Chapter 1: Sets

EXERCISE 1.4

1. Find the union of each of the following pairs of sets:

(i) $X=\{1,3,5\}$ $Y=\{1,2,3\}$

(ii) $A=[a,e,i,o,u\}$ $B=\{a,b,c\}$

(iii) $A=\{x:x$ is a natural number and multiple of $3\}$ $B=\{x:x$ is a natural number less than $6\}$

(iv) $A=\{x:x$ is a natural number and $1<x\leq 6\}$ $B=\{x:x$ is a natural number and $6<x<10\}$

(v) $A=\{1,2,3\}, B=\phi$

Solution

(i) $X=\{1,3,5\}$ $Y=\{1,2,3\}$

The combination of two sets

$X\cup Y=\{1,3,5\}\cup\{1,2,3\}$

$=\{1,2,3,5\}$

(ii) $A=[a,e,i,o,u\}$ $B=\{a,b,c\}$

Combination of two sets

$A\cup B=\{a,e,i,o,u\}\cup\{a,b,c\}$

$=\{a,b,c,e,i,o,u\}$

(iii) $A=\{3,6,9,12,\ldots\}B=\{1,2,3,4,5\}$

$A\cup B=\{3,6,9,12,\ldots\}\cup\{1,2,3,4,5\}$

$=\{1,2,3,4,5,6,9,12,\ldots\}$

$=\{x:x=1,2,4,5$ or a multiple of $3\}$

(iv)

$A=\{2,3,4,5,6\}$
$B=\{7,8,9\}$

$A\cup B=\{2,3,4,5,6\}\cup|\ 7,8,9\}$

$=\{2,3,4,5,6,7,8,9\}$

$=\{x:1<x<10$ and $x\in N\}$

(v)

$A \cup B = \{1, 2, 3\} \cup \phi$

$= \{1, 2, 3\}$

$= A$

The result of this is true in general also.

$A \cup \phi = A$ for every set A.

2. Let $A = \{a, b\}, B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution

Given, $A = \{a, b\}$

$B = \{a, b, c\}$

Every element of A is in B.

So, A is a subset of B

That is $A \subset B$.

$A \cup B = \{a, b\} \cup \{a, b, c\}$

$= \{a, b, c\}$

Yes. $A \cup B = \{a, b, c\} = B$

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution

Let's take an example

$A = \{1, 2\}$

$B = \{1, 2, 3\}$

Every element of A is in B.

Hence, A is a subset of B

That is $A \subset B$.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$= \{x : x \in B\}$

4. If $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$

(ii) $A \cup C$

(iii) $B \cup C$

(iv) $B \cup D$

(v) $A \cup B \cup C$

(vi) $A \cup B \cup D$

(vii) $B \cup C \cup D$

Solution

Given, $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$;

(i) The combination of sets A and B

$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

(ii) The combination of sets A and C

$A \cup C = \{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) The combination of sets B and C

$B \cup C = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$

$= \{3, 4, 5, 6, 7, 8\}$

(iv) The combination of sets B and D

$B \cup D = \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) The combination of sets

$A \cup B \cup C = (A \cup B) \cup C$

$= \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) The combination of sets

$A \cup B \cup D = (A \cup B) \cup D$

$= \{1, 2, 3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$

$=\{1,2,3,4,5,6,7,8,9,10\}$

(vii) The combination of sets

$B\cup C\cup D=(B\cup C)\cup D=\{3,4,5,6,7,8\}\cup\{7,8,9,10\}$

$=\{3,4,5,6,7,8,9,10\}$

5. Find the intersection of each pair of sets of question 1 above.

Solution

From exercise 1

(i) The intersection of sets

$X\cap Y=\{1,3,5\}\cap\{1,2,3\}=\{1,3\}$.

(ii) The intersection of sets

$A\cap B=\{a,e,i,o,u\}\cap|a,b,c|=|a|$.

(iii) $A=\{3,6,9,12,\ldots\}$

$B=\{1,2,3,4,5\}$

The intersection of sets

$A\cap B=\{3,6,9,12,\ldots\}\cap\{1,2,3,4,5\}=\{3\}$

(iv) $A=\{2,3,4,5,6\}$

$B=\{7,8,9\}$

The intersection of sets

$A\cap B=\{2,3,4,5,6\}\cap| 7,8,9\}=\phi$

(v) The intersection of sets

$A\cap B=\{1,2,3|\cap\phi=\phi$

The result of this part is true in general also.

$A\cap\phi=\phi$ for every set A.

6. If $A=\{3,5,7,9,11\}, B=\{7,9,11,13\}, C=\{11,13,15\}$ and $D=\{15,17\}$; find

(i) $A\cap B$

(ii) $B\cap C$

(iii) $A \cap C \cap D$

(iv) $A \cap C$

(v) $B \cap D$

(vi) $A \cap (B \cup C)$

(vii) $A \cap D$

(viii) $A \cap (B \cup D)$

(ix) $(A \cap B) \cap (B \cup C)$

Solution

Given $A = \{3, 5, 7, 9, 11\}, B = \{7, 9, 11, 13\}, C = \{11, 13, 15\}$ and $D = \{15, 17\}$

Intersection of sets

(i) $A \cap B = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\}$

$= \{7, 9, 11\}$

(ii) Intersection of sets

$B \cap C = \{7, 9, 11, 13\} \cap \{11, 13, 15\} = \{11, 13\}$

(iii) Intersection of sets

$A \cap C \cap D = (A \cap C) \cap D$

$= (|3, 5, 7, 9, 11| \cap |11, 13, 15|) \cap |15, 17|$

$= \{11\} \cap | 15, 17\} = \phi$

(iv) Intersection of sets

$A \cap C = \{3, 5, 7, 9, 11\} \cap |11, 13, 15| = |11|$

(v) Intersection of sets

$B \cap D = \{7, 9, 11, 13\} \cap \{15, 17\} = \phi$

(vi) Intersection of sets

$A \cap (B \cup C) = \{3, 5, 7, 9, 11\} \cap (\{7, 9, 11, 13\} \cup \{11, 13, 15\})$

$= \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11\}$

(vii) Intersection of sets

$A \cap D = \{3, 5, 7, 9, 11\} \cap \{15, 17\} = \phi$

(viii) Intersection of sets

$A\cap(B\cup D)=\{3,5,7,9,11\}\cap(\{7,9,11,13\}\cup\{15,17\})$

$=\{3,5,7,9,11\}\cap|7,9,11,13,15,17|$

$=\{7,9,11\}$

(ix) Intersection of sets

$(A\cap B)\cap(B\cup C)=(\{3,5,7,9,11\}\cap\{7,9,11,13\})\cap(\{7,9,11,13\}\cup\{11,13,15\})$

$=\{7,9,11\}\cap\{7,9,11,13,15\}=\{7,9,11\}$

(x) Intersection of sets

$(A\cup D)\cap(B\cup C)=(\{3,5,7,9,11\}\cup\{15,17\})\cap(\{7,9,11,13\}\cup\{11,13,15\})$

$=\{3,5,7,9,11,15,17\}\cap|7,9,11,13,15\}$

$=\{7,9,11,15\}$

7. If $A=\{x:x$ is a natural number $\},B=\{x:x$ is an even natural number $\}$ $C=\{x:x$ is an odd natural number $\}$ and D $=\{x:x$ is a prime number $\}$, find

(i) $A\cap B$

(ii) $A\cap C$

(iii) $A\cap D$

(iv) $B\cap C$

(v) $B\cap D$

(vi) $C\cap D$

Solution

Given

$A=\{x:x$ is a natural number $\}=\{1,2,3,4,5\ldots\}$

$B=\{x:x$ is an even natural number $\}=\{2,4,6,8\ldots\}$

$C=\{x:x$ is an odd natural number $\}=\{1,3,5,7,9\ldots\}$

$D=\{x:x$ is a prime number $\}=\{2,3,5,7\ldots\}$

(i) $A\cap B=B$

Because, $B\subset A$

Every even natural number is a natural number

(ii) $A\cap C=C$

Because, $\mathrm{C} \subset \mathrm{A}$

Every odd natural number is a natural number

(iii) $\mathrm{A} \cap \mathrm{D} = \mathrm{D}$

Because $\mathrm{D} \subset \mathrm{A}$

Every prime number is a natural number

(iv) $\mathrm{B} \cap \mathrm{C} = \phi$ There is no natural number which is both even and odd

(v) $\mathrm{B} \cap \mathrm{D} = \{2\}, 2$ is only even prime number.

(vi) $\mathrm{C} \cap \mathrm{D} = \{x : x$ is an odd prime number $\}$.

8. Which of the following pairs of sets are disjoint

(i) $\{1,2,3,4\}$ and $\{x : x$ is a natural number and $4 \leq x \leq 6\}$

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x : x$ is an even integer $\}$ and $\{x : x$ is an odd integer $\}$

Solution

(i) Let $\mathrm{A} = \{1,2,3,4\}$ and

$\mathrm{B} = \{x : x \in \mathrm{N} \text{ and } 4 \leq x \leq 6\} = \{4,5,6\}$

$\mathrm{A} \cap \mathrm{B} = \{4\} \neq \phi \therefore$ Sets A and B are not disjoint.

(ii) e is a common element of the two sets.

$\therefore$ Sets are not disjoint.

(iii) Given sets are disjoint sets because there is no natural number which is both even and odd.

9. If $A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$ $\mathrm{C} = \{2,4,6,8,10,12,14,16\}$, $\mathrm{D} = \{5,10,15,20\}$; **find**

(i) $\mathrm{A} - \mathrm{B}$

(ii) $\mathrm{A} - \mathrm{C}$

(iii) $\mathrm{A} - \mathrm{D}$

(iv) $\mathrm{B} - \mathrm{A}$

(v) $\mathrm{C} - \mathrm{A}$

(vi) $\mathrm{D} - \mathrm{A}$

(vii) $\mathrm{B} - \mathrm{C}$

(viii) $B - D$

(ix) $C - B$

(x) $D - B$

(xi) $C - D$

(xii) $D - C$

Solution

Given,

$A = \{3, 6, 9, 12, 15, 18, 21\},$
$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\},$

$D = \{5, 10, 15, 20\};$

(i) The sets in difference

$A - B = \{3, 6, 9, 12, 15, 18, 21\} - \{4, 8, 12, 16, 20\}$

$= \{3, 6, 9, 15, 18, 21\}$

(ii) The difference of two sets

$A - C = \{3, 6, 9, 12, 15, 18, 21\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$

$= \{3, 9, 15, 18, 21\}$

(iii) The difference of two sets

$A - D = \{3, 6, 9, 12, 15, 18, 21\} - \{5, 10, 15, 20\}$

$= \{3, 6, 9, 12, 18, 21\}$

(iv) The difference of two sets

$B - A = \{4, 8, 12, 16, 20\} - \{3, 6, 9, 12, 15, 18, 21\}$

$= \{4, 8, 16, 20\}$

(v) The difference of two sets

$C - A = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{3, 6, 9, 12, 15, 18, 21\}$

$= \{2, 4, 8, 10, 14, 16\}$

(vi) The difference of two sets

$D - A = \{5, 10, 15, 20\} - \{3, 6, 9, 12, 15, 18, 21$

$=\{5,10,20\}$

(vii) The difference of two sets

$B-C=\{4,8,12,16,20\}-\{2,4,6,8,10,12,14,16\}$

$=\{20\}$

(viii) The difference of two sets

$B-D=\{4,8,12,16,20\}-\{5,10,15,20\}$

$=\{4,8,12,16\}$

(ix) The difference of two sets

$C-B=\{2,4,6,8,10,12,14,16\}-\{4,8,12,16,20\}$

$=\{2,6,10,14\}$

(x) The difference of two sets

$D-B=\{5,10,15,20\}-\{4,8,12,16,20\}$

$=\{5,10,15\}$

(xi) The difference of two sets

$C-D=\{2,4,6,8,10,12,14,16\}-\{5,10,15,20\}$

$=\{2,4,6,8,12,14,16\}$

(xii) The difference of two sets

$D-C=\{5,10,15,20\}-\{2,4,6,8,10,12,14,16\}$

$=\{5,15,20\}$

10. If $X=\{a,b,c,d\}$ and $Y=\{f,b,d,g\}$, find

(i) $X-Y$

(ii) $Y-X$

(iii) $X\cap Y$

Solution

Given

$X=\{a,b,c,d\}$

$Y=\{f,b,d,g\}$

(i) The difference of two sets

$X-Y=\{a,b,c,d\}-\{f,b,d,g\}$

$=\{a,c\}$

(ii) The difference of two sets

$Y-X=\{f,b,d,g\}-\{a,b,c,d\}$

$=\{f,g\}$

(iii) The intersection of two sets

$X\cap Y=\{a,b,c,d\}\cap|f,b,d,g|$

$=\{b,d\}.$

11. If $\mathbf{R}$ is the set of real numbers and $\mathbf{Q}$ is the set of rational numbers, then what is $\mathbf{R}-\mathbf{Q}$?

Solution

The Set $\mathbf{R}=\{x:x$ is a real number $\}$

The Set $Q\{x:x$ is a rational number $\}$

$R-Q=\{x:x\in R \text{ and } x\notin Q\}$

$=\{x:x$ is a real number and x is not a rational

$=\{x:x$ is an irrational number $\}$

$=T$

Each real number is either rational or irrational but not both

12. State whether each of the following statement is true or false. Justify your answer.

(i) $\{2,3,4,5\}$ and $\{3,6\}$ are disjoint sets.

(ii) $\{a,e,i,o,u\}$ and $\{a,b,c,d\}$ are disjoint sets.

(iii) $\{2,6,10,14\}$ and $\{3,7,11,15\}$ are disjoint sets.

(iv) $\{2,6,10\}$ and $\{3,7,11\}$ are disjoint sets.

Solution

(i) Given sets is

$\{2,3,4,5\}$ and $\{3,6\}$

Intersection sets are:

$\{2,3,4,5\}\cap\{3,6\}=\{3\}$

Hence these are not disjoint sets.

The given disjoint sets are false.

(ii) Given sets is

$\{a,e,i,o,u\}$ and $\{a,b,c,d\}$

Intersection sets are:

$\{a,e,i,o,u\}\cap\{a,b,c,d\}=\{a\}$

Hence these are not disjoint sets.

The given disjoint sets are false

(iii) Given sets is

$\{2,6,10,14\}$ and $\{3,7,11,15\}$

Intersection sets are:

$\{2,6,10,14\}\cap\{3,7,11,15\}=\varnothing$

Then, $A\cap B$ is an empty set.

Hence these are disjoint sets.

The given disjoint sets are True.

(iv) Given sets is

$\{2,6,10\}$ and $\{3,7,11\}$.

Intersection sets are:

$\{2,6,10\}\cap\{3,7,11\}=\varnothing$

Then, $A\cap B$ is an empty set.

Hence these are disjoint sets.

The given disjoint sets are True.

Example 20

Let $U=\{1,2,3,4,5,6,7,8,9,10\}$ and $A=\{1,3,5,7,9\}$. Find A'.

Solution

Given,

$U=\{1,2,3,4,5,6,7,8,9,10\}$

$A=\{1,3,5,7,9\}$

The value of $A'=U-A=\{2,4,6,8,10\}$

$2,4,6,8,10$ are the elements of U which does not belong to A.

Example 21

Let U be universal set of all the students of Class XI of a coeducational school and A be the set of all girls in Class XI. Find A'.

Solution

In a coeducational school, there can be only boys and girls in a school.

A is the set of all girls,

$A' =$ Set of all Students $-$ Set of all girls

$A' =$ Set of all boys in class XI.

Since A is the set of all girls, A' is clearly the set of all boys in the class.

Example 22

Let $U=\{1,2,3,4,5,6\}, A=\{2,3\}$ and $B=\{3,4,5\}$

Find $A', B', A' \cap B', A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.

Solution

Given

$U=\{1,2,3,4,5,6\}, A=\{2,3\}$

$B=\{3,4,5\}$

The complement of Set A

$A' = U - A = \{1,2,3,4,5,6\} - \{2,3\}$

$= \{1,4,5,6\}$

Clearly $A' = \{1,4,5,6\}$,

Similarly

$B' = U - B = \{1,2,3,4,5,6\} - \{3,4,5\}$

$= \{1,2,6\}$

$B' = \{1,2,6\}$

Now,

$A' \cap B' = \{1,4,5,6\} \cap \{1,2,6\}$

$= \{1,6\}$

Hence $A' \cap B' = \{1,6\}$

Also,

$A \cup B = \{2,3\} \cup \{3,4,5\}$

$= \{2,3,4,5\}$

Also $A \cup B = \{2,3,4,5\}$,

So that $(A \cup B)' = \{1,6\}$

Now, to prove $(A \cup B)' = A' \cap B'$

$(A \cup B)' = U - (A \cup B) = \{1,2,3,4,5,6\} - \{2,3,4,5\}$

$= \{1,6\}$

Now, $A' \cap B' = \{1,6\}$

$(A \cup B)' = \{1,6\}$

Thus, $(A \cup B)' = A' \cap B'$

Hence proved