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Chapter 3: Matrices.

Exercise Miscellaneous

1. Let $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, show that $(aI+bA)^n=a^nI+na^{n-1}bA,$ where I is the identity matrix of order 2 and $n\in N$

Solution:

Given, $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$

By using the principle of mathematical induction

For n = 1

$P(1):(aI+bA)=aI+ba^{\circ}A=aI+bA$

Therefore, the result is true for n = 1

Let the result be true for n = k

That is, $P(k):(aI+bA)^k=a^kI+ka^{k-1}bA$

Now, we have to prove that the result is true for $n=k+1$

$(aI+bA)^{k-1}=(aI+bA)^k(aI+bA)$

$=(a^kI+ka^{k-1}bA)(aI+bA)$

$=a^{k-1}+ka^kbAI+a^kbIA+ka^{k-1}b^2A^2$

$=a^{k-1}I+(k+1)a^kbA+ka^{k-1}b^2A^2$ ....(1)

Now, $A^2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=0$

From (1), we have

$$(aI+bA)^{k+1}=a^{k+1}+(k+1)a^kbA+0$$

$$=a^{k+1}+(k+1)a^kbA$$

Thus, the result is true for $n=k+1$

By the principal of mathematical induction, we have

$$(aI+bA)^n=a^nI+na^{n-1}bA \text{ where } A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, n\in N$$

2. If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$, prove that $A^n\begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix}, n\in N$

Solution:

Given, $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

By using the principles of mathematical induction

For n = 1, we have

$$P(1)=\begin{bmatrix}3^{1-1} & 3^{1-1} & 3^{1-1}\\ 3^{1-1} & 3^{1-1} & 3^{1-1}\\ 3^{1-1} & 3^{1-1} & 3^{1-1}\end{bmatrix}=\begin{bmatrix}3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}=A$$

Thus, the result is true for n = 1

Let the result be true for n = k

$$P(k): A^k=\begin{bmatrix}3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\end{bmatrix}$$

Now, we have to prove that the result is true for $n=k+1$

Now, $A^{k+1} = A.A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3.3^{(k+1)-1} & 3.3^{(k+1)-1} & 3.3^{(k+1)-1} \\ 3.3^{(k+1)-1} & 3.3^{(k+1)-1} & 3.3^{(k+1)-1} \\ 3.3^{(k+1)-1} & 3.3^{(k+1)-1} & 3.3^{(k+1)-1} \end{bmatrix}$$

Thus, the result is true for $n = k+1$

By the principal of mathematical induction, we have

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in N$$

3. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer

Solution:

Given, $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

By using the principle of mathematical induction

For n = 1, we have

$$P(1): A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} A$$

Thus, the result is true for n = 1

Let the result be true for n = k

$$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}, n \in N$$

Now, we have to prove that the result is true for $n = k+1$

$$A^{k+1} = A^k.A$$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1+2k)-4k & -4(1+2k)+4k \\ 3k+1-2k & -4k-1(1-2k) \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ 1+k & 1-2(k+1) \end{bmatrix}$$

Thus, the result is true for $n = k+1$

By the principal of mathematical induction, we have

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \notin N$$

4. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix

Solution:

Given, A and B are symmetric matrices. Therefore, we have

$$A' = A \text{ and } B' = B \qquad ....(1)$$

Now, $(AB - BA)' = (AB)' - (BA)'$

$$= B'A' - A'B'$$

$= BA - AB$ [Using (1)]

$= -(AB - BA)$

$\therefore (AB - BA)' = -(AB - BA)$

Hence, $(AB - BA)$ is a skew- symmetric matrix

5. Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution:

Let A is a symmetric matrix, then $A' = A$ .....(1)

$(B'AB)' = \{B'(AB)\}'$

$= (AB)'(B)'$

$= B'A'(B)$

$= B'(AB)$ [Using (1)]

$\therefore (B'AB)' = B'AB$

Thus, if A is symmetric matrix, then $B'AB$ is a symmetric matrix.

Let A is a skew – symmetric matrix

Then, $A' = A$

$(B'AB)' = [B'(AB)]' = (AB)'(B)'$

$= (B'A')B = B'(-A)B$

$= -B'AB$

$\therefore (B'AB)' = -B'AB$

Thus, A is skew – symmetric matrix then $B'AB$ is a skew – symmetric matrix

Therefore, if A is a symmetric or skew – symmetric matrix, then $B'AB$ is a symmetric or skew – symmetric matrix accordingly.

6. Solve system of linear equations, using matrix method.

$2x-y=-2$

$3x+4y=3$

Solution:

The given system of equation can be written in the form of $AX=B$, where

$$A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix} \text{ and } B=\begin{bmatrix}-2\\ 3\end{bmatrix}$$

Now, $|A|=8+3=11\neq 0$

Thus, A is non – singular. Therefore, its inverse exists

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}\begin{bmatrix}-2\\ 3\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-8+3\\ 6+6\end{bmatrix}$$

$$=\frac{1}{11}\begin{bmatrix}-5\\ 12\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-5}{11}\\ \frac{12}{11}\end{bmatrix}$$

Thus, $x=\frac{-5}{11}$ and $y=\frac{12}{11}$

7. For what values of $x, \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$?

Solution:

Given, $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \left[6(0)+2(2)+4(x)\right] = 0$$

$$\Rightarrow \left[4+4x\right] = \left[0\right]$$

$$\therefore 4+4x = 0$$

$$\Rightarrow x = -1$$

Thus, the required value of x is – 1

8. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

Solution:

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix}$$

$$=\begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}=\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$L.H.S = A^2-5A+7I$$

$$=\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}-5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}+7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}-\begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$=0=R.H.S$$

$$\therefore A^2-5A+7I=0$$

9. Find X, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=0$

Solution:

Given, $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=0$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=0$$

$$\Rightarrow \left[x(x-2)-40+2x-8\right]=0$$

$$\Rightarrow \left[x^2-2x-40+2x-8\right]=[0]$$

$$\Rightarrow\left[x^2-48\right]=[0]$$

$$\Rightarrow x^2-48=0$$

$$\Rightarrow x^2=48$$

$$\Rightarrow x=\pm 4\sqrt{3}$$

10. A manufacture produces three products X, Y, Z which he sells in two markets.

Annual sales are indicated below

Market	Products		
I	10000	2000	18000
II	6000	20000	8000

a) If unit sale prices of X, Y and Z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

Solution:

a. Here, the total revenue in market I can be represented in the form of matrix as

$$\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$

$$=10000\times 2.50+2000\times 1.50+18000\times 1.00$$

$$=25000+3000+18000$$

$$=46000$$

And, the total revenue in market II can be represented in the form of a matrix as

$$\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$

$$=6000\times 2.50+20000\times 1.50+8000\times 1.00$$

$$=15000+30000+8000$$

$$=53000$$

Thus, the total revenue in market I is Rs 46000 and the same in market II is Rs 53000

b. Here, the total cost prices of all the products in the market I can be represented in the form of a matrix as

$$\begin{bmatrix} 10000 & 2000 & 180000 \end{bmatrix}\begin{bmatrix} 2.50 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$= 10000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50$

$= 20000 + 2000 + 9000 = 31000$

As, the total revenue in market I is Rs 46000, the gross profit in this market is

Rs 46000 –Rs 31000 = Rs 150000

The total cost prices of all the products in market II can be represented in the form of a matrix as

$$\begin{bmatrix} 6000 & 2000 & 8000 \end{bmatrix}\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$= 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50$

$= 12000 + 20000 + 4000$

$= 36000$

Since the total revenue in market II is Rs 53000, the gross profit in this market is

Rs 53000 – Rs 36000 = Rs 170000

11. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Solution:

Given, $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Here, X has to be a 2×2 matrix

Now, let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Thus, we have $\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Comparing the corresponding elements of two matrices, we have

$a+4c=-7, \quad 2a+5c=-8 \quad 3a+6c=-9$

$b+4d=2, \quad 2b+5d=4, \quad 3b+6d=6$

Now, $a+4c=-7 \Rightarrow a=-7-4c$

$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$

$\Rightarrow -3c=6$

$\Rightarrow c=-2$

$\therefore a=-7-4(-2)=-7+8=1$

Now, $b+4d=2 \Rightarrow b=2-4d$

$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$

$\Rightarrow -3d=0$

$\Rightarrow d=0$

$\therefore b=2-4(0)=2$

$\therefore a=1, b=2, c=-2, d=0$

Thus, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

12. If A and B are square matrices of the same order such that AB = BA, then prove by induction that $AB^n = B^n A$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in N$

Solution:

Given, A and B are square matrices of the same order such that AB = BA

For n = 1, we have

$P(1): AB = BA$ [Given]

$\Rightarrow AB^1 = B^1 A$

Therefore, the result is true for n = 1

Let the result be true for n = k

$P(k): AB^k = B^k A \qquad ....(1)$

Now, we have to prove that the result is true for $n = k+1$

$AB^{k+1} = AB^k.B$

$= (B^k A)B \qquad$ [By (1)]

$= B^k (AB)$

$= B^k (BA)$

$= (B^k B)A$

$= B^{k+1}A$

Therefore, the result is true for $n = k+1$

By the principle of mathematical induction, we have $AB^n = B^n A, n \in N$

13. Choose the correct answer in the following questions.

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

$(A) 1+\alpha^2+\beta\gamma = 0$

$(B) 1-\alpha^2+\beta\gamma = 0$

$(C) 1-\alpha^2-\beta\gamma = 0$

$(D) 1+\alpha^2-\beta\gamma = 0$

Solution:

Given, $A\begin{bmatrix} \alpha & \beta \\ y & -\alpha \end{bmatrix}$

$$\therefore A^2 = A.A = \begin{bmatrix} \alpha & \beta \\ y & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ y & -\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix}$$

Now, $A^2 = I$

$$\Rightarrow \begin{bmatrix} \alpha^2+\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements, we have

$$\alpha^2+\beta\gamma=1$$

$$\Rightarrow \alpha^2+\beta\gamma-1=0$$

$$\Rightarrow 1-\alpha^2-\beta\gamma=0$$

14. If the matrix A is both symmetric and skew symmetric, then

(A) A is diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Solution:

If A is both symmetric and skew – symmetric matrix, then

$$A' = \text{A and A}' = -\text{A}$$

$$A' = A$$

$$A' = -A$$

$$\Rightarrow A = -A$$

$$\Rightarrow A + A = 0$$

$\Rightarrow 2A = 0$

$\Rightarrow A = 0$

15. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A (B) $I - A$ (C) I (D) 3A

Solution:

$$(I + A)^3 - 7A = I^3 + A^3 + 3I^2A + 3A^2I - 7A$$

$$= I + A^3 + 3A + 3A^2 - 7A$$

$$= I + A^2.A + 3A + 3A - 7A \qquad [A^2 = A]$$

$$= I + A.A - A$$

$$= I + A^2 - A$$

$$= I + A - A$$

$$= \text{I}$$

$$\therefore (I + A)^3 - 7A = I$$

Exercise 3.1

1. In the matrix $A=\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$ write

   i. The order of the matrix

   ii. The number of elements

   iii. Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Solution: The given matrix is $A=\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$

i. In the matrix, the number of rows is 3 and the number of columns is 4. Hence the order of the matrix is 3×4

ii. If the order of the matrix is $m\times n$, then the number of elements in the matrix is product of m, n. So that the number of elements of the given matrix is $3\cdot 4=12$

iii. The element a_{ij} is i^{th} row and j^{th} column element.

a. $a_{13}=19$

b. $a_{21}=35$

c. $a_{33}=-5$

d. $a_{24}=12$

2. If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Solution: If a matrix is of the order $m\times n$, then it has $m\cdot n$ elements.

Given that the number of elements in the matrix is 24.

The possible pairs of factors of 24 are S $(1,24),(2,12),(3,8),(4,6)$

Therefore, the possible orders of the matrix having 24 elements are $1\times24, 2\times12, 3\times8$ $4\times6, 6\times4, 8\times3, 12\times2, 24\times1$.

If the matrix has 13 elements, then the possible orders of the matrix are $1\times13, 13\times1$

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution: If a matrix is of the order $m\times n$, then it has $m\cdot n$ elements.

Given that the number of elements in the matrix is 18

The possible pairs of factors of 18 are $(1,18),(2,9),(3,6)$

Therefore, the possible orders of the matrix having 18 elements are $1\times18, 2\times9, 3\times6$ and $18\times1, 9\times2, 6\times3$

If the matrix has 5 elements, then the possible orders of the matrix are $1\times5, 5\times1$

4. Construct a matrix of order 3×4, whose elements are given by

i. $a_{ij}=\frac{1}{2}|-3i+j|$

ii. $a_{ij}=2i-j$

Solution: The general 3×4 matrix is $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

i. Given $a_{ij}=\frac{1}{2}|-3i+j|$

Hence the matrix is $A=\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

ii. Given $a_{ij}=2i-j$

Hence, the matrix is $A=\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

5. Find the values of x, y, z from the following equations

i. $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix}=\begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

ii. $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

iii. $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Solution: Two matrices are said to be equal if the order of those two matrices are equal and each entry must be equal to the corresponding entry**.**

i. Given $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix}=\begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

Orders of both matrices are equal. Each entry equal to corresponding entries

Hence, $x=1, y=3$

ii. Given $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Orders of both matrices are equal. Each entry equal to corresponding entries

Hence, $x+y=6, xy=8, 5+z=5$

Therefore, $z=0$

Consider

$$\begin{aligned}(x-y)^2 &= (x+y)^2-4xy \\ &= 36-32 \\ &= 4 \\ x-y &= \pm 2\end{aligned}$$

Suppose that $x-y=2$

It gives $2x=8 \Rightarrow x=4, y=2$

Suppose that $x-y=-2$

It gives $2x=6 \Rightarrow x=3, y=3$

Therefore $x=4, y=2, z=0$ and $x=3, y=3, z=0$,

iii. Given $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Orders of both matrices are equal. Each entry equal to corresponding entries

Hence, $x+y+z=6, x+y=6, y+z=7$

Therefore, $z=0, y=7, x=-1$

6. Find the value of a, b, c and d from the equation $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Solution: Given, $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Comparing the corresponding elements, we get

$$a-b=-1, 2a+c=5, 2a-b=0, 3c+d=13$$

Solving the above equations

$$a-b-2a+b=-1$$
$$-a=-1$$
$$a=1$$

Hence, $1-b=-1 \Rightarrow b=2$

$$2(1)+c=5 \Rightarrow c=5-2 \Rightarrow c=3$$

$$3c+d=13 \Rightarrow 3(3)+d=13 \Rightarrow d=4$$

Therefore, $a=1, b=2, c=3, d=4$

7. $A=\left[a_{ij}\right]_{mxn}$ is a square matrix, if

A) $m<n$ B) $m>n$ C) $m=n$ D) None

Solution: A matrix having number of rows and columns are equal is called a square matrix

Hence, $m = n$

This is matching with the option (C)

8. Which of the given values of x and y when $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

A) $x = -\frac{1}{3}, y = 7$ B) Not possible to find

C) $y = 7, x = -\frac{2}{3}$ D) $x = -\frac{1}{3}, y = \frac{2}{3}$

Solution: Given, $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

When two matrices are equal the corresponding entries are also equal

$$3x+7=0 \Rightarrow x = -\frac{7}{3}, y-2=5 \Rightarrow y=7$$

But by equating a_{22}, we get $2-3x=4 \Rightarrow x = -\frac{2}{3}$

But there are two different values of x, which is contradiction.

Hence, the above two matrices are not equal

This is matching with the option (B)

9. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

(A) 27 (B) 18 (C) 81 (D) 512

Solution: Given matrix is of the order 3×3 has 9 elements and each of these elements can be either 0 or 1

Now, each of the 9 elements can be filled in two possible ways.

Therefore, the required number of possible matrices is $2^9 = 512$

Exercise 3.2

1. Let $A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}$ Find each of the following

i) $A+B$

ii) $A-B$

iii) $3A-C$

iv) AB

v) BA

Solution: Given matrices are $A=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}, C=\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}$

i) The sum of two matrices is a matrix whose entries are equal to the sum of the corresponding entries

Consider $A+B$

$$A+B=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}+\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}2+1 & 4+3\\3-2 & 2+5\end{bmatrix}$$

$$=\begin{bmatrix}3 & 7\\1 & 7\end{bmatrix}$$

Therefore, $A+B=\begin{bmatrix}3 & 7\\1 & 7\end{bmatrix}$

ii) The difference of two matrices is a matrix whose entries are difference of corresponding entries.

Consider $A-B$

$$A-B=\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}-\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}2-1 & 4-3\\3+2 & 2-5\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1\\5 & -3\end{bmatrix}$$

Therefore, $A-B=\begin{bmatrix}1 & 1\\ 5 & -3\end{bmatrix}$

iii) Consider $3A-C$

$$3A-C=3\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$$
$$=\begin{bmatrix}6+2 & 12-5\\ 9-3 & 6-4\end{bmatrix}$$
$$=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$$

iv) Two matrices are said to multipliable, if the number of columns of the first matrix is equal to the number of rows of the second matrix.

Consider AB

$$A\cdot B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$$
$$=\begin{bmatrix}2-8 & 6+20\\ 3-4 & 9+10\end{bmatrix}$$
$$=\begin{bmatrix}-6 & 26\\ -1 & 19\end{bmatrix}$$

Therefore, $AB=\begin{bmatrix}-6 & 26\\ -1 & 19\end{bmatrix}$

v) Consider BA

$$B\cdot A=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\cdot\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$$
$$=\begin{bmatrix}2+9 & 4+6\\ -4+15 & -8+10\end{bmatrix}$$
$$=\begin{bmatrix}11 & 10\\ 11 & 2\end{bmatrix}$$

Therefore, $BA=\begin{bmatrix}11 & 10\\ 11 & 2\end{bmatrix}$

2. Compute the following

i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

Solution:

i) The sum of two matrices is defined as sum of the corresponding entries

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix} \backslash$$

ii) The sum of two matrices is defined as sum of the corresponding entries

$$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

iii) The sum of two matrices is defined as sum of the corresponding entries

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

iv) The sum of two matrices is defined as sum of its corresponding entries

$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} = \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

3. Compute the indicated products

i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Solution:

i) Two matrices are multipliable if the number of rows of the first matrix is equal to the number of columns of the second matrix

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ba + ab & b^2 + a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & b^2 + a^2 \end{bmatrix}$$

$$= \left(a^2 + b^2\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

ii) Two matrices are multipliable if the number of rows of the first matrix is equal to the number of columns of the second matrix

$$\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}=\begin{bmatrix}1\cdot2 & 1\cdot3 & 1\cdot4\\2\cdot2 & 2\cdot3 & 2\cdot4\\3\cdot2 & 3\cdot3 & 3\cdot4\end{bmatrix}=\begin{bmatrix}2 & 3 & 4\\4 & 6 & 8\\6 & 9 & 12\end{bmatrix}$$

iii) Two matrices are multipliable if the number of rows of the first matrix is equal to the number of columns of the second matrix

$$\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix}=\begin{bmatrix}-3 & -4 & 1\\8 & 13 & 9\end{bmatrix}$$

iv) Two matrices are multipliable if the number of rows of the first matrix is equal to the number of columns of the second matrix

$$\begin{bmatrix}2 & 3 & 4\\3 & 4 & 5\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & -3 & 5\\0 & 2 & 4\\3 & 0 & 5\end{bmatrix}==\begin{bmatrix}2+0+12 & -6+6+0 & 10+12+20\\3+0+15 & -9+8+0 & 15+16+25\\4+0+18 & -12+10+0 & 20+20+30\end{bmatrix}$$

$$==\begin{bmatrix}14 & 0 & 42\\18 & -1 & 56\\22 & -2 & 70\end{bmatrix}$$

v) Two matrices are multipliable if the number of rows of the first matrix is equal to the number of columns of the second matrix

$$\begin{bmatrix}2 & 1\\3 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\-1 & 2 & 1\end{bmatrix}=\begin{bmatrix}2\cdot1+1\cdot-1 & 2\cdot0+1\cdot2 & 2\cdot1+1\cdot1\\3\cdot1+2\cdot-1 & 3\cdot0+2\cdot2 & 3\cdot1+2\cdot1\\-1\cdot1+1\cdot-1 & -1\cdot0+1\cdot2 & -1\cdot1+1\cdot1\end{bmatrix}$$

$$=\begin{bmatrix}2-1 & 0+2 & 2+1\\3-2 & 0+4 & 3+2\\-1-1 & 0+2 & -1+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2 & 3\\1 & 4 & 5\\-2 & 2 & 0\end{bmatrix}$$

vi) Two matrices are multipliable if the number of rows of the first matrix is equal to the number of columns of the second matrix

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3\cdot 2-1\cdot 1+3\cdot 3 & 3\cdot -3-1\cdot 0+3\cdot 1 \\ -1\cdot 2+0\cdot 1+2\cdot 3 & -1\cdot -3+0\cdot 0+2\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

4. If $A=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, b=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C=\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B),(B-C)$. Also verify that $A+(B-C)=(A+B)-C$

Solution: The given matrices are $A=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, b=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C=\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

Consider the matrix $A+B$

$$A+B=\begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}+\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

Consider the matrix $B-C$

$$B-C=\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}-\begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

Consider the matrix $A+(B-C)$

$$A+(B-C)=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+\begin{bmatrix}-1&-2&0\\4&-1&3\\1&2&0\end{bmatrix}$$

$$=\begin{bmatrix}1+(-1)&2+(-2)&-3+0\\5+4&0+(-1)&2+3\\1+1&-1+2&1+0\end{bmatrix}$$

$$==\begin{bmatrix}0&0&-3\\9&-1&5\\2&1&1\end{bmatrix}$$

Consider the matrix $(A+B)-C$

$$(A+B)-C=\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix}+\begin{bmatrix}3&-1&2\\4&2&5\\2&0&3\end{bmatrix}-\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}$$

$$=\begin{bmatrix}4&1&-1\\9&2&7\\3&-1&4\end{bmatrix}-\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix}$$

$$=\begin{bmatrix}0&0&-3\\9&-1&5\\2&1&1\end{bmatrix}$$

Therefore, $A+(B-C)=(A+B)-C$

5. If $A=\begin{bmatrix}\frac{2}{3}&1&\frac{5}{3}\\\frac{1}{3}&\frac{2}{3}&\frac{4}{3}\\\frac{7}{3}&2&\frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5}&\frac{3}{5}&1\\\frac{1}{2}&\frac{2}{5}&\frac{4}{5}\\\frac{7}{5}&\frac{6}{5}&\frac{2}{5}\end{bmatrix}$ the compute $3A-5B$

Solution: The given matrices are $A=\begin{bmatrix}\frac{2}{3}&1&\frac{5}{3}\\\frac{1}{3}&\frac{2}{3}&\frac{4}{3}\\\frac{7}{3}&2&\frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5}&\frac{3}{5}&1\\\frac{1}{2}&\frac{2}{5}&\frac{4}{5}\\\frac{7}{5}&\frac{6}{5}&\frac{2}{5}\end{bmatrix}$

Consider the matrix $3A-5B$

$$3A-5B=3\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}-5\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{2} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Therefore, $3A-5B$ is null matrix.

6. Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Solution: Consider the matrix expression $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Simplifying the above expression as

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Therefore, $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

7. Find the matrices X, Y if

i. $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

ii. $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

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Solution:

i) The given matrices are $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

Adding the above two equations

$$2X=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$
$$=\begin{bmatrix}7+3 & 0+0\\ 2+0 & 5+3\end{bmatrix}$$
$$=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

Hence, $X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$

And

$$\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$
$$Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$
$$=\begin{bmatrix}7-5 & 0-0\\ 2-1 & 5-4\end{bmatrix}$$
$$=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

Therefore, $X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$ and $Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$

ii) Consider the equations $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

Consider the matrix equation: $2(2X+3Y)-3(3X+2Y)=-5X$

Hence,

$$X=-\frac{1}{5}\left(2(2X+3Y)-3(3X+2Y)\right)$$

$$=-\frac{1}{5}\left\{\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix}-\begin{bmatrix}6 & -6\\ -3 & 15\end{bmatrix}\right\}$$

$$=-\frac{1}{5}\begin{bmatrix}-2 & 12\\ 11 & -15\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{5} & -\frac{12}{5}\\ -\frac{11}{5} & 3\end{bmatrix}$$

Substitute the matrix X in the equation $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$

Hence,

$$3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}-\begin{bmatrix}\frac{4}{5} & -\frac{24}{5}\\ -\frac{22}{5} & 6\end{bmatrix}$$

$$3Y=\begin{bmatrix}2-\frac{4}{5} & 3+\frac{24}{5}\\ 4+\frac{22}{5} & 0-6\end{bmatrix}$$

$$=\begin{bmatrix}\frac{6}{5} & \frac{39}{5}\\ \frac{42}{5} & -6\end{bmatrix}$$

$$Y=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$$

8. Find X, if $Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

Solution: Given $Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

$$2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$2X+\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$2X=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$$

$$X=\frac{1}{2}\begin{bmatrix}1-3 & 0-2\\ -3-1 & 2-4\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

9. Find x and y, if $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

Solution: Given $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

It implies that $\begin{bmatrix}2 & 6\\ 0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}\Rightarrow\begin{bmatrix}2+y & 6\\ 1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

Since two matrices are equal, corresponding entries are equal.

$$2+y=5\Rightarrow y=3 \text{ and } 2x+2=8\Rightarrow x=3$$

10. Solve the equation for X, Y, Z and t if $2\begin{bmatrix}x & z\\ y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$

Solution:

Given, $2\begin{bmatrix}x & z\\ y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$

$$\Rightarrow\begin{bmatrix}2x & 2z\\ 2y & 2t\end{bmatrix}+\begin{bmatrix}3 & -3\\ 0 & 6\end{bmatrix}=\begin{bmatrix}9 & 15\\ 12 & 18\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2x+3 & 2z-3\\ 2y & 2t+6\end{bmatrix}=\begin{bmatrix}9 & 15\\ 12 & 18\end{bmatrix}$$

Equating the corresponding elements of these two matrices, we get

$2x+3=9$

$\Rightarrow 2x=6$

$\Rightarrow x=3$

$2y=12$

$\Rightarrow y=6$

$2z-3=15$

$\Rightarrow 2z=18$

$\Rightarrow z=9$

$2t+6=18$

$\Rightarrow 2t=12$

$\Rightarrow t=6$

$\therefore x=3, y=6, z=9 \text{ and } t=6$

11. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, find values of x and y

Solution:

$$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

Equating the corresponding elements of these two matrices, we get

$2x-y=10 \text{ and } 3x+y=5$

Adding these two equations, we have

$5x=15$

$\Rightarrow x=3$

Now, $3x+y=5$

$\Rightarrow y=5-3x$

$\Rightarrow y=5-9=4$

$\therefore x=3\ and\ y=-4$

12. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w

Solution:

Given, $3\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}=\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$

Equating the corresponding elements of these two matrices, we get

$3x = x+4$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

$3x = 6+x+y$

$\Rightarrow 2y = 6+x = 6+2 = 8$

$\Rightarrow y = 4$

$3w = 2w+3$

$\Rightarrow w = 3$

$3z = -1+z+w$

$\Rightarrow 2z = -1+w = -1+3 = 2$

$\Rightarrow z = 1$

$\therefore x=2, y=4, z=1\ and\ w=3$

13. If $F(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y)=F(x+y)$

Solution:

$$F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}, F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$F(x)F(Y)$$

$$=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y+0 & -\cos x\sin y-\sin x\cos y+0 & 0\\ \sin x\cos y+\cos x\sin y+0 & -\sin x\sin y+\cos x\cos y+0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=F(x+y)$$

$$\therefore F(x)F(y)=F(x+y)$$

14. Show that (i) $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\neq\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 1 & 1 & 0\end{bmatrix}$

Solution:

(i) $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$

$$=\begin{bmatrix}5(2)-1(3) & 5(1)-1(4)\\ 6(2)+7(3) & 6(1)+7(4)\end{bmatrix}$$

$$=\begin{bmatrix}10-3 & 5-4\\12+21 & 6+28\end{bmatrix}=\begin{bmatrix}7 & 1\\33 & 34\end{bmatrix}$$

$$\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$$

$$=\begin{bmatrix}2(5)+1(6) & 2(-1)+1(7)\\3(5)+4(6) & 3(-1)+4(7)\end{bmatrix}$$

$$=\begin{bmatrix}10+6 & -2+7\\15+24 & -3+28\end{bmatrix}=\begin{bmatrix}16 & 5\\39 & 25\end{bmatrix}$$

$$\therefore\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$$

(ii) $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}$

$$=\begin{bmatrix}1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4)\\0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4)\\1(-1)+1(0)+0(2) & 1(1)+1(-1)+(3) & 1(0)+1(1)+0(4)\end{bmatrix}$$

$$=\begin{bmatrix}5 & 8 & 14\\0 & -1 & 1\\-1 & 0 & 1\end{bmatrix}$$

Also, $\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

$$=\begin{bmatrix}-1(1)+1(0)+0(1) & -1(2)+1(-1)+0(1) & -1(3)+1(0)+0(0)\\0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0)\\2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -1 & -3\\1 & 0 & 0\\6 & 11 & 6\end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

15. Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Solution:

$$A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)3+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$=\begin{bmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-5+6 & -1+0 & -3+0\\ -1+0 & -7+6 & -10+0\\ -5+0 & 4+0 & -2+6\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1 & -3\\ -1 & -1 & -10\\ -5 & 4 & 4\end{bmatrix}$$

16. If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=0$

Solution:

$$A^2=AA=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}$$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}$$

Now $A^3=A^2.A$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5+0+16 & 0+0+0 & 10+0+24\\ 2+0+10 & 0+8+0 & 4+4+15\\ 8+0+26 & 0+0+0 & 16+0+39\end{bmatrix}$$

$$=\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\therefore A^3-6A^2+7A+21$$

$$=\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}-6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}+7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}+2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}-\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}+\begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix}+\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix}-\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$=\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}-\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=0$$

$$\therefore A^3-6A^2+7A+2I=0$$

17. If $A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2=kA-2I$

Solution:

$$A^2=A.A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 3(3)+(-2)(4) & 3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2) \end{bmatrix}=\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now $A^2=kA-2I$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Equating the corresponding elements, we have

$3k - 2 = 1$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

Thus, the value of k is 1

18. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Solution:

$I + A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \qquad \text{..........(1)}$$

$$(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$=\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}\right)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha+\sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha+\cos\alpha\tan\frac{\alpha}{2}\\ -\cos\alpha\tan\frac{\alpha}{2}+\sin\alpha & \sin\alpha\tan\frac{\alpha}{2}+\cos\alpha\end{bmatrix} \quad \text{.........(2)}$$

$$=\begin{bmatrix}1-2\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}\\ -\left(2\cos^2\frac{\alpha}{2}-1\right)\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2}\end{bmatrix}$$

$$=\begin{bmatrix}1-2\sin^2\frac{\alpha}{2}+2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}-\tan\frac{\alpha}{2}\\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\tan\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2}+1-2\sin^2\frac{\alpha}{2}\end{bmatrix}$$

$$=\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix}$$

Thus, from (1) and (2), we get L.H.S = R.H.S

19. A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of

(A) Rs 1,800 (B) Rs 2,000

Solution:

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second

bond pays Rs $(30000-x)$

It is given that the first bond pays 5% interest per year and the second bond pay 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have

$$\left[x\left(30000-x\right)\right]\begin{bmatrix}\frac{5}{100}\\ \frac{7}{100}\end{bmatrix}=1800 \quad \left[\because S.I\ \ for\ 1\ year=\frac{\Pr incipal\times Rate}{100}\right]$$

$$\Rightarrow\frac{5x}{100}+\frac{7\left(30000-x\right)}{100}=1800$$

$$\Rightarrow 5x+210000-7x=180000$$

$$\Rightarrow 210000-2x=180000$$

$$\Rightarrow 2x=210000-180000$$

$$\Rightarrow 2x=30000$$

$$\Rightarrow x=15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000-x)$.

Therefore, in order to obtain an annual total interest of Rs 2000, we have

$$\left[x\left(30000-x\right)\right]\begin{bmatrix}\frac{5}{100}\\ \frac{7}{100}\end{bmatrix}=2000$$

$$\Rightarrow\frac{5x}{100}+\frac{7\left(30000-x\right)}{100}=2000$$

$$\Rightarrow 5x+210000-7x=200000$$

$$\Rightarrow 210000-2x=200000$$

$$\Rightarrow 2x=210000-200000$$

$$\Rightarrow 2x=10000$$

$\Rightarrow x = 5000$

Therefore, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond.

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solution:

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as

$$12\begin{bmatrix}10 & 8 & 10\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix}$$

$$=12\left[10\times80+8\times60+10\times40\right]$$

$$=12(800+480+400)$$

$$=12(1680)$$

$$=20160$$

$\therefore$ The bookshop will receive Rs 20160 from the sale.

21. Assume X, Y, Z, W and P are the matrices of order $2\times n, 3\times k, 2\times p, n\times 3$ *and* $p\times k$ respectively. The restriction on n, k and p so that $PY+WY$ will be defined are

A. $k=3, p=n$

B. k is arbitrary, p = 2

C. p is arbitrary, k = 3

D. k =2, p = 3

Solution:

Matrices P and Y are of the orders $p\times k$ *and* $3\times k$ respectively.

Therefore, matrix PY will be defined if k = 3.

Also, PY will be of the order $p \times k$.

Matrices W and Y are of the orders $n \times 3$ *and* $3 \times k$ respectively.

Since the number of column is W is equal to the number of rows in Y, matrix WY is well – defined and is of the order $n \times k$

Matrices PY and WY can be added only when their orders are the same.

But, PY is of the order $p \times k$ and WY is of the order $n \times k$

Thus, we must have p = n

$\therefore$ k =3 and p = n. are the restrictions on n, k and p so that $PY + WY$ will be defined

22. Assume X, Y, Z, W and P are the matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ *and* $p \times k$ respectively. If n = p, then the order of the matrix $7X - 5Z$ is

A) $p \times 2$ B) $2 \times n$ C) $n \times 3$ D) $p \times n$

Solution:

Matrix X is of the order $2 \times n$.

Thus, matrix 7X is also of the same order.

Matrix Z is of the order $2 \times p$, i.e., $2 \times n$ [since n = p]

Therefore, matrix 5Z is also of the same order.

Now, both the matrices 7X 5Z are of the order $2 \times n$

Thus, matrix 7X -5Z is well – defined and is of the order $2 \times n$.

Exercise 3.3

1. If $A=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}$ $and\ B=\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}$, then verify that

(i) $(A+B)'=A'+B'$ (ii) $(A-B)'=A'-B'$

Solution:

We have $A'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}, B'=\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}$

(i) $A+B=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}=\begin{bmatrix}-5 & 3 & -2\\ 6 & 9 & 9\\ -1 & 4 & 2\end{bmatrix}$

$\therefore (A+B)'=\begin{bmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}$

$A'+B'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}=\begin{bmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}$

Thus proved $(A+B)'=A'+B'$

$(ii)\ A-B=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}-\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}=\begin{bmatrix}3 & 1 & 8\\ 4 & 5 & 9\\ -3 & -2 & 0\end{bmatrix}$

$\therefore (A-B)'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}-\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{bmatrix}$

Thus proved $(A-B)'=A'-B'$

2. If $A'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ $and\ b=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, then verify that

(i) $(A+B)'=A'+B'$ (ii) $(A-B)'=A'-B'$

Solution:

(i) Since $A=(A')'$

Thus, we have $A=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}$

$$B'=\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$$

$$A+B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{bmatrix}$$

$$\therefore (A+B)'=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$$

$$A'+B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$$

Hence proved $(A+B)'=A'+B'$

$$(ii)\ A-B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}4 & -3 & -1\\ 3 & 0 & -2\end{bmatrix}$$

$$\therefore (A-B)'=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$$

$$A'-B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$$

Hence proved $(A-B)'=A'-B'$

3. If $A'=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix}$ *and* $B=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$, then find $(A+2B)'$

Solution:

Since $A=(A')'$

$$\therefore A=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}$$

$$\therefore A+2B=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+2\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+\begin{bmatrix}-2 & 0\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 1\\ 5 & 6\end{bmatrix}$$

$$\therefore (A+2B)'=\begin{bmatrix}-4 & 5\\ 1 & 6\end{bmatrix}$$

4. For the matrices A and B, verify that $(AB)'=B'A'$ where

i) $A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}, B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$ ii) $A=\begin{bmatrix}0\\ 1\\ 2\end{bmatrix}, B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

Solution:

(i) $AB=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\end{bmatrix}=\begin{bmatrix}-1 & 2 & 1\\ 4 & -8 & -4\\ -3 & 6 & 3\end{bmatrix}$

$$\therefore (AB)'=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$$

Now, $A'=\begin{bmatrix}1 & -4 & 3\end{bmatrix}, B'=\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Thus proved $(AB)' = B'A'$

(ii) $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now, $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Thus $(AB)' = B'A'$

5. If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = 1$

(ii) $A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$, then verify that $A'A=1$

Solution:

$$A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$A'\text{A}=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Thus proved $A'A=1$

(ii) $A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Thus proved $A'A=1$

6. (i) Show that the matrix $A=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ is a skew symmetric matrix

Solution:

(i) We have $A'=\begin{bmatrix}1 & -1 & 5\\ -1 & 2 & 1\\ 5 & 1 & 3\end{bmatrix}=A$

$\because A'=A$

Thus, A is a symmetric matrix

(ii) We have $A'=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & -1\\ -1 & 1 & 0\end{bmatrix}=-\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}=-A$

$\because A'=-A$

Thus, A is a skew – symmetric matrix

7. For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that

(i) $(A+A')$ is a symmetric matrix

(ii) $(A-A')$ is a skew symmetric matrix

Solution:

$A'=\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$

(i) $A+A'=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}+\begin{bmatrix}1 & 6\\ 5 & 7\end{bmatrix}$

$=\begin{bmatrix}2 & 11\\ 11 & 14\end{bmatrix}$

$$\therefore (A+A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A+A'$$

Thus, $(A+A')$ is a symmetric matrix

(ii) $A-A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A-A') = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= -(A-A')$$

Thus, $(A-A')$ is a skew – symmetric matrix

8. Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Solution:

Given, $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$$A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$A+A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A+A')=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

Now, $A-A'=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}-\begin{bmatrix}0&-a&-b\\a&0&-c\\b&c&0\end{bmatrix}$

$$=\begin{bmatrix}0&2a&2b\\-2a&0&2c\\-2b&-2c&0\end{bmatrix}$$

$$=2\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A-A')=\begin{bmatrix}0&a&b\\-a&0&c\\-b&-c&0\end{bmatrix}$$

9. Express the following matrices as the sum of a symmetric and a skew symmetric matrix

(i) $\begin{bmatrix}3&5\\1&-1\end{bmatrix}$ (ii) $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$ (iii) $\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}$

(iv) $\begin{bmatrix}1&5\\-1&2\end{bmatrix}$

Solution:

Let $A=\begin{bmatrix}3&5\\1&-1\end{bmatrix}$, then $A'=\begin{bmatrix}3&1\\5&-1\end{bmatrix}$

Now, $A+A'=\begin{bmatrix}3&5\\1&-1\end{bmatrix}+\begin{bmatrix}3&1\\5&-1\end{bmatrix}$

$$=\begin{bmatrix}6 & 6\\ 6 & -2\end{bmatrix}$$

Let $P=\frac{1}{2}\left(A+A'\right)$

$$=\frac{1}{2}\begin{bmatrix}6 & 6\\ 6 & -2\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}$$

Now, $P'=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}=P$

$\therefore P=\frac{1}{2}\left(A+A'\right)$ is a symmetric matrix

Now, $A-A'=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}-\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}$

$$=\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}$$

Let $Q=\frac{1}{2}\left(A-A'\right)$

$$=\frac{1}{2}\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}$$

Now, $Q'=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}=-Q$

$\therefore Q=\frac{1}{2}\left(A-A'\right)$ is a skew- symmetric matrix

Thus, A as the sum of P and Q

$$P+Q=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}+\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}=A$$

(ii) Let $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$, then $A'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

Now, $A+A'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

$=\begin{bmatrix}12 & -4 & 4\\ -4 & 6 & -2\\ 4 & -2 & 6\end{bmatrix}$

$P=\frac{1}{2}(A+A')$

$=\frac{1}{2}\begin{bmatrix}12 & -4 & 4\\ -4 & 6 & -2\\ 4 & -2 & 6\end{bmatrix}$

$=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

Now, $P'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=P$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix

Now, $A-A'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

Let $Q=\frac{1}{2}(A-A')=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

Now, $Q'=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=-Q$

Thus $Q=\frac{1}{2}(A-A')$ is a skew- symmetric matrix

Thus, A as the sum of P and Q

$$P+Q=\begin{bmatrix}3 & \frac{1}{2} & \frac{-5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}+\begin{bmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ -\frac{5}{2} & 0 & 3\\ -\frac{3}{2} & -3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}$$

= A

(iv) Let $A=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}$, then $A'=\begin{bmatrix}1 & -1\\ 5 & 2\end{bmatrix}$

Now, $A+A'=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1\\ 5 & 2\end{bmatrix}$

$$=\begin{bmatrix}2 & 4\\ 4 & 4\end{bmatrix}$$

Let $P=\frac{1}{2}(A+A')=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$

Now, $P'=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}=P$

Hence, $P=\frac{1}{2}(A+A')$ is symmetric matrix.

Now, $A-A'=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1\\ 5 & 2\end{bmatrix}$

$$=\begin{bmatrix}0 & 6\\ -6 & 0\end{bmatrix}$$

Let $Q=\frac{1}{2}(A-A')=\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}$

Now $Q'=\begin{bmatrix}0 & -3\\ 3 & 0\end{bmatrix}=-Q$

Hence $Q=\frac{1}{2}(A-A')$ is a skew symmetric matrix

Thus, A as the sum of P and Q

$P+Q=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}+\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}$

$=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}$

= A

10. If A, B are symmetric matrices of same order, then AB – BA is a

A. Skew symmetric matrix B. Symmetric matrix

C. Zero matrix D. Identity matrix

Solution:

A and B are symmetric matrices, therefore, we have

$A'=A \; and \; B'=B$ ....(1)

Here, $(AB-BA)'=(AB)'-(BA)'$

$=B'A'-A'B'$

= BA – AB [by (1)]

$=-(AB-BA)$

$\therefore (AB-AB)'=-(AB-BA)$

Hence, $(AB-BA)$ is a skew symmetric matrix

11. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then $A+A'=I$, if the value of α is

A. $\frac{\pi}{6}$ B. $\frac{\pi}{3}$ C. n D. $\frac{3\pi}{2}$

Solution:

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$\Rightarrow A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

Now $A+A'=I$

$$\therefore \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have

$$\cos\alpha=\frac{1}{2}$$

$$\alpha=\cos^{-1}\left(\frac{1}{2}\right)$$

$$\therefore \alpha=\frac{\pi}{3}$$

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Exercise 3.4

1. Find the inverse of each of the matrices, if it exists $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Since $A = IA$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_2 \to \frac{1}{5} R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

2. Find the inverse of each of the matrices, if it exists $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Since A= IA

$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - R_1$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$

$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

3. Find the inverse of each of the matrices, if it exists $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Since, A = IA

$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

4. Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Since, A = IA

$$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \to -2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

5. Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

Since, $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 7R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow\begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}4 & -1\\ -\frac{7}{2} & 1\end{bmatrix}A$$

Applying $R_2 \to 2R_2$

$$\Rightarrow\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}A$$

$$\therefore A^{-1}=\begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}$$

6. Find the inverse of each of the matrices, if it exists. $\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$

Solution:

Let $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$

Since, A = IA

$$\therefore\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\Rightarrow\begin{bmatrix}1 & \frac{5}{2}\\ 0 & 3\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 - R_1$

$$\Rightarrow\begin{bmatrix}1 & \frac{5}{2}\\ 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ -\frac{1}{2} & 1\end{bmatrix}A$$

Applying $R_1 \to R_2 - 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \to 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

7. Find the inverse of each matrices, if exists. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Since, A = IA

$$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $C_1 \to C_2 - 2C_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 - C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^{-1}=\begin{bmatrix}2 & -1\\ -5 & 3\end{bmatrix}$$

8. Find the inverse of each the matrices, if it exists $\begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix}$

Solution:

Let $A=\begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix}$

Since, A= IA

$$\therefore \begin{bmatrix}4 & 5\\ 3 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix}1 & 1\\ 3 & 4\end{bmatrix}=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 - 3R_1$

$$\Rightarrow \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -1\\ -3 & 4\end{bmatrix}A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}4 & -5\\ -3 & 4\end{bmatrix}A$$

$$\therefore A^{-1}=\begin{bmatrix}4 & -5\\ -3 & 4\end{bmatrix}$$

9. Find the inverse of each of the matrices, if it exists. $\begin{bmatrix}3 & 10\\ 2 & 7\end{bmatrix}$

Solution:

Let $A=\begin{bmatrix}3 & 10\\ 2 & 7\end{bmatrix}$

Since, A = IA

$$\therefore \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$$

Applying $R_1 \to R_2 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

10. Find the inverse of each of the matrices, if it exists $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

Since, A = AI

$$\therefore \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $C_1 \to C_1 + 2C_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 + C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$$

Applying $C_2 \to \frac{1}{2} C_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

11. Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

Since, $A = AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $C_2 \to C_2 + 3C_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix}$$

Applying $C_1 \to \frac{1}{2}C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

12. Find the inverse of each of the matrices, if it exists. $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Since, A = IA

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{6} R_1$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A$$

Since, we can see all the zeros in the second row of the matrix on the L.H.S

Therefore, A^{-1} does not exist.

13. Find the inverse of each of the matrices, if it exists $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Since, A= IA

$$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

14. Find the inverse of each of the matrices, if it exist. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

Since, A= IA

$$\therefore \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - \frac{1}{2} R_2$

$$\begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

Since, we can see all the zeros in the first row of the matrix on the L.H.S

Therefore, A^{-1} does not exist

15. Find the inverse of each of the matrices, if it exists $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Since, A = IA

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 3R_1$ *and* $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 3R_3$ *and* $R_2 \to R_2 + 8R_3$

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \to \frac{1}{25} R_3$

$$\begin{bmatrix}1 & 0 & 10\\0 & 1 & 21\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-5 & 0 & 3\\-13 & 1 & 8\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}A$$

Applying $R_1 \to R_1 - 10R_3$, *and* $R_2 \to R_2 - 21R_3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\-\frac{2}{5} & \frac{4}{25} & \frac{11}{25}\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}A$$

$$\therefore A^{-1}=\begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\-\frac{2}{5} & \frac{4}{25} & \frac{11}{25}\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}$$

16. Find the inverse of each of the matrices, if it exists $\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$

Solution:

Let $A=\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$

Since $A = IA$

$$\therefore \begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 - 2R_3$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3, \textit{ and } R_2 \to R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

17. Matrices A and B will be inverse of each other only if

A. AB = BA

B. AB=0,BA=I

C. AB=BA=0

D. AB=BA=I

Solution:

Since, if A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB=BA=I, then B is said to be the inverse of A. In such case, it is clear that A is the inverse of B.

Thus, matrices A and B will be inverse of each other only if AB = BA= I