Chapter 1: Sets

EXERCISE 1.1

1. Which of the following are sets? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world.

Solution

(i) The collection consists of months January, June and July. It is well-defined and therefore, it's a group.

(ii) A writer of India could also be most talented for one person but not for an additional person. Opinion varies from person to person. So, the given collection isn't well-defined and thus, not a set.

(iii) The term 'best cricket batsman' is vague. The same batsman could also be one among the simplest for one person but not for an additional. Opinion varies from person to person. So, the given collection isn't well-defined and thus, not a set.

(iv) Any boy is either in your class or not in your class. There is no ambiguity. The given collection is well-defined and therefore, it's a group.

(v) The collection consists of first 99 natural numbers. It is well-defined and therefore, it is a set.

(vi) it's a well-defined collection and thus, it is a set.

(vii) it's a well-defined collection and thus , it is a set.

(viii) It is a well-defined collection and therefore, it is a set.

(ix) The criterion for determining an animal as most dangerous varies from person to person. For some people, even a lizard is extremely dangerous. So, the given collection isn't well defined and thus , it's not a group ..

2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

(i) $5 \ldots A$

(ii) $8 \ldots A$

(iii) $0 \ldots A$

(iv) $4 \ldots A$

(v) $2 \ldots A$

(vi) $10 \ldots . A$

Solution

Given, $A = \{1, 2, 3, 4, 5, 6\}$

So, Find the symbols that matches

(i) 5 is in set A

So, $5 \in A$

(ii) 8 is not in set A

So, $8 \notin A$

(iii) 0 is not in set A

So, $0 \notin A$

(iv) 4 is in set A

Hence, $4 \in A$

(v) 2 is not in set A

Hence, $2 \in A$

(vi) 10 is not in set A

Hence, $10 \notin A$.

3. Write the following sets in roster form:

(i) $A = \{x : x$ **is an integer and** $-3 \leq x < 7\}$

(ii) $B = \{x : x$ **is a natural number less than 6** $\}$

(iii) $C = \{x : x$ **is a two-digit natural number such that the sum of its digits is 8** $\}$

(iv) $D = \{x : x$ **is a prime number which is divisor of 60** $\}$

(v) $E =$ **The set of all letters in the word TRIGONOMETRY**

(vi) $F =$ **The set of all letters in the word BETTER**

Solution

(i) The integers are $-3,-2,-1,0,1,2,3,4,5,6$ (not 7).

$\therefore$ In roster form, $A=\{-3,-2,-1,0,1,2,3,4,5,6\}$

(ii) Natural numbers less than 6 are $1,2,3,4,5$.

$\therefore$ In roster form, $B=\{1,2,3,4,5\}$

(iii) The numbers are $17,26,35,44,53,62,71,80$.

$\therefore$ In roster form, $C=\{17,26,35,44,53,62,71,80\}$

(iv) Divisors of 60 are $1,2,3,4,5,6,10,12,15,20,30,60$.

Among them, prime numbers are $2,3,5$.

$\therefore$ In roster form, $D=\{2,3,5\}$

(v) In the word TRIGONOMETRY, the letters T, R and O are repeat. Dropping the repetitions, in roster form $E=\{T,R,I,G,O,N,M,E,Y\}$

(vi) In the word BETTER, the letters E and T are repeated. Dropping the repetitions, in roster form $F=\{B,E,T,R\}$

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4. Write the following sets in the set-builder form:

(i) $(3,6,9,12\}$

(ii) $\{2,4,8,16,32\}$

(iii) $\{5,25,125,625\}$

(iv) $\{2,4,6,\ldots\}$

(v) $\{1,4,9,\ldots,100\}$

Solution

(i) $\{3,6,9,12\}=\{3\times1,3\times2,3\times3,3\times4\}$

$=\{x:x=3n,n\in N \text{ and } 1\le n\le4\}$

(ii) $\{2,4,8,16,32\}=\left\{2^1,2^2,2^3,2^4,2^5\right\}$

$=\left\{x:x=2^n,n\in N \text{ and } 1\le n\le5\right\}$

(iii) $\{5,25,125,625\}=\left\{5^1,5^2,5^3,5^4\right\}$

$=\left\{x:x=5^n,n\in N \text{ and } 1\le n\le4\right\}$

(iv) $\{2,4,6,\ldots\}=\{2\times1,2\times2,2\times3,\ldots\}$

$= \{x : x = 2n, n \in N\}$

Alternatively, we can write $\{x : x$ is an even natural number $\}$.

(v) $\{1, 4, 9, \ldots, 100\} = \{1^2, 2^2, 3^2, \ldots\ldots, 10^2\}$

$= \{x : x = n^2, n \in N$ and $1 \le n \le 10\}$

5. List all the elements of the following sets:

(i) $A = \{x : x$ **is an odd natural number** $\}$

(ii) $B = \left\{x : x \text{ is an integer, } -\frac{1}{2} < x < \frac{9}{2}\right\}$

(iii) $C = \{x : x$ **is an integer,** $x^2 \le 4\}$

(iv) $D = \{x : x$ **is a letter in the word "LOYAL"** $\}$

(v) $E = \{x : x$ **is a month of a year not having 31 days** $\}$

(vi) $F = \{x : x$ **is a consonant in the English alphabet which precedes** $k\}$**.**

Solution

(i) Odd natural numbers are $1, 3, 5, \ldots$

$\therefore \quad A = \{1, 3, 5, \ldots\}$

(ii)Integers greater than $-\frac{1}{2}$ and less than $\frac{9}{2}$ are $0, 1, 2, 3, 4$

$\therefore \quad B = \{0, 1, 2, 3, 4\}$

(iii) Integers whose square is less than or equal to 4 are -2,

$-1, 0, 1, 2$

$\therefore C = \{-2, -1, 0, 1, 2\}$

(iv) Dropping the repetition $D = \{L, O, Y, A\}$

(v) Months of a year not having 31 days are: February, April, June, September, November

$\therefore E = \{February, April, June, September, November\}$

(vi) Consonants in the English alphabet which precede k are:

b, c, d, f, g, h, j

$\therefore F = \{b, c, d, f, g, h, j\}$

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) $\{1, 2, 3, 6\}$	(a) $\{x : x$ is a prime number and a divisor of $6\}$
(ii) $\{2, 3\}$	(b) $\{x : x$ is an odd natural number less than 10 $\}$
(iii) $\{M, A, T, H, E, I, C, S\}$	**(c)** $\{x : x$ **is natural number and divisor of 6** $\}$
(iv) $\{1, 3, 5, 7, 9\}$	**(d)** $\{x : x$ **is a letter of the word MATHEMATICS** $\}$**.**

Solution

(i) All the elements of the set are natural numbers similarly well-being the divisors of 6.

Hence, (i) matches with (c).

(ii) It can see the 2 and 3 are prime numbers. They include the divisors of 6.

So, (ii) matches with (a).

(iii) All the elements of the set are letters of the expression MATHEMATICS.

Hence, (iii) matches with (d).

(iv) All the elements of the set are odd natural numbers less than 10.

Hence, (iv) matches with (b).

Example 6

State which of the following sets are finite or infinite:

(i) $\{x : x \in N$ **and** $(x-1)(x-2) = 0\}$

(ii) $\{x : x \in N$ **and** $x^2 = 4\}$

(iii) $\{x : x \in N$ **and** $2x - 1 = 0\}$

(iv) $\{x : x \in N$ **and** x **is prime** $\}$

(v) $\{x : x \in N$ **and** x **is odd** $\}$

Solution

(i) $\{x : x \in N$ **and** $(x-1)(x-2) = 0\}$

$(x-1)(x-2) = 0$

Hence, $x = 1, 2$

Given set $= \{1, 2\}$.

Since the elements has 2 sets

Hence, it is finite.

(ii) $\{x : x \in \mathrm{N}$ **and** $x^2 = 4\}$

$x^2 = 4$

$x = \pm 2$

The x be the natural numbers

Given set $= \{2\}$.

Hence, it is finite.

(iii) $\{x : x \in \mathrm{N}$ and $2x - 1 = 0\}$

$2x - 1 = 0$

$x = \frac{1}{2}$

But x should be a natural number

It cannot be fraction.

The given set has no elements.

Given set $= \phi$.

Since the number of elements of a null set is 0 ,

Hence, it is finite.

(iv) $\{x : x \in \mathrm{N}$ and x is prime $\}$

A given set is a set of all key numbers and as a set of key numbers is unlimited. So, the given set is infinite

(v) $\{x : x \in \mathrm{N}$ and x is odd $\}$

As there is an infinite number of odd numbers, therefore, the set given is infinite.

Example 7

Find the pairs of equal sets, if any, give reasons:

$\mathrm{A} = \{0\},$

$\mathrm{B} = \{x : x > 15$ and $x < 5\}$

$C = \{x : x - 5 = 0\}$,

$D = \{x : x^2 = 25\}$

$E = \{x : x$ **is an integral positive root of the equation** $x^2 - 2x - 15 = 0\}$

Solution

Since $0 \in A$ and 0 does not belong to any of the B, C, D and E sets, it follows, $A \neq B, A \neq C, A \neq D, A \neq E$.

From $B = \phi$ but none no other sets are empty. So $B \neq C, B \neq D$ and $B \neq E$.

And $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.

From $E = \{5\}, C = E$. Additionally $D = \{-5, 5\}$ and $E = \{5\}$, we find that, $D \neq E$. Thus, the only pair of equal sets is C and E.

Example 8

Which of the following pairs of sets are equal? Justify your answer.

(i) X **, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".**

(ii) $A = \{n : n \in Z$ **and** $n^2 \leq 4\}$ **and** $B = \{x : x \in R$ **and** $x^2 - 3x + 2 = 0\}$**.**

Solution

(i) $X = \{A, L, L, O, Y\}, B = \{L, O, Y, A, L\}$. Then X and B are equal sets as repetition of elements in a set does not change a set.

Every element of X is same as every element of B.

So, $X = \{A, L, O, Y\} = B$

(ii) $A = \{n : n \in Z$ **and** $n^2 \leq 4\}$

Combine the terms

$n^2 = 4$

$n = \pm 2$

So, $-2 \leq n \leq 2$

$A = \{-2, -1, 0, 1, 2\}$,

Similarly, $B = \{x : x \in R$ **and** $x^2 - 3x + 2 = 0\}$**.**

$x^2 - 3x + 2 = 0$

Solving we get

$(x-2)(x-1)=0$

$x=2$ or $x=1$

$B=\{1,2\}$

Since $0 \in A$ and $0 \notin B, A$ and B are not equal sets.