Chapter 1: Sets

EXERCISE 1.3

1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces :

(i) $\{2,3,4\}\ldots\{1,2,3,4,5\}$

(ii) $\{a,b,c\}\ldots\{b,c,d\}$

(iii) $\{x : x$ is a student of Class XI of your school $\}\ldots\{x : x$ student of your school $\}$

(iv) $\{x : x$ is a circle in the plane $\}\ldots\{x : x$ is a circle in the same plane with radius 1 unit $\}$

(v) $\{x : x$ is a triangle in a plane $\}\ldots\{x : x$ is a rectangle in the plane $\}$

(vi) $\{x : x$ is an equilateral triangle in a plane $\}\ldots\{x : x$ is a triangle in the same plane $\}$

(vii) $\{x : x$ is an even natural number $\}\ldots\{x : x$ is an integer $\}$

Solution

(i) Every element of the set $\{2,3,4\}$ is also an element of the set $\{1,2,3,4,5\}$

$\therefore \{2,3,4\} \subset \{1,2,3,4,5\}$.

(ii) $a \in \{a,b,c\}$ but $a \notin b,c,d\}$ $\therefore \{a,b,c\} \subset \{b,c,d\}$.

(iii) Every student of class XI of your school is a student of your school.

$\therefore \{x : x$ is a student of class XI of your school $\} \subset \{x : x$ is a student of your school $\}$.

(iv) Every circle in a plane is not a circle with radius 1 unit, as it can have any radius $r,(r>0)$.

$\therefore \{x : x$ is a circle in the plane $\} \not\subset \{x : x$ is a circle in the same plane with radius 1 unit $\}$.

(v) Any triangle is never a rectangle.

$\therefore \{x : x$ is a triangle in the plane $\} \not\subset \{x : x$ is a rectangle in the plane $\}$.

(vi) Every equilateral triangle in a plane is a triangle in the plane.

$\therefore \{x : x$ is an equilateral triangle in a plane $\} \subset \{x : x$ is a triangle in the same plane 1 .

(vii) Every even natural number is an integer.

$\therefore \{x : x$ is an even natural number $\} \subset \{x : x$ is an integer $\}$.

2. Examine whether the following statements are true or false:

(i) $\{a,b\} \not\subset \{b,c,a\}$

(ii) $\{a,e\} \subset \{x : x$ is a vowel in the English alphabet $\}$

(iii) $\{1,2,3\} \subset \{1,3,5\}$

(iv) $\{a\} \subset \{a,b,c\}$

(v) $\{a\} \in \{a,b,c\}$

(vi) $\{x : x$ is an even natural number less than $6\} \subset \{x : x$ is a natural number which divides $36\}$

Solution

(i) False, since every element of the set $\{a,b\}$ is also an element of the set $\{b,c,a\}$, therefore, $\{a,b\} \subset \{b,c,a\}$.

(ii) True, since every element of the set $\{a,e\}$ is also an element of the set of vowels $\{a,e,i,o,u\}$, therefore, $\{a,e\} \subset \{a,e,i,o,u\}$.

(iii) False, since $2 \in \{1,2,3\}$ but $2 \notin \{1,3,5\}$.

(iv) True, since $a \in \{a,b,c\}$.

(v) False, since $\{a\}$ is subset of the set $\{a,b,c\}$ but not an element of the set $\{a,b,c\}$.

(vi) True, since $\{x : x$ is an even natural number less than $6\} = \{2,4\}$ and $\{x : x$ is a natural number which divides $36\} = \{1,2,3,4,6,9,12,18,36\}$.

Clearly, $\{2,4\} \subset \{1,2,3,4,6,9,12,18,36\}$

3. Let $A = \{1,2,\{3,4\},5\}$. Which of the following statements are incorrect and why?

(i) $\{3,4\} \subset A$

(ii) $\{3,4\} \in A$

(iii) $\{\{3,4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1,2,5\} \subset A$

(vii) $\{1,2,5\} \in A$

(viii) $\{1,2,3\} \subset A$

(ix) $\phi \in A$

(x) $\phi \subset A$

(xi) $\{\phi\} \subset A$

Solution

Given $A=\{1,2,\{3,4\},5\}$

(i) The statement $\{3,4\}\subset A$ is incorrect because $3\in\{3,4\}$; however, $3\notin A$.

(ii) The statement $\{3,4\}\in A$ is correct because $\{3,4\}$ is an element of A.

(iii) The statement $\{\{3,4\}\}\subset A$ is correct because $\{3,4\}\in\{\{3,4\}\}$ and, $\{3,4\}\in A$.

(iv) The statement $1\in \mathrm{A}$ is correct because 1 is an element of A.

(v) The statement $1\subset \mathrm{A}$ is incorrect because an element of a set can never be a subset of itself.

(vi) The statement $\{1,2,5\}\subset A$ is correct because each element of $\{1,2,5\}$ is also an element of A.

(vii) The statement $\{1,2,5\}\in A$ is incorrect because $\{1,2,5\}$ is not an element of A.

(viii) The statement $\{1,2,3\}\subset A$ is incorrect because $3\in\{1,2,3\}$; however, $3\notin A$.

(ix) The statement $\Phi\in A$ is incorrect because Φ is not an element of A.

(x) The statement $\Phi\subset A$ is correct because Φ is a subset of every set.

(xi) The statement $\{\Phi\}\subset A$ is incorrect because $\Phi\in\{\Phi\}$; however, $\Phi\in A$.

4. Write down all the subsets of the following sets

(i) $\{a\}$

(ii) $\{a,b\}$

(iii) $\{1,2,3\}$

(iv) ϕ

Solution

(i) Let $\mathrm{A}=|a|$, then A has one element.

$\therefore$ Number of subsets of the set $A=2^n=2^1=2$.

Subset of A will have either no element or one element.

Subset of A having no element is ϕ

Subset of A having one element is $\{a\}$ $\therefore$ The subsets of A are $\phi,\{a\}$.

(ii) Let $\mathrm{A}=\{a,b\}$, then A has 2 elements.

$\therefore$ Number of subsets of $\mathrm{A}=2^n=2^2=4$.

Subset of A having no element is ϕ

Subsets of A having one element are $\{a|,|b|$,

Subset of A having two elements is $\{a, b\}$

$\therefore$ The subsets of A are $\phi, \{a\}, \{b\}, \{a, b\}$.

(iii) Let $\mathrm{A}=\{1,2,3\}$, then A has 3 elements.

$\therefore$ Number of subsets of $\mathrm{A}=2^{n}=2^{3}=8$.

Subsets of A having no element is ϕ

Subsets of A having one element are $\{1\},\{2\},\{3\}$

Subsets of A having two elements are $\{1,2\},\{1,3\},\{2,3\}$

Subset of A having three elements is $\{1,2,3\}$

$\therefore$ The subsets of Aare $\phi,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}$, $\{1,2,3\}$.

(iv) The only subset of the empty set ϕ is ϕ itself.

5. How many elements has $\mathrm{P}(\mathrm{A})$, if $\mathrm{A}=\phi$?

Solution

$A=\varnothing$

Set A does not contain any elements

Use the formula to calculate the number of Subsets,

$n(P(A))=2^{n}$

Here, Number of elements (n) in Set A is 0

Hence,

$n(P(A))=2^{0}$

$n(P(A))=1$

Since, Set A has no elements, The number of elements is its power set is 1 .

6. Write the following as intervals:

(i) $\{x : x \in \mathrm{R}, -4 < x \leq 6\}$

(ii) $\{x : x \in \mathrm{R}, -12 < x < -10\}$

(iii) $\{x : x \in \mathrm{R}, 0 \leq x < 7\}$

(iv) $\{x : x \in \mathrm{R}, 3 \leq x \leq 4\}$

Solution

(i) The Set $\mathrm{A} = \{x : x \in R, -4 < x \leq 6\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

$\{x : x \in R, -4 < x \leq 6\}$:

This represents that the interval is an open interval including 6 but excluding -4, therefore the interval will be $(-4, 6]$.

(ii) The Set $\mathrm{A} = \{x : x \in R, -12 < x \leq -10\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

This represents that the interval is an open interval excluding both the end points, that is, -12 and -10, therefore the interval will be $(-12, -10)$.

(iii) The Set $\mathrm{A} = \{x : x \in R, 0 \leq x < 7\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

$\{x : x \in R, 0 \leq x < 7\}$

This represents that the interval is an open interval including 0 but excluding the end point 7, therefore the interval will be $[0, 7)$.

(iv) The Set $\mathrm{A} = \{x : x \in R, 3 \leq x \leq 4\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

$\{x : x \in R, 3 \leq x \leq 4\}$:

This represents that the interval is a closed interval including both the end points, that is, 3 and 4, therefore the interval will be $[3, 4]$.

7. Write the following intervals in set-builder form:

(i) $(-3, 0)$

(ii) $[\mathbf{6,12}]$

(*iii*) $(\mathbf{6,12}]$

(*iv*) $[\mathbf{-23,5})$

Solution

The following interval in set builder form

(i) Set builder form $\{x : x \in \mathrm{R}, -3 < x < 0\}$

(ii) Set builder form $\{x : x \in \mathrm{R}, 6 \le x \le 12\}$

(iii) Set builder form $\{x : x \in \mathrm{R}, 6 < x \le 12\}$

(iv) Set builder form $\{x : x \in \mathrm{R}, -23 \le x < 5\}$.

8. What universal set(s) would you propose for each of the following:

(i) The set of right triangles.

(ii) The set of isosceles triangles.

Solution

(i) In the right-hand triangle set, the universal set may be a set of triangles or a set of polygons.

(ii) With a set of isosceles triangles, a universal set could be a set of triangles or a set of polygons or a set of two-dimensional figures.

9. Given the sets $\mathrm{A} = \{1,3,5\}, \mathrm{B} = \{2,4,6\}$ and $\mathrm{C} = \{0,2,4,6,8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C

(i) $\{0,1,2,3,4,5,6\}$

(ii) ϕ

(iii) $\{0,1,2,3,4,5,6,7,8,9,10\}$

(iv) $\{1,2,3,4,5,6,7,8\}$

Solution

(i) It can be seen that $A \subset \{0,1,2,3,4,5,6\}$

$B \subset \{0,1,2,3,4,5,6\}$

However, $C \not\subset \{0,1,2,3,4,5,6\}$

Hence, the set $\{0,1,2,3,4,5,6\}$ cannot be the universal set of $A, B,$ and C sets.

(ii) $A \not\subset \Phi, B \not\subset \Phi, C \not\subset \Phi$

Therefore, Φ cannot be the universal set for the sets $A, B,$ and C.

(iii) $A \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

$\mathrm{B} \subset \{0,1,2,3,4,5,6,7,8,9,10\}$ $\mathrm{C} \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

Therefore, the set $\{0,1,2,3,4,5,6,7,8,9,10\}$ is the universal set for the sets $A, B,$ and C.

(iv) $A \subset \{1,2,3,4,5,6,7,8\}$

$B \subset \{1,2,3,4,5,6,7,8\}$

However, $C \not\subset \{1,2,3,4,5,6,7,8\}$

Therefore, the set $\{1,2,3,4,5,6,7,8\}$ cannot be the universal set for the sets $A, B,$ and C.

Example 12

Let $\mathrm{A} = \{2,4,6,8\}$ and $\mathrm{B} = \{6,8,10,12\}$. Find $\mathrm{A} \cup \mathrm{B}$.

Solution

Given,

$\mathrm{A} = \{2,4,6,8\}$

$\mathrm{B} = \{6,8,10,12\}$

The combination of two sets

$A \cup B = \{2,4,6,8\} \cup \{6,8,10,12\}$

Common elements 6,8 should be taken once

The combination of the sets the $A \cup B = \{2,4,6,8,10,12\}$

The common elements 6 and 8 have been taken only once while writing $A \cup B$

Example 13

Let $\mathrm{A} = \{a,e,i,o,u\}$ and $\mathrm{B} = \{a,i,u\}$. Show that $\mathrm{A} \cup \mathrm{B} = \mathrm{A}$

Solution

Given,

$A = \{a, e, i, o, u\}$

$B = \{a, i, u\}$.

The combination of sets

$A \cup B = \{a, e, i, o, u\} = A$.

$\therefore A \cup B = A$

The example illustrates that union of sets A and its subset B is the set A itself,

if $B \subset A$, then $A \cup B = A$

Example 14

Let $X = \{$ Ram, Geeta, Akbar $\}$ be the set of students of Class XI, who are in school hockey team. Let $Y = \{$ Geeta, David, Ashok $\}$ be the set of students from Class XI who are in the school football team. Find $X \cup Y$ and interpret the set.

Solution

Given $X = \{$ Ram, Geeta, Akbar $\}$

$Y = \{$ Geeta, David, Ashok $\}$

Common elements (Geeta) should be taken once

$X \cup Y = \{$ Ram, Geeta, Akbar, David, Ashok $\}$.

This is the group of students from Class XI who are on the hockey team or the football team or both.

Example 15

Consider the sets A and B of Example 12. Find $A \cup B$.

Solution

From example 12

Given, $A = \{2, 4, 6, 8\}$

$B = \{6, 8, 10, 12\}$

6,8 are the only elements which are common to both A and B.

Intersection of common sets

Hence $A \cap B = \{6, 8\}$.

Example 16

Consider the sets X **and** Y **of Example 14. Find** $\mathrm{X} \cap \mathrm{Y}$**.**

Solution

Given $X = \{$ Ram, Geeta, Akbar $\}$

$\mathrm{Y} = \{$ Geeta, David, Ashok $\}$

So, the intersection of sets

$\mathrm{X} \cap \mathrm{Y} = \{$ Ram, Geeta, Akbar $\} \cap \{$ Geeta, David, Ashok $\}$

'Geeta' is the only element common to both.

Hence, $\mathrm{X} \cap \mathrm{Y} = \{$ Geeta $\}$.

Example 17

Let $\mathrm{A} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **and** $\mathrm{B} = \{2, 3, 5, 7\}$**. Find** $\mathrm{A} \cap \mathrm{B}$ **and hence show that** $\mathrm{A} \cap \mathrm{B} = \mathrm{B}$**.**

Solution

Given, $\mathrm{A} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\mathrm{B} = \{2, 3, 5, 7\}$

The intersection of sets

The sets of $A \cap B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cap \{2, 3, 5, 7\}$

$A \cap B = \{2, 3, 5, 7\} = B$.

So $\mathrm{B} \subset \mathrm{A}$ and that $\mathrm{A} \cap \mathrm{B} = \mathrm{B}$ is proved.

Example 18

Let $\mathrm{A} = \{1, 2, 3, 4, 5, 6\}, \mathrm{B} = \{2, 4, 6, 8\}$**. Find** $\mathrm{A} - \mathrm{B}$ **and** $\mathrm{B} - \mathrm{A}$**.**

Solution

Given,

$\mathrm{A} = \{1, 2, 3, 4, 5, 6\}, \mathrm{B} = \{2, 4, 6, 8\}$

Intersection of common sets

$A - B = A - (A \cap B)$

The intersection of sets

$$A \cap B = \{1,2,3,4,5,6\} \cap \{2,4,6,8\}$$

$$= \{2,4,6\}$$

Substitute the values

$$A - B = A - (A \cap B)$$

$$= \{1,2,3,4,5,6\} - \{2,4,6\}$$

$$= \{1,3,5\}$$

Finally,

$$B - A = B - (B \cap A)$$

$$= B - (A \cap B)$$

$$= \{2,4,6,8\} - \{2,4,6\}$$

$$= \{8\}$$

So, $\mathrm{A} - \mathrm{B} = \{1,3,5\}$, since the elements $1,3,5$ belong to A but not to B and $\mathrm{B} - \mathrm{A} = \{8\}$,

Hence the element 8 belongs to B and not to A.

Therefore, $\mathrm{A} - \mathrm{B} \neq \mathrm{B} - \mathrm{A}$.

Example 19

Let $\mathrm{V} = \{a,e,i,o,u\}$ **and** $\mathrm{B} = \{a,i,k,u\}$. **Find** $\mathrm{V} - \mathrm{B}$ **and** $\mathrm{B} - \mathrm{V}$

Solution

Given, $\mathrm{V} = \{\mathrm{a,e,i,o,u}\}$ and $\mathrm{B} = \{\mathrm{a,i,k,u}\}$

$\mathrm{V} - \mathrm{B} =$ is a set which contain elements which are only present in V

$\therefore \mathrm{V} - \mathrm{B} = \{\mathrm{e,o}\}$

$\mathrm{B} - \mathrm{V} =$ is a set which contain elements which are only present in B $\therefore \mathrm{B} - \mathrm{V} = \{\mathrm{k}\}$

Hence, $\mathrm{V} - \mathrm{B} \neq \mathrm{B} - \mathrm{V}$

Since the elements e, o belong to V but not to B and $\mathrm{B} - \mathrm{V} = \{k\}$, since the element k belongs to B but not to V.