Chapter: Three dimensional Geometry.

Exercise: 11.1

1. Find the direction cosines if the line makes angles $90°, 135°, 45°$ with x,y and z axes respectively.

Solution: Let us consider 1,m and n be the direction cosines of line

Then,

$1 = \cos 90° = 0,$

$m = \cos 135°$

$= \cos(90° + 45°)$

$= \sin 45°$

$= -\frac{1}{\sqrt{2}}$

And, $n = \cos 45° = \frac{1}{\sqrt{2}}$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$

2. Find the direction cosines if the line makes equal angles with the coordinates axes.

Solution: Let us consider that the line makes an angle α with coordinates axes

Which means $1 = \cos\alpha, m = \cos\alpha, n = \cos\alpha$ Now, we know that

$1^2 + m^2 + n^2 \Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = \frac{1}{3} \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$

Therefore, the direction cosines of the line are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.

3. Find the direction cosines of a line having direction ratios -18,12,-4

Solution: We have the direction ratios as -18, 12-4, Now, the direction cosines will be as

$$1=\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}},$$

$$m=\frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}},$$

$$n=\frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$$

$$\frac{-18}{22},\frac{12}{22},\frac{-4}{22}\Rightarrow\frac{-9}{11},\frac{6}{11},\frac{-2}{11}$$

Therefore, direction cosines of the line are $\frac{-9}{11},\frac{6}{11}$ and $\frac{-2}{11}$.

4. Show that $(2,3,4),(-1,-2,1),(5,8,7)$ are collinear.

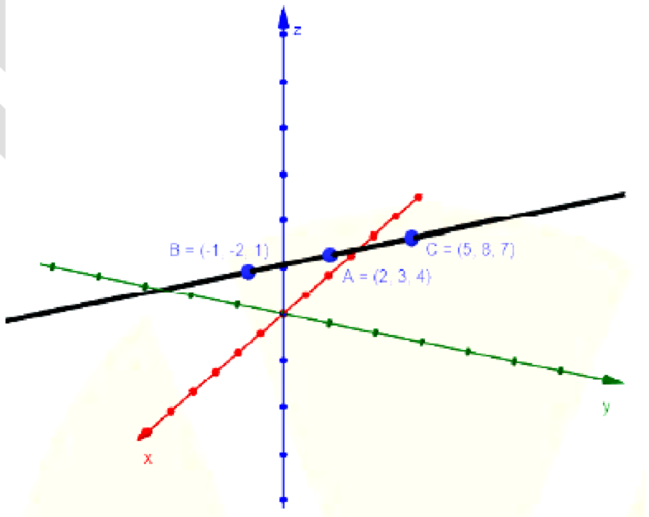
Solution: Let us consider the points be A(2, 3, 4), B(-1, -2, 1) and C(5, 8, 7).

Now, as we know that direction cosines can be found by $(x_2-x_1),(y_2-y_1)$, and (z_2-z_1)

Therefore,

Direction ratios of AB and BC be -3, -5, -3 and 6, 10, 6 respectively.

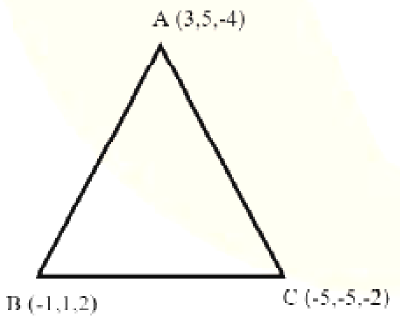
As we can see that AB and BC are proportional, we get that AB is parallel to BC.



Therefore, the points are collinear.

5. If the vertices of a triangles are $(3,5,-4),(-1,1,2),(-5,-5,-2)$, find its directions cosines.

Solution: Let us consider the points be $A(3,5,-4), B(-1,1,2)$ and $C(-5,-5,-2)$.



Now, the direction ratios of AB will be -4, -4 and 6, we get

$$\sqrt{(-4)^2+(-4)^2+(6)^2}=\sqrt{68}\Rightarrow 2\sqrt{17}$$

Now,

$$1=\frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}}, m=\frac{-4}{\sqrt{(-4)^2(-4)^2+(6)^2}}, n=\frac{6}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$$

$$\Rightarrow 1=\frac{-2}{\sqrt{17}}, m=\frac{-2}{\sqrt{17}}, n=\frac{3}{\sqrt{17}}$$

Therefore, the direction cosines of AB are $\frac{-2}{\sqrt{17}},\frac{-2}{\sqrt{17}},\frac{3}{\sqrt{17}}$

Similarly, the direction ratios of side BC will be -4, -6 and -4.

Now,

$$1=\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, m=\frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, n=\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$1=\frac{-4}{2\sqrt{17}}, m=\frac{-6}{2\sqrt{17}}, n=\frac{-4}{2\sqrt{17}}$$

Therefore, the direction cosines of BC is $\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$

Similarly, the direction ratios of CA will be -8, -10 and 2.

Now,

$$1 = \frac{-8}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, m = \frac{10}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, n = \frac{2}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}$$

$$1 = \frac{-8}{2\sqrt{42}}, m = \frac{-10}{2\sqrt{42}}, n = \frac{2}{2\sqrt{42}}.$$

Therefore, the direction cosines of CA is $\frac{-4}{\sqrt{42}}, \frac{-5}{\sqrt{42}}, \frac{1}{\sqrt{42}}$