive and exhaustive.**

Answer: Let us assume that $1,2,3,4,5 \text{ and } 6$ are the possible outcomes when the die is thrown.

Here we have given that when a pair of die is thrown, so sample space will be,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5)\times(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

A be the sample space of getting an even number is on the 1^{st} die.

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5)\times(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

B be the sample space of getting an odd number is on the 1^{st} die.

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

C be the sample space of getting the sum of the numbers when the dice ≤ 5

$C=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$

Then,

(i) $\mathrm{A\ and\ B}$ are mutually exclusive

So, $(A\cap B)=\varphi$

So, $\mathrm{A\ and\ B}$ are mutually exclusive

Hence, the given statement is true.

(ii) $\mathrm{A\ and\ B}$ are mutually exclusive and exhaustive

$$A\cup B=\left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5)\cdot(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}=S$$

$\Rightarrow A\cup B\ =S$

From statement (i) we have $\mathrm{A\ and\ B}$ are mutually exclusive.

therefore $\mathrm{A\ and\ B}$ are mutually exclusive and exhaustive.

Hence, the statement is true.

(iii) $A=B'$

$$B'=\left\{\begin{matrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\end{matrix}\right\}=A$$

Hence, the statement is true.

(iv) $\mathrm{A\ and\ C}$ are mutually exclusive

We have,

$A\cap C=\{(2,1),(2,2),(2,3),(4,1)\}$

$A\cap C\neq\varphi$

$\mathrm{A\ and\ C}$ are not mutually exclusive

Hence, the given statement is false

(v) $A \text{ and } B'$ are mutually exclusive.

We have,

$$A \cap B' = A \cap A = A$$
$$A \cap B \neq \varphi$$

So, $A \text{ and } B'$ are mutually exclusive.

Hence, the given statement is false

(vi) A', B', C are mutually exclusive and exhaustive.

$$A = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \end{Bmatrix}$$

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

$$\text{And } C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$$
$$A' \cap B' = \varphi$$

Hence there is no common element $A' \text{ and } B'$

$$B' \cap C = \{(2,1),(2,2),(2,3),(4,1)\}$$
$$B' \cap C' \neq \varphi$$

They are not mutually exclusive.

Now, since $B' \text{ and } C$ are not mutually exclusive,

Therefore A', B', C are mutually exclusive and exhaustive.

So, the given statement is false.

Exercise 16.3

1: Which of the following cannot be valid assignment of probabilities for outcomes of sample Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	**0.1**	**0.01**	**0.05**	**0.03**	**0.01**	**0.2**	**0.6**
(b)	**1/7**	**1/7**	**1/7**	**1/7**	**1/7**	**1/7**	**1/7**
(c)	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**

(d)	**-0.1**	**0.2**	**0.3**	**0.4**	**-0.2**	**0.1**	**0.3**
(e)	**1/14**	**2/14**	**3/14**	**4/14**	**5/14**	**6/14**	**15/14**

Answer: (a) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities $0.01+0.05+0.03+0.01+0.2+0.6=1$

Therefore, the given assignment is valid.

b) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero.

Condition (ii): Sum of probabilities = (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7)

= 7/7

= 1

Therefore, the given assignment is valid.

c) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities $=0.1+0.2+0.3+0.4+0.5+0.6+0.7$

$=2.8>1$

Therefore, the 2nd condition is not satisfied

Which states that $p(w_i)\le 1$

So, the given assignment is not valid.

d) The conditions of axiomatic approach don't hold true in the given assignment, that is

1) Each of the number $p(\omega_i)$ is less than zero but also negative

To be true each of the number $p(\omega_i)$ should be less than zero and positive

So, the assignment is not valid

e) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero.

Condition (ii): Sum of probabilities

$=(1/14)+(2/14)+(3/14)+(4/14)+(5/14)+(6/14)+(7/14)$

$=(28/14)\ge 1$

The second condition does not hold, So the given assignment is not valid

2: A coin is tossed twice, what is the probability that at least one tail occurs?

Answer: Since either coin can turn up Head(H) or Tail(T), are the possible outcomes.

When coin is tossed twice,

then sample space is $S = (TT,HH,TH,HT)$

Therefore, Number of possible outcomes $n(S) = 4$

Let A be the event of forming at least one tail

$\therefore n(A) = 3$

P(Event) = Number of outcomes favorable to event / Total number of possible outcomes

$P(A) = n(A) / n(S)$

$= 3/4$

3: A die is thrown, find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

Answer: Let us suppose that $1,2,3,4,5,$ and 6 are the possible outcomes when the die is thrown.

Here, $S = 1,2,3,4,5,6$

$\therefore n(S) = 6$

(i) A prime number will appear,

Let us suppose 'A' be the event of getting a prime number,

$A = \{2,3,5\}$

Then, $n(A) = 3$

P(Event) = Number of outcomes favorable to event / Total number of possible out comes

$P(A) = n(A)/ n(S)= 3/6$

$= 1/2$

(ii) That number which is greater than or equal to 3 will appear,

Let us suppose 'B' be the event of getting a number greater than or equal to 3, B = {3,4,5,6}

Then, n(B) = 4

P(Event) = Number of outcomes favorable to event / Total number of possible out comes

$\therefore$ P(B) = n(B) / n(S)

= 4 / 6

= 2/3

(iii) That number which is less than or equal to one will appear,

Let us suppose 'C' be the event of getting a number less than or equal to 1,

C = {1}

Then, n(C) = 1

P(Event) = Number of outcomes favorable to event / Total number of possible out comes

$\therefore$ P(C) = n(C) / n(S)

= 1/6

(iv) That number which is more than 6 will appear,

Let us suppose 'D ' be the event of getting a number more than 6, then

D = {0)}

Then, n(D) = 0

P(Event) = Number of outcomes favorable to event / Total number of possible out comes

$\therefore$ P(D) = n(D) / n(S)

= 0/6

= 0

(v) That number which less than 6 will appear.

Let us suppose 'E' be the event of getting a number less then 6, then

E = (1,2,3,4,5)

Then, n(E) = 5

P(Event) = Number of outcomes favorable to event / Total number of possible out comes

∴ P(E) = n(E) / n(S)

= 5/6

4: A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card

Answer: From the question it is given that, there are 52 cards in the desk

(a) Number of cases in the sample space = 52(given)

∴ n(S) = 52

(b) Let us suppose ' A ' be the event of drawing an ace of spades.

A = 1 Then, n(A) = 1

P(Event) = Number of outcomes favorable to event / Total number of possible outcomes

∴P(A) = n(A) / n(S)

= 1/52

(c) Let us suppose 'B' be the event of drawing an ace. There are four aces.

Then, n(B) = 4

P(Event) = Number of outcomes favorable to event / Total number of possible outcomes

∴P(B) = n(B) / n(S) = 4/52

= 1 / 13

(d) Let us suppose ' C ' be the event of drawing a black card. There are 26 black cards.

Then, n(C) = 26

P(Event) = Number of outcomes favorable to event / Total number of possible outcomes

∴P(C) = n(C) / n(S)

= 26/52

1/2

5: A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 **(ii)** 12

Answer: Let us suppose that 1,2,3,4,5 and 6 are the possible outcomes when the die is thrown.

So, the sample space

$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

Then, $n(S) = 12$

(i) Let us suppose ' P ' be the event having sum of numbers as 3.

$P = \{(1,2)\}$

Then, $n(P) = 1$

P(Event)= Number of outcomes favorable to event/ Total number of possible outcomes

$\therefore P(P) = n(P)/n(S)$

$= 1/12$

(ii) Let us suppose ' Q ' be the event having sum of number as 12.

Then $Q = \{(6,6)\}$, $n(Q) = 1$

P(Event)= Number of outcomes favorable to event/ Total number of possible outcomes

$\therefore P(Q) = n(Q)/n(S)$

$= 1/12$

6: There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Answer: From the question it is given that,

Here total members in the council $= 4 + 6 = 10$,

Hence, the sample space has 10 points

$\therefore n(S) = 10$

Number of women are 6 [given]

Let us suppose 'A' be the event of selecting a woman

Then $n(A) = 6$

P(Event)= Number of outcomes favorable to event/ Total number of possible outcomes

$\therefore P(A) = n(A)/n(S)$

$= 6/10$

$= 3/5$

7: A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.50 for each tail that turns up.

From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Answer: Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

After toss a coin four times, then sample space is

S=(HHHH,HHHT,HHTH,HTHH,THHH,HHTT,HTHT,THHT,HTTH,THTH,
TTHH,,TTTH,TTHT,THTT,HTTT,TTTT)

According to the condition that are given in the question, a person will win or lose money depending up on the face of the coin so,

(i) For 4 heads $=1+1+1+1=\mathrm{Rs}4$

So, he wins $\mathrm{Rs}4$

(ii) For 3 heads and 1 tail $= 1 + 1 + 1 - 1.50$

$= 3 - 1.50$

$= \mathrm{Rs}1.50$

So, he will be winning $= \mathrm{Rs}1.50$

(iii) For 2 heads and 2 tail $= 1 + 1 - 1.50 - 1.50$

$= 2 - 3 = \mathrm{Rs}{-1}$

(iv) For 1 head and 3 tails $= 1 - 1.50 - 1.50 - 1.50 = 1 - 4.50$

$= - \mathrm{Rs}3.50$

So, he will be losing Rs 3.50

(v) For 4 tails $= - 1.50 - 1.50 - 1.50 - 1.50$

$= - \mathrm{Rs}\ 6$

So, he will be losing $\mathrm{Rs}\ 6$

Now the sample space of amounts is

S={4,1.50,1.50,1.50,1.50,-1,-1,-1,-1,-1,-1,-3.50,-3.50,-3.50,-3.50,-6}

Then, n(S) = 16

P(winning Rs 4) = 1/16

P(winning Rs1.50) = 4/16

= 1/4

P(winning Rs1) = 6 /16

= 3/8

P(winning Rs3.50) = 4/16

= 1/4

P(winning Rs6) = 1/16

= 3/8

8: Three coins are tossed once. Find the probability of getting

(i) heads

(ii) 2 heads

(iii) at least 2 heads

(iv) At most 2 heads

(v) no head

(vi) 3 tails

(vii) Exactly two tails

(viii) no tail

(ix) at most two tails

Answer: Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

Here coin is tossed three times, then sample space is

S = {HHH,HHT,HTH,THH,TTH,HTT,TTT,THT}

Where S is sample space and n(S) = 8

(i) 3 Heads

Let us suppose 'A' be the event of getting 3 heads

n(A) = 1

∴ P(A) = n(A) / n(S)

= 1/8

(ii) 2 heads

Let us suppose 'B' be the event of getting 2 heads

n(A) = 3

P(B) = n(B) / n(S)

= 3/8

(iii) At least 2 heads

Let us suppose 'C' be the event of getting at least 2 heads

n(C) = 4

P(C) = n(C) / n(S)

= 4/8

= 1/2

(iv) At most 2 heads

Let us suppose 'D' be the event of getting at most 2 heads

n(D) = 7

P(D) = n(D) / n(S)

= 7/8

(v) No head

Let us suppose 'E' be the event of getting at most no heads

n(E) = 1

P(E) = n(E) / n(S)

= 1 / 8

(vi) 3 Tails

Let us suppose 'F' be the event of getting 3 Tails

n(F) = 1

∴ P(F) = n(F) / n(S)

= 1 / 8

(vii) Exactly two tails

Let us suppose 'G' be the event of getting exactly two tails

n(G) = 3

∴ P(G) = n(G) / n(S)

= 3 / 8

(viii) No tail

Let us suppose 'H' be the event of getting No tail

n(H) = 1

∴ P(H) = n(H) / n(S)

= 1 / 8

(ix) At most two tails

Let us suppose 'I' be the event of getting at most two tails

n(I) = 7

∴ P(I) = n(I) / n(S)

= 7 / 8

9: If $2/11$ is the probability of an event, what is the probability of the event 'not A'.

Answer: From the question it is given that, $2/11$ is the probability of an event A,

i.e. P(A) = 2 / 11

Then, P(not A) = 1 - P(A)

= 1 - (2 / 11)

= (11 - 2) / 11

= 9 / 11

10: A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) a consonant

Answer: The 'ASSASSINATION' word given in the question

Total letters in the given word $=13$

Vowels in the given word = 6

Number of consonants in the given word $=7$

Then, the sample space n $=13$

(i) A vowel

Let us suppose ' A ' be the event of selecting a vowel

$n(A) = 6$

$\therefore P(A) = n(A) / n(S)$

$= 6 / 13$

(ii) Let us suppose 'B' be the event of selecting the consonant letter

$n(B) = 7$

$\therefore P(B) = n(B) / n(S)$

$= 7 / 13$

11: In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint order of the numbers is not important.]

Answer: From the question it is given that,

Total numbers of numbers in the draw $= 20$

Numbers to be selected $= 6$

$\therefore n(S) = {}^{20}C_6$

Let us suppose ' A ' be the event those that six are numbers match with the six numbers already fixed by the lottery committee.

$\therefore n(A) = {}^{6}C_6 = 1$

Probability of winning the prize

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^{6}C_6}{{}^{20}C_6} = \frac{6!14!}{20!}$$

$$= \frac{6\times5\times4\times3\times2\times1\times14!}{20\times19\times18\times17\times16\times15\times14!}$$

$$= \frac{1}{38760}$$

12: Check whether the following probabilities P(A) and P(B) are consistently defined

(i) $P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$

(ii) $P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

Answer: (i) $P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$

$P(A \cap B) > P(A)$

Therefore, the given probability are not consistently defined

(ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

Then,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.5 + 0.4 - P(A \cap B)$

Transposing $- P(A \cap B)$to LHS and it becomes $P(A \cap B)$ and 0.8 to RHS and it becomes - 0.8

$P(A \cap B) = 0.9 - 0.8$

$= 0.1$

Therefore, $P(A \cap B) < P(A)$ and $P(A \cap B) < P(B)$

Therefore, the given probability are consistently defined

13: Fill in the blanks in following table:

	P(A)	P(B)	$P(A \cap B)$	$P(A \cup B)$
(i)	1/3	1/5	1/15	……….
(ii)	0.35	……….	0.25	0.6
(iii)	0.5	0.35	…………..	0.7

Answer: From the given table

(i) $P(A) = 1/3$, $P(B) = 1 / 5$, $P(A \cap B) = 1 / 15$, $P(A \cup B) = ?$

We know that,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= (1/3) + (1/5) - (1 / 15)$

$= ((5+ 3)/15) - (1/15)$

$= (8 / 15) - (1 / 15)$

$= (8 - 1) / 15$

$= 7 / 15$

(ii) $P(A) = 0.35$, $P(B) = ?$, $P(A \cap B) = 0.25$, $P(A \cup B) = 0.6$

Then,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 0.35 + P(B) - 0.25$

Transposing- 0.25,0.35 to LHS and it becomes 0.25 and - 0.35

$P(B) = 0.6 + 0.25 - 0.35$

$= 0.5$

(iii) $P(A) = 0.5$, $P(B) = 0.35$, $P(A \cup B) = 0.7$, $P(A \cap B) = ?$

Then,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.5 + 0.35 - P(A \cap B)$

Transposing - $P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.7 to RHS and it becomes - 0.7

$P(A \cap B) = 0.85 - 0.7$

$= 0.15$

14: Given $P(A) = 5/3$ and $P(B) = 1/5$. Find P(A or B), if A and B **are mutually exclusive events.**

Answer: From the question it is given that,

$P(A) = 5/3$ and $P(B) = 1/5$

Then, P(A or B) , if A and B are mutually exclusive

$P(A \cup B)$ or P(A or B) = P(A) + P(B)

$= (3/5) + (1/5)$

$= 4/5$

15: If E and F are events such that $P(E)=1/4$, $P(F) = 1/2$ and P(E and F) = $1 / 8$, find

(i) P(E or F),

(ii) P(not E and not F)

Answer: From the question, we have

$P(E) = 1/4,\ P(F) = 1/2 \text{ and } P(E \cap F) = 1/8$

(i) $P(E \text{ or } F) \text{ i.e. } P(E \cup F) = P(E) + P(F) - P(E \cup F)$

$= 1/4 + 1/2 - (1/8)$

$= 5/8$

(ii) $P(\text{ not } E \text{ and not } F) = P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

$= 1 - (5/8)$

$= (8 - 5)/8$

$= 3/8$

16: Events E and F are such that $P(\text{ not } E \text{ or not } F) = 0.25$**, State whether E and F are mutually exclusive.**

Answer: From the question it is given that, $P(\text{ not } E \text{ or not } F) = 0.25$

We have,

$\Rightarrow P(\overline{E \cap F}) = 0.25$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\Rightarrow P(E \cap F) = 1 - 0.25$

$= 0.75$

$P(E \cap F) \neq 0$

Hence, E and F are mutually exclusive.

17: A and B are events such that $P(A) = 0.42,\ P(B) = 0.48 \text{ and } P(A \text{ and } B) = 0.16$ **Determine (i) P(not A), (ii) P(not B) and (iii) P(A or B)**

Answer: From the question it is given that,

$P(A) = 0.42,\ P(B) = 0.48 \text{ and } P(A \text{ and } B) = 0.16$

(i) $P(\text{ not } A) = 1 - P(A)$

$= 1 - 0.42$

$= 0.58$

(ii) $P(\text{ not } B) = 1 - P(B)$

$= 1 - 0.48$

$= 0.52$

(iii) $P(AvnotvB) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.74$

18. In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Answer: Let us take ' A' be the event of those student who are studying mathematics and 'B' be the event of those student who are studying biology.

So, P(A) = 40 / 100

= 2 / 5

And, P(B) = 30 / 100

= 3/10

Then, $P(A \cap B) = (10/100)$

$= 1/10, P(A \cap B)$ is probability of studying both mathematics and biology.

Here, let us suppose the Probability of the student studying mathematics or biology

= P (AUB)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

= (2/5) + (3/10) - (1/10)

= 6 / 10

= 3 / 5

Hence, (3 / 5) is the probability of those student who will studying mathematics or biology

19: In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Answer: Let us take the probability of a continuously chosen student passing the first examination is 0.8 be P(A). And also take the probability of passing the 2^{nd}

examination is 0.7 be P(B)

Then,

$P(A \cup B)$ be the probability of those who are passing at least one of the examinations

Now,

$P(A \cup B) = 0.95, P(A) = 0.8, P(B) = 0.7$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.95 = 0.8 + 0.7 - P(A \cap B)$

Transposing $-P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.95 to RHS and it becomes -0.95

$P(A \cap B) = 1.5 - 0.95$

$= 0.55$

Hence, $= 0.55$ is the probability of those student who will pass both examination

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Answer: Let us assume probability of passing the English examination is 0.75 be P(A).

And let us assume the probability of passing the Hindi examination is P(B).

Here given, $P(A) = 0.75, P(A \cap B) - 0.5, P\left(A' \cap B'\right) = 0.1$ We know that,

$P\left(A' \cap B'\right) = 1 - P(A \cup B)$

Then, $P(A \cup B) = 1 - P\left(A' \cap B'\right)$

$= 1 - 0.1$

$= 0.9$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.9 = 0.75 + P(B) - 0.5$

Transposing $0.75, -0.5$ to LHS and it becomes $-0.75, 0.5$

$P(B) = 0.9 + 0.5 - 0.75$

$= 0.65$

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS.

(ii) The student has opted neither NCC nor NSS.

(iii) The student has opted NSS but not NCC.

Answer: From the question it is given that,

The total number of students in class $= 60$

Thus, the sample space consists of $n(S) = 60$

Let us take the students which are opted for NCC be 'A'

Let us take the students which are opted for NSS be 'B'

So, $n(A) = 30$, $n(B) = 32$, $n(A \cap B) = 24$

We know that, $P(A) = n(A) / n(S)$

$= 30/60$

$= 1/2$

$P(B) = n(B) / n(S)$

$= 32/60$

$= 8/15$

$P(A \cap B) = n(A \cap B) / n(S)$

$= 24 / 60$

$= 2/5$

Therefore, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(i) Those students which are opted for NCC or NSS.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1/2 + (8 / 15) - (2/5)$

$= 19/30$

(ii) The student which are opted for neither NCC nor NSS.

$P(\text{ not } A \text{ and not } B) = P\left(A' \cap B'\right)$

We know that, $P\left(A' \cap B'\right) = 1 - P(A \cup B)$

$= 1 - (19 / 30)$

$= 11 / 30$

(iii) The student which is opted for NSS but not NCC.

$(B - A) = n(B) - n(A \cap B)$

$\Rightarrow 32 - 24 = 8$

The probability that the selected student has opted NSS but not NCC

$= (8 / 60)$

$= 2/15$

Miscellaneous Exercise

1: A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(i) All will be blue? (ii) At least one will be green?

Answer: From the question it is given that,

Number of red marbles in the box $=10$

Number of blue marbles in the box $=20$

Number of green marbles in the box $=30$

Adding all the marbles then, number of marbles in the box $=10+20+30=60$

Number of ways of getting 5 marbles from the 60 marbles $= {}^{60}C_5$,

(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles.

We have,

Number of ways of drawing 5 blue marbles from 20 blue marbles $= {}^{20}C_5$

Then,

Probability that all marbles will be blue $= {}^{20}C_5 \ / \ {}^{60}C_5$

(ii) Number of ways if the getting marble is not green $= {}^{(20+10)}C_5$

We have,

Probability that marble if there is no green marble $= {}^{30}C_5 / {}^{60}C_5$

Then,

Probability that at least one marble is green $=1-{}^{30}C_5 / {}^{60}C_5$

2: 4 cards are drawn from a well - shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Answer: From the question it is given that,

4 Cards are getting out from a box of 52 cards

Number of ways of drawing 4 cards from 52 cards $= {}^{52}C_4$

In a deck of 52 cards, there are 13 diamonds and 13 spades.

Number of ways of drawing 3 diamonds and one spade $= {}^{13}C_3 \times {}^{13}C_1$

So, the probability of getting 3 diamonds and one spade

$=\left({}^{13}C_3 \times {}^{13}C_1\right) / {}^{52}C_4$

3. A die has two faces each with number ' 1 ', three faces each with number ' 2 ' and one face with number ' 3 '. If die is rolled once, determine

(i) P(2)
(ii) P(1 or 3)
(iii) P(not 3)

Answer: From the question it is given that,

Die has two faces each with number ' 1 ',

Three faces each with number ' 2 '

And one face with number ' 3 '

We know the Number of faces of a die = 6

(i) P(2) Number faces with number′ 2′ = 3…[given]
So, P(2) = 3/6
= 1/2

(ii) P(1 or 3)
We know that, P(1 or 3) = P(not 2) = 1 - P(2)
So, P(1 or 3) = 1 - 1 / 2
= (2 - 1) / 2 = 1 / 2

(iii) P(not 3)
Number of faces with number '3' = 1
P(3) = 1/6
P(not 3) = 1 - P(3)
= 1 - 1/6
= (6 - 1) / 6
= 5 / 6

4. In a certain lottery 10000 tickets are sold, and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets?

Answer: From the question it is given that,

Number of lottery tickets sold = 10000

Number prizes awarded = 10

(a) if not getting a prize if we buy one ticket, then probability

P(getting a prize) = 10/10000

= 1/1000

Then,

P(not getting a prize) = 1 - (1/1000)

= (1000 - 1) / 1000

= 999 / 1000

(b) if not getting a prize if we buy two tickets, then probability

Then,

Number of tickets not awarded $=10,000-10=9990$

Therefore, $P(\text{not getting a prize}) = {}^{9990}C_2 / {}^{10000}C_2$

(iii) if not getting a prize if we buy 10 tickets, then probability

Number of tickets not awarded $=10,000-10=9990$

Therefore, $P(\text{not getting a prize}) = {}^{9990}C_{10} \ / \ {}^{10000}C_{10}$

5. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(a) You both enter the same section?

(b) You both enter the different sections?

Answer: From the question,

Total number of students $=100$,

I and my friend are among the 100 students.

Then, two sections of 40 and 60 are formed.

Total number of ways of selecting 2 students out of 100 students

(a) Let $S =$ the two of us will enter the same section if both of us are among 40 students or among 60 students.

Let Number of ways if both of them enter the same section $= P(S)$

$P(S) = \left({}^{40}C_2 + {}^{60}C_2\right) / {}^{100}C_2$

$$P(S) = \frac{\frac{40!}{2! \times 38!} + \frac{60!}{2! \times 58!}}{\frac{100!}{2! \times 98!}} = \frac{39 \times 40 + 59 \times 60}{99 \times 100}$$

$$= \frac{5100}{9900} = \frac{17}{33}$$

(b) P (we enter different sections)

= 1 - P(we enter the same section)

P(we enter different sections) = 1 - (17 / 33)

= (33 - 17) / 33

= 16 / 33

6. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Answer: Let us assume L_1,L_2,L_3 be three letters and E_1,E_2,E_3 be their corresponding envelops respectively.

Then, Sample space is

L_1E_1,L_2E_3,L_3E_2
L_2E_2,L_1E_3,L_3E_1
L_3E_3,L_1E_2,L_2E_1
L_1E_1,L_2E_2,L_3E_3
L_1E_2,L_2E_3,L_3E_1
L_1E_3,L_2E_1,L_3E_2

Hence, there are 6 ways of inserting 3 letters in 3 envelops.

And there are 4 ways in which at least one letter is inserted in proper envelope. (First 4 rows of sample space) Probability that at least one letter is inserted in proper envelope**,**

$= 4/6$
$= 2/3$

7. A and B are two events such that $P(A) = 0.54, P(B) = 0.69$ and $P(A \cap B) = 0.35$**. Find**

(i) $P(A \cup B)$

(ii) $P(A' \cap B')$

(iii) $P(A \cap B')$

(iv) $P(B \cap A')$

Answer: From the question it is given that, A and B are two events such that,

$P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$

(i) $P(A \cup B)$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.54 + 0.69 - 0.35 = 0.88$

$\therefore P(A \cup B) = 0.88$

(ii) $P(A' \cap B')$

We know that $A' \cap B' = (A \cup B)'$

So, $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - 0.88 = 0.12$

$\therefore P(A' \cap B') = 0.12$

(iii) (ii) $P(A \cap B')$

We have

$P(A \cap B') = P(A) - P(A \cap B)$

$= 0.54 - 0.35$

$= 0.19$

$\therefore P(A \cap B') = 0.19$

(iv) $P(B \cap A')$

We know that $P(B \cap A') = P(B) - P(A \cap B)$

$\Rightarrow P(B \cap A') = 0.69 - 0.35$

$\therefore P(B \cap A') = 0.34$

8: From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S.NO.	Name	Sex	Age in years
1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alis	F	28
5	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Answer: From the given table,

The number of persons $= 5$

Out of 5 persons 3 are Male

Out of 5 persons 2 are 35 years of age.

Let ‘ A ' be that event in which the spokesperson are male and B be that event in which the spokesperson are from the over the age of 35.

Accordingly, $P(A) = 3/5$ & $P(B) = 2/5$

Since there is only one male who is over 35 years of age,

$P(A \cap B) = 1/5$

We know that:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= (3/5) + (2/5) - (1/5)$$
$$= 4/5$$

So, the probability of those spokesperson who are either be a male or over 35 years of age $= \frac{4}{5}$.

9. If 4-digit numbers greater than 5,000 are randomly formed from the digits $0,1,3,5,$ and 7, what is the probability of forming a number divisible by 5 when, (i) the digits are repeated? (ii) The repetition of digits is not allowed?

Answer: (i) When the digits are repeated

If four-digit numbers greater than 5000 are formed, then

The thousand's place digit is either 7 or 5

Total number of 4-digit numbers greater than $5000 = 2 \times 5 \times 5 \times 5 - 1$

$$= 250 - 1$$
$$= 249$$

A number is divisible by 5 if the digit at its unit's place is either 0 or 5.

Therefore, Total number of 4-digit numbers greater than 5000 that are divisible by 5

$= 2 \times 5 \times 5 \times 2 - 1$
$= 100 - 1 = 99$

Hence, the probability of forming a number divisible by 5 when the digits are repeated is = P (number divisible by 5 when digits repeated)

P (number divisible by 5 when digits repeated) $= 99 / 249$

$= 33/83$

(ii) For repetition of digits is not allowed

The thousands place can be filled with either of the two digits 5 or 7 i.e. by 2 ways.

The remaining 3 places can be filled with any of the remaining 4 digits.

Total number of 4-digit numbers greater than $5000 = 2 \times 4 \times 3 \times 2 = 48$

Here, number of 4-digit numbers starting with 5 and divisible by $5 = 1 \times 3 \times 2 \times 1 = 6$

Here, number of 4-digit numbers starting with 7 and divisible by 5

$= 1 \times 2 \times 3 \times 2 = 12$

Total number of 4-digit numbers greater than 5000 that are divisible by 5

$= 6 + 12 = 18$

Thus, the probability for a number when formed is divisible by 5 when the repetition of digits is not allowed

= P (number divisible by 5 when digits are not repeated)

P (number divisible by 5 when digits are not repeated) $= 18/48 = 3/8$

10. The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Answer: From the question it is given that,

The number lock of a suitcase 4 wheels, each designed with ten digits i.e., from 0 to 9

Then,

Selecting of way of a number in 4 different digits from the 10 digits $= {}^{10}C_4$

So, combination of each 4 different digits number arranged by the $4!$ Ways.

Also, it is given

Number of four digits in which have no repetitions $= 4! \times {}^{10}C_4 = 5040$

In this have only one number that can open the suitcase.

Hence, the probability $= 1/5040$