Infinity Learn by Sri Chaitanya Educational Institutions

Chapter: 1. Relations and functions

Exercise 1.1

1. Determine whether each of the following relations are reflexive, symmetric and transitive

(i) Relation R in the set $A=\{1,2,3,...14\}$ defined as $R=\{(x,y):3x-y=0\}$

(ii) Relation R in the set N as $R=\{(x,y):y=x+5 \text{ and } x<4\}$

(iii) Relation R in the set $A=\{1,2,3,4,5,6\}$ as $R=\{(x,y):y \text{ is divisible by } x\}$

(iv) Relation R in the set Z of all integers as $R=\{(x,y):x-y \text{ is an integer}\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

a. $R=\{(x,y):x,y \text{ work at the same place}\}$

b. $R=\{(x,y):x,y \text{ live in the same locality}\}$

c. $R=\{(x,y):x \text{ is } 7\text{ cm taller than } y\}$

d. $R=\{(x,y):x \text{ is wife of } y\}$

e. $R=\{(x,y):x \text{ is father of } y\}$

Solution: A relation R is defined on the set A is said to be

- Reflexive relation if $(a,a)\in R$ for all $a\in A$
- Symmetric relation if $(a,b)\in R \Rightarrow (b,a)\in R$
- Transitive relation if $(a,b)\in R,(b,c)\in R \Rightarrow (a,c)\in R$

(i) Given a relation R in the set $A=\{1,2,3,...14\}$ defined as $R=\{(x,y):3x-y=0\}$

The relation R in the roster form is $R=\{(1,3),(2,6),(3,9),(4,12)\}$,

Observing the above relation,

- It is not reflexive because $(1,1)\notin A$

- It is not symmetric because $(1,3) \in A$ and it does not implies that $(3,1) \notin A$
- If is not transitive because $(1,3),(3,9) \in A$ but $(1,9) \notin A$

Therefore, the relation R is neither Reflexive, nor symmetric nor transitive.

(ii) Relation R in the set N as $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$

The relation R in the roster form is $R = \{(1,6),(2,7),(3,8)\}$,

Observing the above relation,

- It is not reflexive because $(1,1) \notin R$
- It is not symmetric because $(1,6) \in R$ and it does not implies that $(6,1) \notin R$
- It is transitive because $(1,6),(2,7),(3,8) \in R$ but there is not image for $6, 7, 8$

Therefore, the relation R is neither reflexive nor symmetric, but it is transitive.

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(x, y) : y \text{ is divisible by } x\}$

Observing the above relation,

- It is reflexive because $(x, x) \in R$, since x is divisible by x
- It is not symmetric because $(x, y) \in R$ and it does not implies that x $(y, x) \notin R$, it means x is divisible by y does not implies that y divisible by x
- If x is divisible by y and y is divisible by z it implies that x is divisible by z, For any $(x, y),(y, z) \in R$ it implies that $(x, z) \in R$. Hence, the relation R is transitive

Therefore, the relation R is reflexive and transitive but not symmetric.

(iv) Relation R in the set Z of all integers as $R = \{(x, y) : x - y \text{ is an integer}\}$

- The relation R is reflexive because 0 is an integer, so that $(x, x) \in R$

- The relation R is symmetric because for any integer a, $-a$ is also integer so that for any $(x,y)\in R$ it implies that $(y,x)\in R$.
- The relation R is transitive because for any $(x,y),(y,z)\in R$ implies that $(x,z)\in R$

  The ordered pairs $(x,y),(y,z)\in R$, implies that

  $x-y$ is an integer and $y-z$ is an integer

  Sum of the above two values $x-y+y-z=x-z$ is also an integer. Hence, $(x,y),(y,z)\in R$ implies that $(x,z)\in R$

Therefore, the relation R is reflexive, symmetric and transitive, it means that the relation R is equivalence relation.

v) Suppose that the relation R in the set A of human beings in a town at a particular time

a. Given that $R=\{(x,y): x,y \text{ work at the same place}\}$

- It is reflexive, because $(x,x)\in R$ each person, himself work at the same place
- It is Symmetric, because if (x,y) working in the same place, then (y,x) also work in the same place
- It is transitive, because if x,y working in the same place and y,z also working in the same place then x,z work in the same place
- Therefore, the relation R is reflexive, symmetric and transitive. It implies that the relation R is equivalence relation.

b. $R=\{(x,y): x,y \text{ live in the same locality}\}$

- The relation R is reflexive, because $(x,x)\in R$, each person, himself lives in the same locality

- It is Symmetric, because if (x, y) staying in the same place, then (y, x) also stay in the same place
- It is transitive, because if x, y staying in the same place and y, z also staying in the same place then x, z staying in the same place
- Therefore, the relation R is reflexive, symmetric and transitive. It implies that the relation R is equivalence relation.

c. $R = \{(x, y): x \text{ is } 7\text{cm taller than } y\}$

- It is not reflexive, no person is 7 cm taller than him self
- If is not symmetric, if a person is 7 cm taller than other person, the other person is not 7 cm taller than first one
- It is not transitive. If $(x, y), (y, z)$ belongs to the relation means y is 7 cm taller than x and z is 7 cm taller than y it concludes that z is 14 cm taller than x

  Therefore, (x, z) is not an element of R

d. $R = \{(x, y): x \text{ is wife of } y\}$

- It is not reflexive, because no person is wife itself
- It is not symmetric, because if x is wife of y then y is not wife of x
- It is transitive, because if x is wife of y then y is male, so y is not wife of any one

e. $R = \{(x, y): x \text{ is father of } y\}$

- It is not reflexive, because no person is father to himself
- It is not symmetric, because if x is father of y then y is not father of x
- It is not transitive, because if x is father of y and y is father of z then x is not father of z

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2. Show that the relation R in $\mathbb{R}$, defined as $R=\{(a,b):a\le b^2\}$ is neither reflexive nor symmetric nor transitive.

Solution: Given that the relation is defined as $R=\{(a,b):a\le b^2\}$

- Relation is not reflexive, $\frac{1}{2}$ is a real numbers, the square of $\frac{1}{2}$ is $\frac{1}{4}$

  But $\frac{1}{2}\le\frac{1}{4}$ is false, hence $\left(\frac{1}{2},\frac{1}{2}\right)\notin R$

  Hence, the relation is not reflexive.

- Relation is not symmetric, $\left(\frac{1}{2},1\right)\in R$ it implies that $\frac{1}{2}\le 1^2$ but $\left(1,\frac{1}{2}\right)$ does not belongs to R because $1\le\frac{1}{4}$ is false.

- Relation is not transitive, $(2,1)\in R$ and $(1,1.1)\in R$ but $(2,1.1)$ does not belongs to R because $2\ge 1.21$

Therefore, the relation R is neither reflexive, not symmetric nor transitive relation.

3. Check whether the relation R defined in set $\{1,2,3,4,5,6\}$ as $R=\{(a,b):b=a+1\}$ is reflexive, symmetric or transitive

Solution: Given that the relation R defined in set $\{1,2,3,4,5,6\}$ as $R=\{(a,b):b=a+1\}$

The roster form of the relation is $R=\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

Observing the set,

- $(1,1)\notin R$, so that the relation R is not reflexive.
- $(1,2)\in R$ but $(2,1)\notin R$, so that R is not symmetric
- In the relation R, $(1,2),(2,3)\in R$, but $(1,3)\notin R$. So that the relation is not transitive.

Therefore, the relation R is neither reflexive, nor symmetric nor transitive relation.

4. Show that the relation R in $\mathbb{R}$ defined as $R=\{(a,b):a\le b\}$, is reflexive and transitive but not symmetric.

Solution:

The relation R is defined as $R=\{(a,b):a\le b\}$

- For any real number a, $a\le a$, so that $(a,a)\in R$

  Hence the relation R is reflexive.

- Suppose that $(1,2)\in R$ but $(2,1)\notin R$ because $2>1$

  So that the relation R is not symmetric

- Suppose that $(a,b),(b,c)\in R$

  Hence, $a\le b, b\le c \Rightarrow a\le c$, it implies that $(a,c)\in R$

  Hence, the relation R is transitive.

Therefore, the relation R is reflexive, not symmetric and transitive.

5. Check whether the relation R in $\mathbb{R}$ defined as $R=\{(a,b):a\le b^3\}$ is reflexive, symmetric or transitive.

Solution:

The given relation is $R=\{(a,b):a\le b^3\}$

- $\left(\frac{1}{2},\frac{1}{2}\right)\notin R$ because $\frac{1}{2}\ge\frac{1}{8}$, so that the relation is not reflexive.

- We know that $(1,2)\in R$ but $(2,1)\notin R$ because $2\le 1$ is false. So that the relation is not symmetric.

- We know that $(3,2)\in \mathrm{R},(2,1.5)\in \mathrm{R}$ but $3\le(1.5)^3$ is false. So that the relation is not transitive.

Therefore, the relation is not reflexive, not symmetric, and not transitive.

6. Show that the relation R in the set $\{1,2,3\}$ given by $R=\{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive

Solution: Given that the relation $R=\{(1,2),(2,1)\}$ is defined on the set $A=\{1,2,3\}$

Observing the relation $R=\{(1,2),(2,1)\}$

- $(1,1),(2,2),(3,3)$ does not belongs to the relation R, so that the relation R is not reflexive relation
- $(1,2)\in R$ and $(2,1)\in R$ hence the relation is symmetric
- $(1,2)\in R$ and $(2,1)\in R$ but $(1,1)$ does not belongs to R, hence the relation is not transitive.

Therefore, the relation R is not reflexive, symmetric but not transitive

7. Show that the relation R in the set A of all the books in a library of a college, given by $R=\{(x,y): x$ and y have the same number of pages$\}$ is an equivalence relation.

Solution: Given that the relation $R=\{(x,y): x$ and y have the same number of pages$\}$

- For any $x\in A$, $(x,x)\in R$ because x and x have the same number of pages, so that the relation is reflexive.
- Suppose that $(x,y)\in R$, it implies that both x,y have the same number of pages. Hence $(y,x)\in R$. Hence the relation is symmetric.
- Suppose that $(x,y)\in R,(y,z)\in R$, it implies that both x,y have the same number of pages, and both y,z have the same number of pages Hence, both x,z have the same number of pages, hence $(x,z)\in R$. Hence the relation is transitive.

Therefore, the relation is reflexive, symmetric and transitive. So that the relation is equivalence relation

8. Show that the relation R in the set $A=\{1,2,3,4,5\}$ given by $R=\{(a,b):|a-b| \text{ is even}\}$ is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each

other and all the elements of $\{2,4\}$ are related to each other. But no element of $\{1,3,5\}$ is related to any element of $\{2,4\}$.

Solution: Given that the relation R in the set A is defined as d $R=\{(a,b):|a-b| \text{is even}\}$

- For any element $a \in A$, $a-a=0$ is an even number, hence $(a,a)\in R$, so that the relation R is reflexive
- Suppose that $(a,b)\in R$,

  Hence, $|a-b|=2k \Rightarrow |b-a|=2k$, It implies that $(b,a)\in R$

  Hence, the relation R is symmetric.
- Suppose that $(a,b)\in R,(b,c)\in R$, Hence, $|a-b|=2p,|b-c|=2q$

  Adding the above two equations

$$|a-b+b-c|=2p+2q$$
$$|a-c|=2(p+q)$$
$$|c-a|=2(p+q)$$

It implies that $(a,c)\in R$, Hence, the relation R is transitive.

Therefore, the relation is equivalence.

All elements of $\{1,3,5\}$ are related to each other because they are all odd. So, the modulus of the difference between any two elements is even.

Similarly, all elements of $\{2,4\}$ are related to each other because they are all even.

Because the difference of two odd numbers or the difference of two even numbers is even, so that no element of {1, 3, 5} can be related to any element of {2, 4} and vice versa.

9. Show that each of the relation R in the set $A=\{x\in z;\ 0\le x\le 12\}$ given by

   (i) $R=\{(a,b):|a-b| \text{is multiple of } 4\}$

   (ii) $R=\{(a,b):a=b\}$ is an equivalence relation.

Find the set of all elements related to 1 in each case.

Solution: Consider the set $A=\{0,1,2,3,4,5,6,7,8,9,10,11,12\}$

(i) Relation is defined as $R=\{(a,b):|a-b|\text{is multiple of }4\}$

- For any $a\in A$, $|a-a|=0$ is multiple of 4, so that (a,a) is an element of R is reflexive
- Suppose that $(a,b)\in R$, it means $|a-b|$ is multiple of 4, $\Rightarrow|a-b|=4k$

  It implies $|b-a|=4k\Rightarrow(b,a)\in R$

  Hence the relation R is symmetric
- Suppose that $(a,b)\in R,(b,c)\in R$

  $$\Rightarrow|a-b|=4p,|b-c|=4q$$

  Adding the above two equations

  $$\Rightarrow|a-b+b-c|=4(p+q)$$
  $$\Rightarrow|a-c|=4\mathrm{Q}$$

  It implies that $(a,c)\in R$

  Hence, the relation R is transitive.

Therefore, the relation is equivalence relation.

The set of all elements which are related to 1 are $\{1,5,9\}$

(ii) Relation is defined as $R=\{(a,b):a=b\}$

- For any $a\in A$, $a=a$, so that (a,a) is an element of R is reflexive
- Suppose that $(a,b)\in R$, it means $a=b$ $b=a$

  It implies $(b,a)\in R$

  Hence the relation R is symmetric
- Suppose that $(a,b)\in R,(b,c)\in R$

  $$\Rightarrow a=b,b=c\Rightarrow a=c$$

  It implies that $(a,c)\in R$

  Hence, the relation R is transitive.

Therefore, the relation is equivalence relation.

The set of all elements which are related to 1 is $\{1\}$

10. Given an example of a relation. Which is

 a. Symmetric but neither reflexive nor transitive
 b. Transitive but neither reflexive nor symmetric.
 c. Reflexive and symmetric but not transitive.
 d. Reflexive and transitive but not symmetric.
 e. Symmetric and transitive but not reflexive

Solution:

(a) Suppose that $A=\{1,2,3\}$ and relation R is defined as $R=\{(1,2),(2,1)\}$

This is an example for the relation which is symmetric but not reflexive and not transitive.

(b) Suppose that $A=\{1,2,3\}$ and relation R is defined as $R=\{(1,2)\}$

This is an example for the relation which is transitive but not reflexive and not symmetric.

(c) Suppose that $A=\{1,2,3\}$ and relation R is defined as

$$R=\{(1,1),(2,2),(3,3),(1,3),(3,2),(3,1),(2,3)\}$$

This is an example for the relation which is reflexive, symmetric but not transitive.

(d) Suppose that $A=\{1,2,3\}$ and relation R is defined as

$$R=\{(1,1),(2,2),(3,3),(1,3)\}$$

This is an example for the relation which is reflexive, transitive but not symmetric.

(e) Suppose that $A=\{1,2,3\}$ and relation R is defined as

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$$R=\{(3,1),(1,3),(1,1)\}$$

This is an example for the relation which is symmetric, transitive but not reflexive.

11. Show that the relation R in the set A of points in a plane given by $R=\{(P,Q):\mathrm{OP}=\mathrm{OQ}\}$, is an equivalence relation. Further, show that the set of all point related to a point $P\neq(0,0)$ is the circle passing through P with origin as centre

Solution:

Consider the set A of points in a plane

The relation R on the set A is defined as $R=\{(P,Q):\mathrm{OP}=\mathrm{OQ}\}$

- For any $P\in A$, $OP=OP$, so that (P,P) is an element of R, so that R is reflexive
- Suppose that (P,Q), it means $OP=OQ$ it gives $OQ=OP$

  It implies $(Q,P)\in R$

  Hence the relation R is symmetric
- Suppose that $(P,Q)\in R,(Q,S)\in R$

  $$\Rightarrow OP=OQ,OQ=OS$$
  $$\Rightarrow OP=OS$$

  It implies that $(P,S)\in R$

  Hence, the relation R is transitive.

Therefore, the relation is equivalence relation.

The set of all elements which are related to P is the set of all points on the circle having Centre at origin and radius OP

12. Show that the relation R defined in the set A of all triangles as $R=\{(T_1,T_2):T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle

triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2, T_3 are related?

Solution: Consider the set A of all triangles in a plane

The relation R on the set A is defined as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$

- For any $T_1 \in A$, the triangle T_1 is similar itself, so that (T_1, T_1) is an element of R, so that R is reflexive
- Suppose that (T_1, T_2), it means T_1, T_2 are similar triangles it gives T_2, T_1 Are similar triangles. It implies $(T_2, T_1) \in R$

  Hence the relation R is symmetric
- Suppose that $(T_1, T_2) \in R, (T_2, T_3) \in R$, it implies that T_1, T_2 are similar triangles and T_2, T_3 are similar triangles

  Hence, T_1, T_3 are similar triangles, it implies that (T_1, T_3) is an element of R

  It implies that $(T_1, T_3) \in R$

  Hence, the relation R is transitive.

Therefore, the relation is equivalence relation.

Consider three right angle triangles T_1 with sides 3, 4, 5, and T_3 with sides 6, 8, 10 are related, so that (T_1, T_3) is an element in R

13. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have the same number of sides}\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Solution: Consider the set A of all polygons in a plane

The relation R on the set A is defined as

$$R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have the same number of sides}\}$$

- For any $P_1 \in A$, the number of sides of polygon P_1 is same as the number of sides of the polygon P_2, so that (P_1, P_2) is an element of R, so that R is reflexive
- Suppose that (P_1, P_2), it means P_1, P_2 have same number of sides and P_2, P_1 have the same number of sides

  It implies $(P_2, P_1) \in R$

  Hence the relation R is symmetric
- Suppose that $(P_1, P_2) \in R, (P_2, P_3) \in R$, it implies that the number of sides of the polygon P_1, P_2 are same and the number of sides of the polygon P_2, P_3 are same

  Hence, the number of sides of polygons P_1, P_3 is same, it implies that (P_1, P_3) is an element of R

  It implies that $(P_1, P_3) \in R$

  Hence, the relation R is transitive.

Therefore, the relation is equivalence relation.

The set of all elements in A related to the right angle triangle T with sides 3, 4 and 5 is the set of all triangles

14. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$

Solution: Consider the set A of all lines in a plane.

The relation R on the set A is defined as

$$R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$$

- For any $L_1 \in A$, any line L_1 is parallel to the line itself. So that (L_1, L_1) is an element of R, so that R is reflexive

- Suppose that (L_1, L_2), it means L, L_2 are parallel, and it implies that L_2, L_1 are parallel lines It implies $(L_2, L_1) \in R$

  Hence the relation R is symmetric
- Suppose that $(L_1, L_2) \in R, (L_2, L_3) \in R$, it implies that L_1 is parallel to L_2 and L_2 is parallel to L_3

  It implies that L_1, L_3 are parallel

  Hence, (L_1, L_3) is an element of R

  It implies that $(L_1, L_3) \in R$

  Hence, the relation R is transitive.

Therefore, the relation is equivalence relation.

The set of all elements in A related to the line $y = 2x + 4$ is the set of all the lines having the equations $y = 2x + c$

15. Let R be the relation in the set $A = \{1, 2, 3, 4\}$

$$R = \{(1,1), (1,2), (2,2), (4,4), (1,3), (3,3), (3,2)\}$$

Choose the correct answer

(A) R is reflexive and symmetric but not transitive

(B) R is reflexive and transitive but not symmetric

(C) R is symmetric and transitive but not reflexive

(D) R is an equivalence relation

Solution:

Consider the given relation

$$R = \{(1,1), (1,2), (2,2), (4,4), (1,3), (3,3), (3,2)\}$$

- The relation is reflexive
- But the relation is not symmetric because $(1,2) \in R, (2,1) \notin R$
- The relation is transitive

This is matching with the option (B)

16. Let R be the relation in the set N given by $R=\{(a,b): a=b-2, b>6\}$

Choose the right answer

(A) $(2,4)\in R$

(B) $(3,8)\in R$

(C) $(6,8)\in R$

(D) $(8,7)\in R$

Solution:

Given that the relation is $R=\{(a,b): a=b-2, b>6\}$

The roster form of the relation contains $(6,8)$

Hence the option (C) is correct.

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