EXERCISE 1.2

1. Which of the following are examples of the null set

(i) Set of odd natural numbers divisible by 2

(ii) Set of even prime numbers

(iii) $\{x : x$ **is a natural number,** $x < 5$ **and** $x > 7\}$

(iv) $\{y : y$ **is a point common to any two parallel lines** $\}$

Solution

(i) No odd natural number which is divisible by 2. So, the given set is a empty set.

(ii) Set of numbers $= \{2\} \neq \phi$. So, the given set is not a empty set. It is a singleton set.

(iii) No natural number which is either than 5 and greater than 7. So, the given set is an empty set.

(iv) Two parallel do not have the common point. Therefore, the given set is a null set.

Hence $(i), (iii)$ and (iv) are examples of the null set.

2. Which of the following sets are finite or infinite?

(i) The set of months of a year.

(ii) $\{1, 2, 3, \ldots\}$

(iii) $\{1, 2, 3, \ldots, 99, 100\}$

(iv) The set of positive integers greater than 100

(v) The set of prime numbers less than 99.

Solution

(i) Since there are 12 months (i.e., actual number of months) per year, the given set is finite.

(ii) As per number of elements in the set is infinite, the given set is infinite.

(iii) Since the number of elements in the set is 100 (i.e., exact number), the given set is finite.

(iv) Since there are infinitely more numbers than 100, the given set is infinite.

(v) Since the number of primes less than 99 is a straight number, the given set is finite.

3. State whether each of the following set is finite or infinite:

(i) The set of lines which are parallel to the x-axis

(ii) The set of letters in the English alphabet

(iii) The set of numbers which are multiple of 5

(iv) The set of animals living on the earth

(v) The set of circles passing through the origin $(0,0)$

Solution

(i) Since there are infinite number of lines corresponding to the $x-$ axis, the given set is infinite.

(ii) Since there are 26 letters, that is, a exact number of letters, in the English alphabet, the given set is finite.

(iii) Since there are many multiples of 5, the given set is infinite.

(iv) The process of counting land animals ends. Therefore, a certain number of animals live in the world and that is why a given set is finite.

(v) There is no end to the number of circles that exceeds the origin $(0,0)$.

Hence, the given set is infinite.

4. In the following, state whether $A = B$ or not:

(i) $A = \{a,b,c,d\}; B = \{d,c,b,a\}$

(ii) $A = \{4,8,12,16\}; B = \{8,4,16,18\}$

(iii) $A = \{2,4,6,8,10\}; B = \{x : x$ **is positive even integer and** $x \le 10\}$

(iv) $A = \{x : x$ **is a multiple of 10** $\}, ;$ **B** $= \{10,15,20,25,30,\ldots\}$

Solution

(i) A and B have exactly same elements, though not in the same order.

$A = B$

(ii) $12 \in A$ but $12 \notin B$

$\therefore \quad A \neq B$

(iii)In roster form $B = \{2,4,6,8,10\}$

Since A and B have exactly same elements, therefore, $A = B$

(iv) In roster form $A = \{10,20,30,\ldots\}$

Since $15 \in B$ but $15 \notin A$, therefore $A \neq B$.

5. Are the following pair of sets equal? Give reasons.

(i) $A=\{2,3\}, \quad B=\{x:x$ **is solution of** $x^2+5x+6=0\}$

(ii) $A=\{x:x$ **is a letter in the word FOLLOW** $\}$

$B=\{y:y$ **is a letter in the word WOLF** $\}$

Solution

(i) $B=\{x:x$ is solution of $(x+2)(x+3)=0\}$

$[\because x^2+5x+6=x^2+2x+3x+6$

$=x(x+2)+3(x+2)$

$=(x+2)(x+3)]$

$=\{-2,-3\}$

$2 \in A$ but $2 \notin B$

$\therefore \quad A \neq B$

(ii) Dropping repetitions $A=\{F,O,L,W\}$

$B=\{W,O,L,F\}$

Sets A and B have exactly same elements, though not in the same order.

$A = B.$

6. From the sets given below, select equal sets:

$A=\{2,4,8,12\}, \quad B=\{1,2,3,4\}, \quad C=\{4,8,12,14\}, \quad D=\{3,1,4,2\}$

$E=\{-1,1\}, \quad F=\{0,a\}, \quad G=\{1,-1\}, \quad H=\{0,1\}$

Solution

$A=\{2,4,8,12\}; B=\{1,2,3,4\}; C=\{4,8,12,14\}$

$D=\{3,1,4,2\}; E=\{-1,1\}; F=\{0,a\}$

$G=\{1,-1\}; A=\{0,1\}$

Set A has 4 elements,

So, A is not equal to sets E,F,G,H

Also, elements of Set A are not in B, C, D

So, A is not equal to set B, C, D

Set B has 4 elements,

And all elements of set B are in set D.

Hence, B = D

Now,

C is not equal to any set

And D is already equal to C

Elements of set E and set G are the same

So, $E = G$

And there are no other equal sets

Hence, among the given sets, $B = D$ and $E = G$

Example 9

Consider the sets

$\phi, A = \{1, 3\}, \quad B = \{1, 5, 9\}, \quad C = \{1, 3, 5, 7, 9\}$

Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets:

(i) $\phi \ldots B$

(ii) $A \ldots B$

(iii) $A \ldots C$

(iv) $B \ldots C$

Solution

(i) $\phi \subset B$ as ϕ is a subset of every set.

(ii) $A \not\subset B$ as $3 \in A$ and $3 \notin B$

(iii) $A \subset C$ as $1, 3 \in A$ also belongs to C

(iv) $B \subset C$ as each element of B is also an element of C.

Example 10

Let $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d\}$. Is A a subset of B ? No. (Why?). Is B a subset of A? No. (Why?)

Solution

(i) For a set to be a subset of another set, it needs to have all elements present in another set.

Set $A, \{e, i, o, u\}$ elements exists but these are not set in B Hence A is not a subset of B.

(ii)For this condition to be true, are elements of sets B should be present in set A

In set $B, \{b, c, d\}$ elements exists but these elements are not in set A

So, B is not a subset of A

Example 11

Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$? . If not, give an example.

Solution

$A \in B$ shows an element A belongs to set B.

Let $A = 3, B = (1, 2, 3)$

$B \subset C$ shows B is a proper subset of C.

Let $C = (1, 2, 3, 4, 5)$

So, Element A is an element of Set C.

Hence correct depiction will be $A \in C$.

$A \subset C$ is incorrect.