Chapter 1: Sets

EXERCISE 1.6

1. If X and Y are two sets such that $n(X)=17, n(Y)=23$ and $n(X\cup Y)=38$, find $n(X\cap Y)$.

Solution

Given that

$n(X)=17,$
$n(Y)=23$

$n(X\cup Y)=38$

Using $n(X\cup Y)=n(X)+n(Y)-n(X\cap Y),$

Substitute the values

$38=17+23-n(X\cap Y)$

$n(X\cap Y)=40-38$

$n(X\cap Y)=2$

2. If X and Y are two sets such that $X\cup Y$ has 18 elements, X has 8 elements and Y has 15 elements ; how many elements does $X\cap Y$ have?

Solution

According to given information,

$n(X\cup Y)=18,$
$n(X)=8$
$n(Y)=15$

Putting values in $n(X\cup Y)=n(X)+n(Y)-n(X\cap Y)$,

we have $18=8+15-n(X\cap Y)$

$\therefore n(X\cap Y)=23-18$

$n(X\cap Y)=5.$

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution

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Let H be the set of people who can speak Hindi and E be the set of people who can speak English. Then,

$n(\mathrm{H} \cup \mathrm{E}) = 400,$
$n(\mathrm{H}) = 250,$
$n(\mathrm{E}) = 200$

Have to find $n(\mathrm{H} \cap \mathrm{E})$.

Using

$n(\mathrm{H} \cup \mathrm{E}) = n(\mathrm{H}) + n(\mathrm{E}) - n(\mathrm{H} \cap \mathrm{E}),$

Substitute the values

$400 = 250 + 200 - n(\mathrm{H} \cap \mathrm{E})$

$n(\mathrm{H} \cap \mathrm{E}) = 450 - 400$

$n(\mathrm{H} \cap \mathrm{E}) = 50$

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and $\mathrm{S} \cap \mathrm{T}$ has 11 elements, how many elements does $\mathrm{S} \cup \mathrm{T}$ have?

Solution

According to given information,

$n(\mathrm{S}) = 21,$
$n(\mathrm{T}) = 32$
$n(\mathrm{S} \cap \mathrm{T}) = 11$

Using $n(\mathrm{S} \cup \mathrm{T}) = n(\mathrm{S}) + n(\mathrm{T}) - n(\mathrm{S} \cap \mathrm{T})$, we get

Substitute the given data

$n(\mathrm{S} \cup \mathrm{T}) \ = 21 + 32 - 11$

$= 53 - 11 = 42$

$\therefore \mathrm{S} \cup \mathrm{T}$ has 42 elements.

5. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Solution

Here, given

$n(X)=40,$
$n(X\cup Y)=60,$
$n(X\cap Y)=10$

Have to find $n(Y)$.

Using $n(X\cup Y)=n(X)+n(Y)-n(X\cap Y)$,

Substitute the values

$60=40+n(Y)-10$

$60=30+n(Y)$

$n(Y)\;=30$

6. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution

Let C be the set of people who like coffee and T be the set of people who like tea.

Given data

$n(C\cup T)=70,$
$n(C)=37,$
$n(T)=52$

Using $n(C\cup T)=n(C)+n(T)-n(C\cap T)$, we get

Substitute the values

$70=37+52-n(C\cap T)$

$n(C\cap T)=89-70=19$

19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Solution

Let C mean a set of people liked cricket and T mean a set of people liked tennis.

Given information

$n(C \cup T) = 65,$
$n(C) = 40,$
$n(C \cap T) = 10$

Using $n(C \cup T) = n(C) + n(T) - n(C \cap T)$,

Substitute the values

$65 = 40 + n(T) - 10$

$65 = 30 + n(T)$

$n(T) = 35$

$\therefore$ 35 people like tennis.

Now, $n(T - C) = n(T) - n(C \cap T)$

Substitute the values

$= 35 - 10$

$= 25$

25 people like tennis only and not cricket.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Solution

Let F means a set of people speaking French and S means a set of people speaking Spanish.

Given information

$n(F) = 50,$
$n(S) = 20,$
$n(F \cap S) = 10$

To find the number of people speaking at least one of the two languages,

That is $n(F \cup S)$

$n(F \cup S) = n(F) + n(S) - n(F \cap S)$

Substitute the values

$= 50 + 20 - 10 = 60$

The required number of people speaking at least one of the two languages is 60.

Example 28

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Show that the set of letters needed to spell " CATARACT " and the set of letters needed to spell " TRACT" are equal.

Solution

Let X be the set of letters in "CATARACT".

Then $X = \{C, A, T, R\}$

Let Y be the set of letters in "TRACT". Then

$Y = \{T, R, A, C, T\} = \{T, R, A, C\}$

Since every element in X is in Y and every element in Y is in X.

It follows that $X = Y$.

Example 29

List all the subsets of the set $\{-1, 0, 1\}$.

Solution

Given Set $A = \{-1, 0, 1\}$.

The element A-free element is an empty set of ϕ. The sub-sets of A have one item in $\{-1\}, \{0\}, \{1\}$. The sub-sets of A have two element $\{-1, 0\}, \{-1, 1\}, \{0, 1\}$. A subset of A with three elements of A is A itself.

Subsets of given set $= \phi, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{0, 1\}, \{1, -1\}, \{-1, 0, 1\}$

Thus, all sets under A are $\phi, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{-1, 1\}$, $\{0, 1\}$ and $\{-1, 0, 1\}$

Example 30

Show that $A \cup B = A \cap B$ implies $A = B$

Solution

Let $a \in$ A. Then $a \in A \cup B$. Since $A \cup B = A \cap B, a \in A \cap$ B. So $a \in B$.

Therefore, $A \subset B$. Similarly, if $b \in B$, then $b \in A \cup B$.

Since $A \cup B = A \cap B, b \in A \cap$ B. So, $b \in$ A.

Therefore, $B \subset$ A. Thus, $A = B$

Example 31

For any sets A and B, show that $P(A\cap B)=P(A)\cap P(B)$

Solution

Let a set X belong to Power set $P(A\cap B)$

$X\in P(A\cap B)$

As set X is in the power set of $A\cap B, X$ is a subset of $A\cap B$ because power set is the set of all subsets

Thus, X is a subset of $A\cap B$

$X\subset A\cap B$.

So, $X\subset A$ and $X\subset B$.

Therefore, Since X is a subset of $A \,\&\, B, X$ is in power set of A and X is in power set of B

$X\in P(A)$ and $X\in P(B)$

$X\in P(A)$ and $X\in P(B)$

$\Rightarrow X\in P(A)\cap P(B)$

So, if $X\in P(A\cap B)$, then $X\in P(A)\cap P(B)$

All elements of set $P(A\cap B)$ are in set $P(A)\cap P(B)$

Thus, gives $\mathbf{P(A\cap B)\subset P(A)\cap P(B)}$.

Similarly,

Let a set Y belong to Power set $P(A)\cap P(B)$

i.e. $Y\in P(A)\cap P(B)$

Then $Y\in P(A)$ and $Y\in P(B)$

As set Y is in the power set of $A \,\&B, Y$ is a subset of $A \,\&Y$ is a subset of B because power set is the set of all subsets

Thus, $\mathrm{Y\subset A}$ and $\mathrm{Y\subset B}$

$\therefore Y\subset A\cap B$

Therefore, since Y is a subset of $A\cap B, Y$ is in power set of $A\cap B$

$\Rightarrow Y\in P(A\cap B)$

So, if $Y\in P(A)\cap P(B)$, then $Y\in P(A\cap B)$

This gives $\mathbf{P(A)\cap P(B)\subset P(A\cap B)}$

Now,

Since $P(A\cap B)\subset P(A)\cap P(B)$ & $P(A)\cap P(B)\subset P(A\cap B)$

Hence, $P(A\cap B)=P(A)\cap P(B)$.

Example 32

A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both products?

Solution

Set of buyers who liked the product A & product B to A & B respectively

Number of buyers who liked product $A=n(A)=720$,

Number of buyers who liked product $B=n(B)=450$

We know that

$n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$n(A\cup B)=720+450-n(A\cap B)$

$n(A\cup B)=1170-n(A\cap B)$

$n(A\cap B)=1170-n(A\cup B)$

Therefore, $n(A\cap B)$ is less than $n(A\cup B)$ is maximum

Number of people who liked product A or B (i.e. n (A U B)) is higher than the number of people surveyed

So, maximum value of $n(A\cup B)$ is 1000.

Thus, the least value of $n(A\cap B)$ is

$n(A\cap B)=1170-n(A\cup B)$

$=1170-1000$

$=170$

Hence, the minimum number of consumers who prefer both products is 170.

Example 33

Out of 500 car owners investigated, 400 owned car A and 200 owned car $B, 50$ owned both A and B cars. Is this data correct?

Solution

Let the set of owners of car A & car B became A & B respectively

Numbers of car owned by A $= n(A) = 400$

Numbers of car owned by B $= n(B) = 200$

Number of car owned by A and B $= n(A \cap B) = 50$

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 200 + 400 - 50$

$= 550$

Thus, number of people who own car A or car B $= 550$

But total number of car owners is only 500 (given in question)

Number of persons owning car A or car B is less than total number of car owners investigated

But, here $550 > 500$

This is a contradiction.

So, the data provided is incorrect.

Example 34

A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports?

Solution

Let F, B, C means a set of people received medals in football, basketball & cricket respectively

Given,

Number of medals won in football $= n(F) = 38$,

Number of medals won in basketball $= n(B) = 15$,

Number of medals won in cricket $= n(C) = 20$

Number of medals won in either football, basketball or cricket $= n(F \cup B \cup C) = 58$

Number of medals won in all football, basketball and cricket $= n(F \cap B \cap C) = 3$

We know

$n(F \cup B \cup C) = n(F) + n(B) + n(C) - n(F \cap B) - n(F \cap C) - n(B \cap C) + n(F \cap B \cap C)$

Substitute the values

$58 = 38 + 15 + 20 - n(F \cap B) - n(F \cap C) - n(B \cap C) + 3$

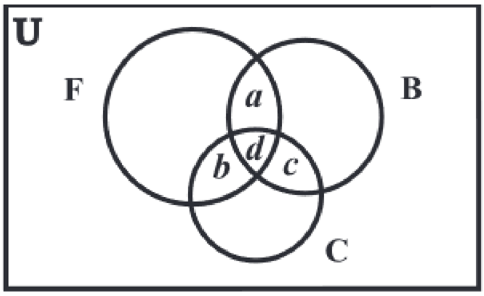
$$58 = 38 + 15 + 20 + 3 - n(F \cap B) - n(F \cap C) - n(B \cap C)$$

$$58 = 76 - n(F \cap B) - n(F \cap C) - n(B \cap C)$$

Rearrange the equation

$$n(F \cap B) + n(F \cap C) + n(B \cap C) = 76 - 58$$

$$n(F \cap B) + n(F \cap C) + n(B \cap C) = 18$$



Also, In the Venn diagram,

Let **a** means the number of men who win medals in football & basketball but not cricket

Let **b** means the number of men win medals in football & cricket but not basketball

Let **c** means the number of men win got medals in cricket & basketball but not football

Let **d** means the number of men who win medals in football & basketball & cricket

Here $d = n(F \cap B \cap C)$

$\therefore d = 3$

Number of people who received exactly two medals $= a + b + c$

From equation

$$n(F \cap B) + n(F \cap C) + n(B \cap C) = 18$$

Rearrange the equation as per diagram

$$(a + d) + (b + d) + (c + d) = 18$$

$$a + b + c + d + d + d = 18$$

Rearrange the equation

$$a + b + c + 3d = 18$$

$$a + b + c + 3 \times 3 = 18$$

$$a + b + c + 9 = 18$$

Arrange in common terms

$$a + b + c = 18 - 9$$

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$a + b + c = 9$

$\therefore 9$ people who won medals in two of the three sports