Chapter 1: Sets

EXERCISE 1.5

1. Let $U=\{1,2,3,4,5,6,7,8,9\}, A=\{1,2,3,4\}, B=\{2,4,6,8\}$ **and** $C=\{3,4,5,6\}$**. Find**

(i) A'

(ii) B'

(iii) $(A\cup C)'$

(iv) $(A\cup B)'$

(v) $\left(A'\right)'$

(vi) $(B-C)'$

Solution

Given,

$U=\{1,2,3,4,5,6,7,8,9\}$,

$A=\{1,2,3,4\}$,

$B=\{2,4,6,8\}$

$C=\{3,4,5,6\}$

(i) The complement of set

$A'=U-A=\{1,2,3,4,5,6,7,8,9\}-\{1,2,3,4\}$

$=\{5,6,7,8,9\}$

(ii) The complement of set

$B'=U-B=\{1,2,3,4,5,6,7,8,9\}-\{2,4,6,8\}$

$=\{1,3,5,7,9\}$

(iii) The complement of set

$A\cup C=\{1,2,3,4\}\cup\{3,4,5,6\}=\{1,2,3,4,5,6\}$

$(A\cup C)'=U-(A\cup C)$

$=\{1,2,3,4,5,6,7,8,9\}-\{1,2,3,4,5,6\}$

$=\{7,8,9\}$

(iv) The complement of set

$A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6, 8\} = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = U - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 6, 8\}$

$= \{5, 7, 9\}$

(v) The complement of set

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$

$= \{5, 6, 7, 8, 9\}$

So,

$(A')' = U - A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{5, 6, 7, 8, 9\}$

$= \{1, 2, 3, 4\} = A$

(vi) The difference of two sets

$B - C = \{2, 4, 6, 8\} - \{3, 4, 5, 6\} = \{2, 8\}$

$(B - C)' = U - (B - C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 8\}$

$= \{1, 3, 4, 5, 6, 7, 9\}$.

2. If $U = \{a, b, c, d, e, f, g, h\}$, find the complements of the following sets:

(i) $A = \{a, b, c\}$

(ii) $B = \{d, e, f, g\}$

(iii) $C = \{a, c, e, g\}$

(iv) $D = \{f, g, h, a\}$

Solution

Given,

$U = \{a, b, c, d, e, f, g, h\}$

(i) The complement of sets

$A' = U - A = \{a, b, c, d, e, f, g, h\} - \{a, b, c\}$

$= \{d, e, f, g, h\}$

(ii) The complement of sets

$B' = U - B = \{a,b,c,d,e,f,g,h\} - \{d,e,f,g\}$

$= \{a,b,c,h\}$

(iii) The complement of sets

$C' = U - C = \{a,b,c,d,e,f,g,h\} - \{a,c,e,g\}$

$= \{b,d,f,h\}$

(iv) The difference of universal sets ad set D

$D' = U - D = \{a,b,c,d,e,f,g,h\} - \{f,g,h,a\}$

$= \{b,c,d,e\}$

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) $\{x : x$ is an even natural number $\}$

(ii) $\{x : x$ is an odd natural number $\}$

(iii) $\{x : x$ is a positive multiple of $3\}$

(iv) $\{x : x$ is a prime number $\}$

(v) $\{x : x$ is a natural number divisible by 3 and $5\}$

(vi) $\{x : x$ is a perfect square $\}$

(vii) $\{x : x$ is a perfect cube $\}$

(viii) $\{x : x + 5 = 8\}$

(ix) $\{x : 2x + 5 = 9\}$

(x) $\{x : x \geq 7\}$

(xi) $\{x : x \in N$ and $2x + 1 > 10\}$

Solution

(i) Complement of the set = Universal set (set of all natural numbers) - Set of even natural numbers.

The complement of $\{x : x$ is an even natural number $\}$ is $\{x : x$ is an odd natural number $\}$

(ii) Complement of the set = Universal set (set of all natural numbers) - Set of odd natural numbers

The complement of $\{x : x$ is an odd natural number $\}$ is $\{x : x$ is an even natural number $\}$

(iii) Complement of the set $=$ Universal set (set of all natural numbers) - Set of all positive multiples of 3

The complement of $\{x : x$ is a positive multiple of $3\}$ is $\{x : x \in \mathrm{N}$ and x is not a multiple of $3\}$

(iv) Complement of the set = Universal set (set of all natural numbers) - Set of all prime numbers

The complement of $\{x : x$ is a prime number $\}$ is $\{x : x$ is positive composite number and $x = 1\}$

(v) Complement of the set = Universal set (set of all natural numbers) - Set of all numbers divisible by 3 and 5

The complement of $\{x : x$ is a natural number divisible by 3 and $5\}$ is $\{x : x \in \mathrm{N}$ and x is not divisible by 3 or not divisible by $5\}$

(vi) Complement of the set = Universal set (set of all natural numbers) - Set of all perfect squares

The complement of $\{x : x$ **is a perfect square** $\}$ is $\{x : x \in \mathrm{N}$ and x is not a perfect square $\}$

(vii) Complement of the set = Universal set (set of all natural numbers) - Set of all perfect cubes

The complement of $\{x : x$ **is a perfect cube** $\}$ **is** $\{x : x \in \mathrm{N}$ and x is not a perfect cube $\}$

(viii) The complement of $\{x : x+5=8\}$ **is** $\{x : x=3\}' = \{x : x \in \mathrm{N}$ and $x \neq 3\}$

(ix) The complement of $\{x : 2x+5=9\}$ **is** $\{x : x \in \mathrm{N}$ and $x \neq 2\}$

(x) The complement of $\{x : x \geq 7\}$ **is** $\{x : x \in \mathrm{N}$ and $x < 7\} = \{1,2,3,4,5,6\}$

(xi) The complement of $\{x : x \in \mathrm{N}$ **and** $2x+1>10\}$ **is** $\left\{x : x \in \mathrm{N} \text{ and } x > \frac{9}{2}\right\} = \left\{x : x \in \mathrm{N} \text{ and } x \leq \frac{9}{2}\right\} = \{1,2,3,4\}$.

4. If $\mathrm{U} = \{1,2,3,4,5,6,7,8,9\}, \mathrm{A} = \{2,4,6,8\}$ and $\mathrm{B} = \{2,3,5,7\}$. Verify that

(i) $(\mathrm{A} \cup \mathrm{B})' = \mathrm{A}' \cap \mathrm{B}'$

(ii) $(\mathrm{A} \cap \mathrm{B})' = \mathrm{A}' \cup \mathrm{B}'$

Solution

Given

$\mathrm{U} = \{1,2,3,4,5,6,7,8,9\}$,

$\mathrm{A} = \{2,4,6,8\}$

$B = \{2, 3, 5, 7\}$

Here $A' = U - A = \{1, 3, 5, 7, 9\}$ and $B' = U - B = \{1, 4, 6, 8, 9\}$

(i) The union of the sets

$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = U - (A \cup B) = \{1, 9\}$

$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$

Therefore the $(A \cup B)' = A' \cap B'$.

(ii) The union of sets

$A \cap B = \{2\}$

$(A \cap B)' = U - (A \cap B) = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore (A \cap B)' = A' \cup B'$

5. Draw appropriate Venn diagram for each of the following:

(i) $(A \cup B)'$,

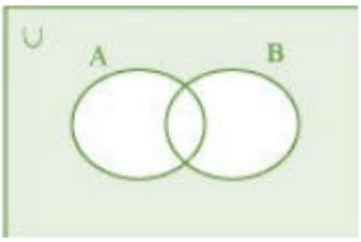
(ii) $A' \cap B'$,

(iii) $(A \cap B)'$,

(iv) $A' \cup B'$

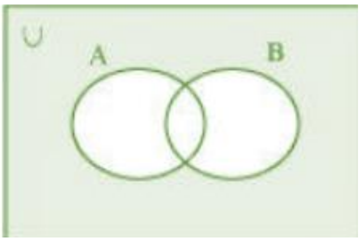
Solution

The venn diagram of $(A \cup B)'$



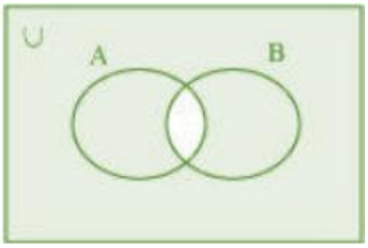
The green region is $(A \cup B)'$

(ii) The venn diagram $A' \cap B'$,

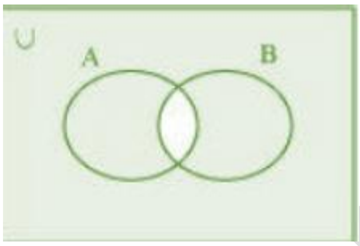


The required region is the green region

(iii) The venn diagram $(A \cap B)'$,



(iv) The venn diagram $A' \cup B'$



6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A' ?

Solution

$U =$ Set of all the triangles in a plane (Universal Set)

$A =$ Set of all the triangles with at least one angle different from $\angle 60$

Use $A' = U - A$

$=$ Set of all triangles with no different angles to 60°

$=$ Set of all triangles per angle 60°

$=$ Set of all equal triangles.

A' is the set of all equal triangles.

7. Fill in the blanks to make each of the following a true statement :

(i) $A \cup A' = \ldots$

(ii) $\phi' \cap A = \ldots$

(iii) $A \cap A' = \ldots$

(iv) $U' \cap A = \ldots$

Solution

(i) $A \cup A'$ (elements of set A) $+$ (elements of universal set that does not belong to set $A =$ Universal Set

The statement of $A \cup A' = U$ [Property of complement sets]

(ii) $\phi' \cap A = U \cap A = A$

The statement of $\phi' \cap A = U \cap A = A$

(iii) No common element between the two sets

The statement of $A \cap A' = \phi$

(iv) The complement of the universal set.

ϕ is the null set.

The statement of $U' \cap A = \phi \cap A = \phi$

Example 23

If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have ?

Solution

Given that

$n(X \cup Y) = 50,$
$n(X) = 28,$
$n(Y) = 32,$
$n(X \cap Y) = ?$

By using the formula

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$$

Substitute the values

$= 28 + 32 - 50$

$= 10$

Example 24

In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics?

Solution

Let M mean a set of math teachers and P mean a set of physics teachers

In the problem statement, the word 'or' gives us a clue as well as the word 'and' gives us a clue.

Number of teachers teaching Maths or Physics $= n(M \cup P) = 20$

Number of teachers teaching Maths $= n(M) = 12$

Number of teachers teaching Maths and physics $= n(M \cap P) = 4$

Number of teachers teaching physics $= n(P) = ?$

Using the result

$n(\mathrm{M} \cup \mathrm{P}) = n(\mathrm{M}) + n(\mathrm{P}) - n(\mathrm{M} \cap \mathrm{P})$

$20 = 12 + n(\mathrm{P}) - 4 \quad n(\mathrm{P}) = 12$

So, $n(\mathrm{P}) = 12$

Hence 12 teachers teach physics.

Example 25

In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

Solution

Let X be the set of students who like to play cricket and Y be the set of students who like to play football.

Then $X \cup Y$ is the set of students who like to play at least one game, and $X \cap Y$ is the set of students who like to play both games.

Given

$n(\mathrm{X}) = 24,$

$n(\mathrm{Y}) = 16,$

$n(\mathrm{X} \cup \mathrm{Y}) = 35,$

$n(\mathrm{X} \cap \mathrm{Y}) = ?$

Using the formula

$n(\mathrm{X} \cup \mathrm{Y}) = n(\mathrm{X}) + n(\mathrm{Y}) - n(\mathrm{X} \cap \mathrm{Y}),$

$35 = 24 + 16 - n(\mathrm{X} \cap \mathrm{Y})$

Thus, $n(\mathrm{X} \cap Y) = 5$

5 students like to play both games.

Example 26

In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Solution

Let U mean a set of surveyed students and A mean a set of students taking apple juice and B means a set of students taking orange juice. Then

Number of students taking apple juice $n(\mathrm{A}) = 100$

Number of students taking orange juice $n(\mathrm{B}) = 150$

Number of students taking both orange juice and apple juice $n(\mathrm{A} \cap \mathrm{B}) = 75.$

Total students $n(\mathrm{U}) = 400,$

Now $n\left(\mathrm{A}' \cap \mathrm{B}'\right) = n(\mathrm{A} \cup \mathrm{B})'$

$= n(\mathrm{U}) - n(\mathrm{A} \cup \mathrm{B})$

$= n(\mathrm{U}) - n(\mathrm{A}) - n(\mathrm{B}) + n(\mathrm{A} \cap \mathrm{B})$

$= 400 - 100 - 150 + 75 = 225$

So, 225 students were not taking apple juice or orange juice.

Example 27

There are 200 individuals with a skin disorder, 120 had been exposed to the chemical $\mathrm{C}_1, 50$ to chemical C_2, and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(i) Chemical C_1 but not chemical C_2

(ii) Chemical C_2 but not chemical C_1

(iii) Chemical C_1 or chemical C_2

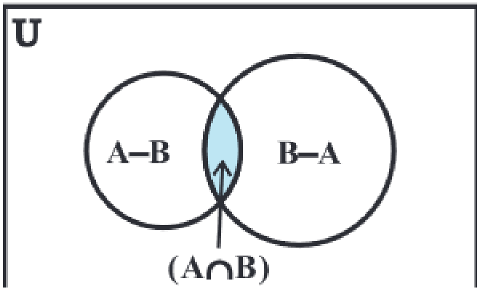
Solution

Let U refers universal set that includes people with skin disorders,

A mean set of people exposed to the chemical C $_1$

B means a set of people exposed to the chemical C_2.

Then, $n(u) = 200, n(A) = 120, n(B) = 50$ and $n(A \cap B) = 30$



(i) From the Venn diagram, we have $A = (A - B) \cup (A \cap B)$ $\therefore n(A) = n(A - B) + n(A \cap B) A - B$ and $A \cap B$ are disjoint

$\therefore n(A - B) = n(A) - n(A \cap B)$

$= 120 - 30 = 90$

Thus, the number of people exposed to the chemical C_1 but not to chemical C_2 is 90 .

(ii) Similarly, $B = (B - A) \cup (A \cap B)$

$n(B) = n(B - A) + n(A \cap B)$

$\because (B - A) \cap (A \cap B) = \varphi$

$\therefore n(B - A) = n(B) - n(A \cap B)$

$= 50 - 30 = 20$

Thus, the number of people exposed to the chemical C_2 and not to chemical C_1 is 20

(iii) Number of people exposed to both chemicals C_1 and $C_2 = n(A \cap B)$

$= 30$

Number of people exposed to the chemical C_1 or to chemical $C_2 = n(A \cup B)$

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 120 + 50 - 30$

$= 140$

Thus, 140 people are exposed to the chemical C_1 or to chemical C_2