Chapter: Three-dimensional Geometry.

Exercise: Miscellaneous Exercise

1. We need to show that the line determined by the points (3,-5,1), (4,3,-1) is perpendicular to the line joining the origin to the point (2,1,1)

Solution: Let us consider the points be $B(3,5,-1)$ and $C(4,3,-1)$ and the line joining the origin $O(0,0,0)$ and $A(2,1,1)$

We can tell that the direction ratios of OA and BC will be 2, 1, 1 and $(4-3)=1,(3-5)=-2$ and $(-1+1)=0$ respectively

As we know that for lines to be perpendicular, then $a_1a_2+b_1b_2+c_1c_2=0$

Now, $a_1a_2+b_1b_2+c_1c_2=2\times1+1\times(-2)+1\times0$

$=2-2+0$

$=0$

Therefore, the lines are perpendicular

2. We needed to show that direction cosines of the perpendicular to both of the lines $m_1n_2-m_2n_1, n_1l_2-n_2l_1, l_1m_2-l_2\,\mathrm{m}_1$ when l_1,m_1,n_1 and l_2,m_2,n_3 are the direction cosines of two mutually perpendicular lines.

Solution: Let us take $l_1l_2+m_1m_2+n_1n_2=0...(1), l_1^2+m_1^2+n_1^2=1...(2), l_1^2+m_1^2+n_1^2=1.....(3)$

Let us consider l,m,n be the direction cosines of the line with direction cosines l_1,m_1,n_1 and l_2,m_2,n_2

We got, $ll_1+mm_1+nn_1=0$ and $ll_2+mm_2+nn_2=0$

$$\therefore \frac{1}{m_1n_2-m_2n_1}=\frac{m}{n_1l_2-n_2l_1}=\frac{n}{l_1m_2-l_2m_1}$$

$$\Rightarrow \frac{l^2}{(m_1n_2-m_2n_1)^2}=\frac{m^2}{(n_1l_2-n_2l_1)^2}=\frac{n^2}{(l_1m_2-l_2m_1)^2}$$

$$=\frac{l^2+m^2+n^2}{(m_1n_2-m_2n_1)^2+(n_1l_2-n_2l_1)^2+(l_1m_2-l_2m_1)^2}......(4)$$

As we know that l, m, n are the direction cosines of the line, we get that

$l^2+m^2+n^2=1......(5)$

As we know that,

$$\left(l_1^2+m_1^2+n_1^2\right)\left(l_2^2+m_2^2+n_2^2\right)-\left(l_1l_2+m_1m_2+n_1n_2\right)$$

$$=\left(m_1n_2-m_2n_1\right)^2+\left(n_1l_2-n_2l_1\right)^2+\left(l_1m_2-l_2m_1\right)^2$$

Now, from $(1),(2),(3)$ we get

$$\Rightarrow 1.1-0=\left(m_1n_2-m_2n_1\right)^2+\left(n_1l_2-n_2l_1\right)^2+\left(l_1m_2-l_2m_1\right)^2$$

$$\therefore\left(m_1n_2-m_2n_1\right)^2+\left(n_1l_2-n_2l_1\right)^2+\left(l_1m_2-l_2m_1\right)^2=1....(6)|$$

By putting the values from equation (5) and (6) in equation (4), we get

$$\frac{l^2}{\left(m_1n_2-m_2n_1\right)^2}=\frac{m^2}{\left(n_1l_2-n_2l_1\right)^2}=\frac{n^2}{\left(l_1m_2-l_2m_1\right)^2}=1$$

$$\Rightarrow 1=m_1n_2-m_2n_1,$$

$$m=n_1l_2-n_2l_1,$$

$$n=l_1m_2-l_2m_1$$

Therefore, the direction cosines of the required line are $m_1n_2-m_2n_1, n_1l_2-n_2l_1, l_1m_2-l_2m_1$.

3. The direction ratios are a, b, c and b-c, c-a, a-b, find the angle between the lines

Solution: As we know that, for any angle $\theta,$ with direction cosines, a, b, c and b-c, c-a, a-b can be found by,

$$\cos\theta=\left|\frac{a(b-c)+b(b-c)+c(c-a)}{\sqrt{a^2+b^2+c^2+\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}}\right|$$

Solving this we get, $\cos\theta=0$

$$\theta=\cos^{-1}0$$

$$\Rightarrow\theta=90^\circ$$

Therefore, the angle between the two lines will be 90°.

4. Find the equation of a line passing through the origin and line parallel to x-axis

Solution: As it is given that the line is passing through the origin and is also parallel to x-axis is x-axis,

Now,

Let us consider a point on x-axis be A

So, the coordinates of A will be (a, 0, 0)

Now, the direction ratios of OA will be,

$\Rightarrow (a-0) = a, 0, 0$

The equation of $OA \Rightarrow \frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0} \Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = a$

Therefore, the equation of the line passing through origin and parallel to x-axis is

$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$.

5. Find the angle between the lines AB and CD if the coordinates of the points A, B, C, D be(1, 2, 3), (4, 5, 7), (-4,3,-6) and (2, 9, 2) respectively.

Solution: It is given that coordinates A, B, C, D are (1, 2, 3), (4, 5, 7), (-4, 3, -6) and (2, 9, 2) respectively

We know that,

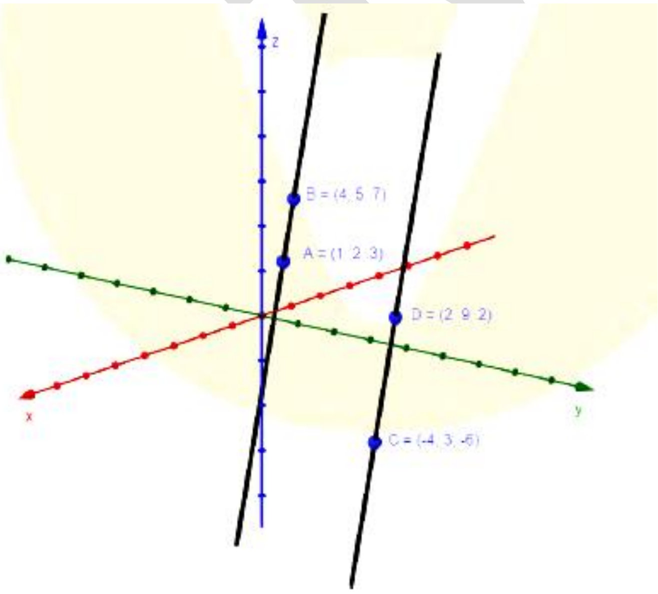
$a_1 = (4-1) = 3, b_1 = (5-2) = 3, c_1 = (7-3) = 4$

$a_2 = (2-(-2)) = 6, b_2 = (9-3) = 6, c_2 = (2-(-6)) = 8$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

$\Rightarrow AB \parallel CD$

We get to know that the lines are parallel to each other.



Therefore, the angle between AB and CD is either $0°$ or $180°$

6. Find the value of k if the lines $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ and $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ are perpendicular.

Solution: From the given equation we can say that $a_1 = -3, b_1 = 2k, c_1 = 2$ and

$a_2 = 3k, b_2 = 1, c_2 = -5$.

We know that the two lines are perpendicular, if $a_{1=0}a_2 + b_1b_2 + c_1c_2 = 0$

$-3(3k) + 2k \times 1 + 2(-5) = 0$

$\Rightarrow -9k + 2k - 10 = 0$

$\Rightarrow 7k = -10$

$\Rightarrow k = \frac{-10}{7}$

Therefore, the value of k is $-\frac{-10}{7}$

7. Find the vector equation of the perpendicular to the plane $\vec{r}.\left(\hat{i} + 2\hat{j} - 5\hat{k}\right) + 9 = 0$ and passing through (1, 2, 3)

Solution: According to the question, we can say that we have

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$\vec{N} = \hat{i} + 2\hat{j} - 5\hat{k}$

As we know we can express the equation of a line passing through a point and perpendicular to the plane in form $\vec{l} = \vec{r} + \lambda\vec{N}, \lambda \in R$

We got,

$\vec{l} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$

Therefore, the vector equation to the plane will be $\vec{l} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$.

8. Find the equation of the plane parallel to the plane $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$ and passing through (a, b, c)

Solution: According to the question, plane is parallel to plane $\vec{r}_1.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$ and it also passes through point (a, b,c)

From this we get the equation,

$\left(a\hat{i} + b\hat{j} + c\hat{k}\right).\left(\hat{i} + \hat{j} + \hat{k}\right) = \lambda$

$\Rightarrow a + b + c = \lambda$

Now, putting value in equation, we get,

$$\vec{r}_1.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c$$

Now we will put $\vec{r}_1.\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$ in equation, we get

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}+\hat{j}+\hat{k}\right)+a+b+c \Rightarrow x+y+z-a+b+c$$

Therefore, the equation of the plane will be $x+y+z=a+b+c$.

9. What is the shortest distance between these two lines

$$\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

$$\vec{r}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$$

Solution: According to the question, we need to find the distance between the lines,

$$\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

$$\vec{r}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$$

As we know we can find the shortest distance by,

$$d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_1-\vec{a}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$

Now, from the equation of lines we get

$$\vec{a}_1=6\hat{i}+2\hat{j}+2\hat{k}$$

$$\vec{b}_1=\hat{i}-2\hat{j}+2\hat{k}$$

$$\vec{a}_2=-4\hat{i}-\hat{k}$$

$$\vec{b}_2=3\hat{i}-2\hat{j}-2\hat{k}$$

$$\Rightarrow \overrightarrow{a_2}-\overrightarrow{a_1}=\left(-4\hat{i}-\hat{k}\right)-\left(6\hat{i}+2\hat{j}+2\hat{k}\right)=-10\hat{i}-2\hat{j}-3\hat{k}$$

$$\Rightarrow \overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}=(4+4)\hat{i}-(-2-6)\hat{j}+(-2+6)\hat{k}$$

$$=8\hat{i}+8\hat{j}+4\hat{k}$$

$$\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2-\vec{a}_1\right)=\left(8\hat{i}+8\hat{j}+4\hat{k}\right).\left(-10\hat{i}-2\hat{j}-3\hat{k}\right)$$

$$=-80-16-12$$

$$=-108$$

Now, putting these values in $d=\left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_1-\vec{a}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$, we get

$$d=\left|\frac{-108}{12}\right|=9$$

Therefore, shortest distance between the above two lines is of 9 units.

10. Find the point of intersection where the line passing through (5, 1, 6) and (3, 4, 1) intersecting through the YZ plane.

Solution: We know that the equation of the line passing through the points is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1},$$

Now, according to the question, the line is passing through the point, (5, 1, 6) and (3, 4, 1), we get

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}\Rightarrow\frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=k$$

$$\Rightarrow x=5-2k, y=3k+1, z=6-5k$$

Now we know that any point on the line will be of form $(5-2k,3k+1,6-5k)$,

Now for YZ plane, x=0, we get

$$x=5-2k=0\Rightarrow k=\frac{5}{2}$$

$$\Rightarrow y=3k+1=3\times\frac{5}{2}+1=\frac{17}{2}$$

$$\Rightarrow z=6-5k=6-5\times\frac{5}{2}=-\frac{13}{2}$$

Therefore, the required point of intersection $\left(0,\frac{17}{2},\frac{-13}{2}\right)$

11. Find the point of intersection where the line crosses through the ZX plane and through (5, 1, 6), (3, 4, 1)

Solution: As we know that the equation of the line passing through the point is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1},$$

According to the question, the line passing through (5, 1, 6) and (3, 4, 1), we got

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}\Rightarrow\frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=k$$

$$\Rightarrow x = 5 - 2k, y = 3k + 1, z = 6 - 5k$$

As we know for any point on the line, it will be in the form of (5-2k,3k+1,6-5k),

Now, for ZX plane, y=0

$$\Rightarrow y = 3k + 1 = 0 \Rightarrow k = \frac{-1}{3}$$

$$\Rightarrow x = 5 - 2k = 5 - 2\times\left(\frac{-1}{3}\right) = \frac{17}{3}$$

$$\Rightarrow z = 6 - 5k = 6 - 5\times\left(\frac{-1}{3}\right) = \frac{23}{3}$$

Therefore, the required point of intersection $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

12. Find the point of intersection where the line crosses through the plane $2x + y + z = 7$ and through (3, -4, -5), (2,-3,1)

Solution: As we know that the equation of the line passing through the points

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1},$$

According to the question, the line passing through (3, -4, -5) and (2, -3, 1), we get,

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} \Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = k$$

$$\Rightarrow x = 3 - k, y = k - 4, z = 6k - 5$$

As we know that the point on the line will be in the form of $(3-k, k-4, 6k-5)$.

As the point lies on $2x + y + z = 7,$ we get

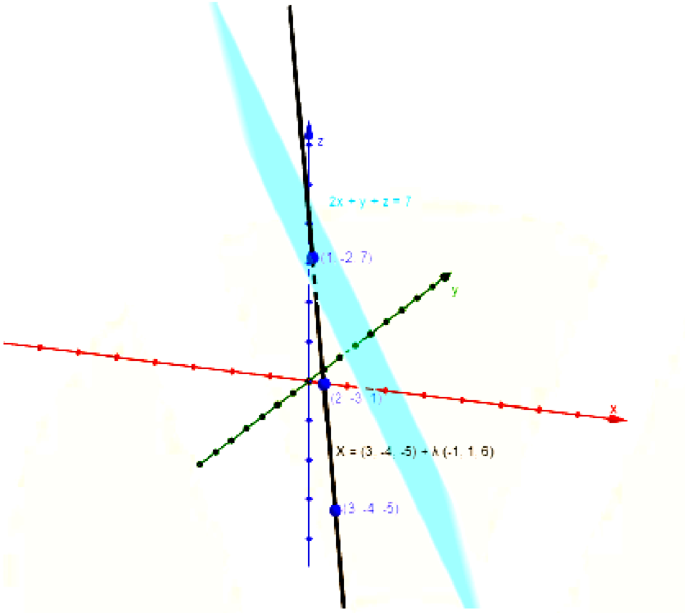
$$2(3-k) + (k-4) + (6k-5) = 7$$

$$\Rightarrow 5k - 3 = 7$$

$$\Rightarrow k = 2$$

Now, by putting the value of k in equation, we get

$$(3-k, k-4, 6k-5) = (3-2, 2-4, 6(2)-5) = (1, -2, 7)$$



Therefore, the point on the required plane (1, -2, 7)

13. Find the equation of the plane passing through the points (-1, 3,2) and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$.

Solution: As we know that the equation for plane passing through the point can be given as,

$a(x+1)+b(y-3)+c(z-2)=0$

Now that we know a, b, c are direction ratios of normal to the plane,

We know that $a_1a_2+b_1b_2+c_1c_2=0$ if the lines are perpendicular to each other,

Now, if $x+2y+3z=5$ is perpendicular then,

$a.1+b.2+c.3=0$

$\Rightarrow a+2b+3c=0$

And if $3x+3y+z=0$ is perpendicular then,

$a.3+b.3+c.1=0$

$\Rightarrow 3a+3b+c=0$

Now,

$$\frac{a}{2\times1-3\times3}=\frac{b}{3\times3-1\times1}=\frac{c}{1\times3-2\times3}$$

$$\Rightarrow \frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=k$$

$\Rightarrow a=-7k, b=8k, c=-3k$

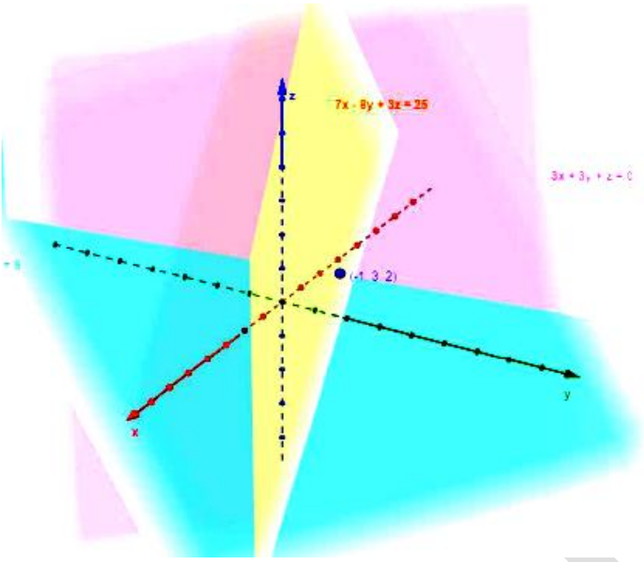
By putting values of a, b, c we get

$$-7k(x+1)+8k(y-3)-3k(z-2)=0$$

$$\Rightarrow(-7x-7)+(8y-24)-3z+6=0$$

$$\Rightarrow -7x+8y-3z-25=0$$

$$\Rightarrow 7x-8y+3z=25$$



Therefore, the equation of the plane will be $7x-8y+3z=25$

14. For $\vec{r}=\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0,$ the points (1, 1, p) and (-3,0,1) are at equal distance from the plane, then find the value of p

Solution: According to the question the vectors are,

$$\vec{a}_1=\hat{i}+\hat{j}+p\hat{k}, \vec{a}_2=-4\hat{i}+\hat{k}$$

And the plane 's equation is $\vec{r}=(3\hat{i}+4\hat{j}-12\hat{k})+13=0$

As we know that the perpendicular distance between vector and the plane can be found by $\vec{r}.\vec{N}=d$,

Now,

$$D=\left|\frac{\vec{a}.\vec{N}-d}{\vec{N}}\right|$$

$\vec{N}=3\hat{i}+4\hat{j}-12\hat{k}$ and $d=-13$

Then, the distance between the point (1, 1, p) and the given plane is

$$D_1=\left|\frac{\left(\hat{i}+\hat{j}+p\hat{k}\right).\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13}{3\hat{i}+4\hat{j}-12\hat{k}}\right|$$

$$\Rightarrow D_1 = \left| \frac{3+4-12p+13}{\sqrt{3^2+4^2+(-12)^2}} \right|$$

$$\Rightarrow D_1 = \left| \frac{20-12p}{13} \right| \text{.......}(i)$$

Similarly, the distance between the point (-3,0,1) and the given plane is

$$D_2 = \left| \frac{\left(-3\hat{i}+\hat{k}\right).\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13}{3\hat{i}+4\hat{j}-12\hat{k}} \right|$$

$$\Rightarrow D_2 = \left| \frac{-9-12+13}{\sqrt{3^2+4^2+(-12)^2}} \right|$$

$$\Rightarrow D_1 = \frac{8}{13} \text{..............}(ii)$$

Now, from the given conditions,

$$D_1 = D_2$$

$$\Rightarrow \frac{|20-12p|}{13} = \frac{8}{13}$$

$$\Rightarrow 20-12p = 8, -(20-12p) = 8$$

$$\Rightarrow 12p = 12, 12p = 28$$

$$\Rightarrow p = 1, p = \frac{7}{3}$$

Therefore, the value will be, $p = 1, p = \frac{7}{3}$

15. Find the equation of the plane parallel to x-axis and passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$

Solution: We have been given the two planes, $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1 \Rightarrow \vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1=0$ and

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$$

Now, we know that the equation of line passing through the line of intersection will be,

$$\left[\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right]+\lambda\left[\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4\right]=0$$

$\vec{r}.\left[(2\lambda+1)\hat{i}+(3\lambda+1)\hat{j}+(1-\lambda)\hat{k}\right]+(4\lambda+1)=0$

$a_1=(2\lambda+1), b_1=(3\lambda+1), c_1=(1-\lambda)$.

As we know that the required plane is to be parallel to x-axis, the normal will perpendicular to x-axis,

Now, the direction ratios of x-axis will be 1, 0, and 0, which means

$a_2=1, b_2=0, c_2=0$

$1.(2\lambda+1)+0(3\lambda+1)+0(1-\lambda)=0$

$\Rightarrow 2\lambda+1=0$

$\Rightarrow \lambda=-\frac{1}{2}$.

By putting $\lambda=-\frac{1}{2}$ in (1)

$\Rightarrow \vec{r}.\left[-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right]+(-3)=0 \Rightarrow \vec{r}\left(\hat{j}-3\hat{k}\right)+6=0$

Therefore, the required Cartesian equation of the plane is $y-3z+6=0$

16. If O be the origin and the coordinates of P be (1, 2, -3), then find the equation of the plane parallel to x-axis and passing though P.

Solution: From the question we know that the direction ratios of OP will be

$a=(1-0)=1, b=(2-0)=2, c=(-3-0)=-3$

Now, we know that the equation will be as,

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

As we can tell, the direction ratios of the normal are 1,2,3

Therefore, the point is P(1,2,-3)

Therefore, the equation of the plane is

$1(x-1)+2(y-2)-3(z+3)=0$

$\Rightarrow x+2y-3z-14=0$

17. Find the equation of the plane which holds the line of intersection of the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0, \vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$

Solution: According to the question, it is given that,

$\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ ....(1)

$\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ ....(2)

Now, we know that the equation of the required plane will be,

$$\left[\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4\right]+\lambda\left[\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5\right]=0$$

$$\Rightarrow \vec{r}.\left[(2\lambda+1)\hat{i}+(\lambda+2)\hat{j}+(3-\lambda)\hat{k}\right]+(5\lambda-4)=0.....(3)$$

Now according to the question, the plane is perpendicular to the plane,

So, $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$

$$\therefore 5(2\lambda+1)+3(\lambda+2)-6(3-\lambda)=0$$

$$\Rightarrow 19\lambda-7=0$$

$$\Rightarrow \lambda=\frac{7}{19}$$

By putting value of $\lambda=\frac{7}{19}$ in equation(3)

$$\Rightarrow \vec{r}.\left[\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right]-\frac{41}{9}=0$$

Therefore, the required Cartesian Equation of the plane is $33x+45y+50z-41=0$

18. Find the distance of the point $(-1,5,-10)$ from the point of intersection of line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

Solution: According to the question, it is given that line is,

$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)....(1)$ and the plane is , $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5....(2)$

Now, we will put the value of $\vec{r}$ from (1) into (2), we get

$$\left[2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$

$$\Rightarrow \left[(3\lambda+2)\hat{i}+(4\lambda+1)\hat{j}+(2\lambda+2)\hat{k}\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$

$$\Rightarrow (3\lambda+2)-(4\lambda-1)+(2\lambda+2)=5$$

$$\Rightarrow \lambda=0$$

Now if we put the value in equation, we will get the equation of the line as

$$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$$

Therefore, the required distance between both the points is,

$$d=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}=\sqrt{9+16+144}=\sqrt{169}=13$$

Therefore, the distance between the |points is 13 units.

19. Find the vector equation of the line parallel to the planes $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$ and passing through (1, 2, 3)

Solution: Let us consider that line parallel to vector $\vec{b}$ is given by, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

Now, the position vector of point (1, 2, 3) will be $\hat{a}=\hat{i}+2\hat{j}+3\hat{k}$

From this we get that the equation of line passing through (1, 2, 3) and is parallel to vector $\vec{b}$ will be,

$$\vec{r}=\hat{a}+\lambda\hat{b}$$

$$\Rightarrow \vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right).....(1)$$

Therefore, the equation of the given planes are

$$\vec{r}.\left(\hat{i}-\hat{j}+5\hat{k}\right)=5 \qquad ....(2)$$

$$\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6 \qquad .....(3)$$

As we can tell that the line in equation (1) and plane in equation (2) are parallel, so we get that the normal to the plane of equation (2) and the given line are perpendicular

$$\Rightarrow\left(\hat{i}-2\hat{j}+2\hat{k}\right).\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)=0$$

$$\Rightarrow(b_1-b_2+2b_3)=0$$

$$\Rightarrow b_1-b_2+2b_3=0 \qquad ......(4)$$

Similarly,

$$\left(3\hat{i}+\hat{j}+\hat{k}\right).\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)=0$$

$$\Rightarrow\lambda(3b_1+b_2+b_3)=0$$

$$\Rightarrow 3b_1+b_2+b_3=0 \qquad .....(5)$$

From equation (4) and (5), we get

$$\frac{b_1}{(-1)\times1-1\times2}=\frac{b_2}{2\times3-1\times1}=\frac{b_3}{1\times1-3(-1)}$$

$$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$$

Now, the direction ratios of $\vec{b}$ are $-3, 5, 4$

Which means $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$

Putting the value of $\vec{b}$ in equation (1)

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

20. Find the vector equation of the line passing through the points (1, 2, -4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Solution: According to the question, we get that $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

We know that the equation of the line passing through point and also parallel to vector, we get

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) \ldots (1)$$

Now, the equation of the two lines will be

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \ldots .(2)$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \ldots .(3)$$

As we know that line (1) and (2) are perpendicular to each other, we get

$$3b_1 - 16b_2 + 7b_3 = 0 \ldots ..(4)$$

Also, we know that the line (1) and (3) are perpendicular to each other, we get

$$3b_1 + 18b_2 - 5b_3 = 0 \ldots ..(5)$$

Now, from equation (4) and (5) we get that

$$\frac{b_1}{(-16)(-5) - 8(7)} = \frac{b_2}{7(3) - 3(-5)} = \frac{b_3}{3(8) - 3(-16)}$$

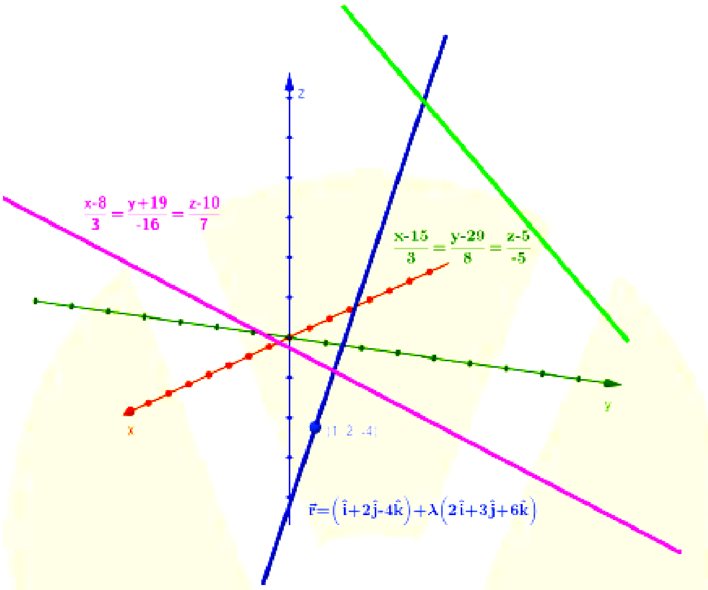
$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{36} = \frac{b_3}{72} \Rightarrow \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$$

Therefore, direction ratios of $\vec{b}$ are 2, 3, 6

Which means $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Putting $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ in equation (1), we get

$$\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$



Therefore, the vector equation will be $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$

21. Prove that if a plane has the intercepts a,b,c and is a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

Solution: We know that the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

The distance of the plane will be,

$$p=\left|\frac{\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}\right|\Rightarrow p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\Rightarrow\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

Therefore, we have proved that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

22. Distance between the two planes : $2x+3y+4z=4$ and $4x+6x+8z=12$ is

(A) 2 units

(B) 4 units

(C) 8 units

(D) $\frac{2}{\sqrt{29}}$ units

Solution: According to the question, the equation of the planes are

$$2x+3y+4z=4$$

$$4x+6y+8z=12$$

We get $2x+3y+4z=6$

As we can tell, the given planes are parallel,

We know that the distance between two parallel planes, is given by,

$$D=\left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right| \Rightarrow D=\left|\frac{6-4}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow D=\frac{2}{\sqrt{29}}$$

Therefore, the distance between two parallel planes is $\frac{2}{\sqrt{29}}$ units.

Therefore, the correct answer is D.

23. The planes : $2x-y+4z=5$ and $5x-2.5y+10z=6$

(A) Perpendicular

(B) Parallel

(C) Intersect y axis

(D) Passes through $\left(0,0,\frac{5}{4}\right)$

Solution: According to the question we get,

$$2x-y+4z=5$$

$$5x-2.5y+10z=6$$

As we can see that,

$$\frac{a_1}{a_2}=\frac{2}{5},\frac{b_1}{b_2}=\frac{-1}{-2.5}=\frac{2}{5},\frac{c_1}{c_2}=\frac{4}{10}=\frac{2}{5}$$

$$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

As we can see that the given lines are parallel,

Therefore, the correct answer is B.