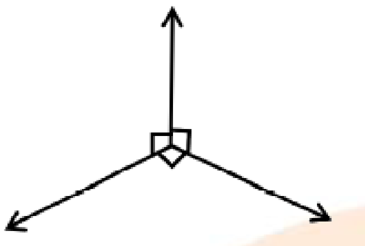
Chapter: Three dimensional Geometry.

Exercise: 11.2

1. Show that the three lines are mutually perpendicular if they have direction cosines be $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$



**Solution:** As we know, if $l_1l_2 + m_1m_2 + n_1n_2 = 0,$ the lines are perpendicular

**i.** Now, from direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$ and $\frac{4}{13}, \frac{12}{13}, \frac{3}{13},$ we get

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{12}{13} \times \frac{4}{13} + \left(\frac{-3}{13}\right) \times \frac{12}{13} + \left(\frac{-4}{13}\right) \times \frac{3}{13}$$

$$\Rightarrow \frac{48}{169} - \frac{36}{169} - \frac{12}{169}$$

$$\Rightarrow 0$$

Therefore, the lines are perpendicular.

**ii.** Similarly, if we take $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$ and $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13},$ we get

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{4}{13} \times \frac{3}{13} + \left(\frac{12}{13}\right) \times \left(\frac{-4}{13}\right) + \frac{3}{13} \times \left(\frac{12}{13}\right)$$

$$\Rightarrow \frac{12}{169} - \frac{48}{169} - \frac{36}{169} = 0$$

Therefore, the lines are perpendicular.

**iii.** Again, if we consider $\frac{-3}{13}, \frac{-4}{13}, \frac{12}{13}$ and $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13},$ we get

$$l_1l_2 + m_1m_2 + n_1n_2 = \frac{3}{13} \times \frac{12}{13} + \left(\frac{-4}{13}\right) \times \left(\frac{-4}{13}\right) + \frac{12}{13} \times \left(\frac{-4}{13}\right)$$

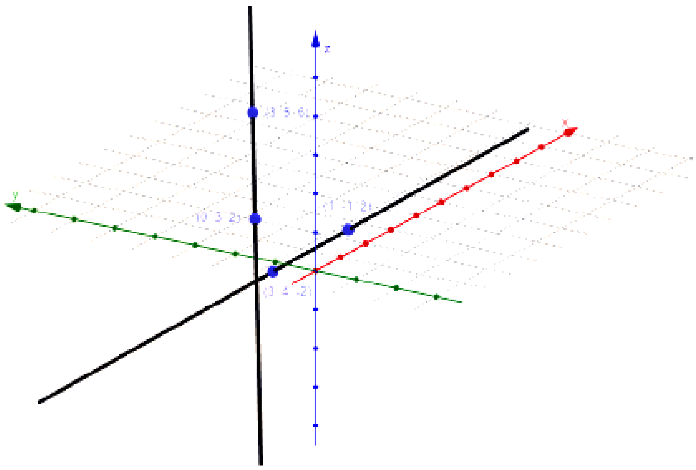
$$\Rightarrow \frac{36}{169} - \frac{12}{169} - \frac{48}{169} = 0$$

Therefore, the lines are perpendicular.

Therefore, we can say that all the lines are mutually perpendicular.

2. How can you show that the line passing through the points $(1,-1,2)(3,4,-2)$ is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6) ?

**Solution:** Let us consider that AB and CD are the lines that pass through the points, $(1,-1,2),(3,4,-2)$ and $(0,3,2),(3,5,6),$ respectively,



Now, we have $a_1 = (2), b_1 = (5), c_1 = (-4)$ and $a_2 = (3), b_2 = (2), c_2 = (4)$

As we know that if $AB \perp CD$ then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Now,

$a_1a_2 + b_1b_2 + c_1c_2 = 2\times3 + 5\times2 + (-4)\times4$

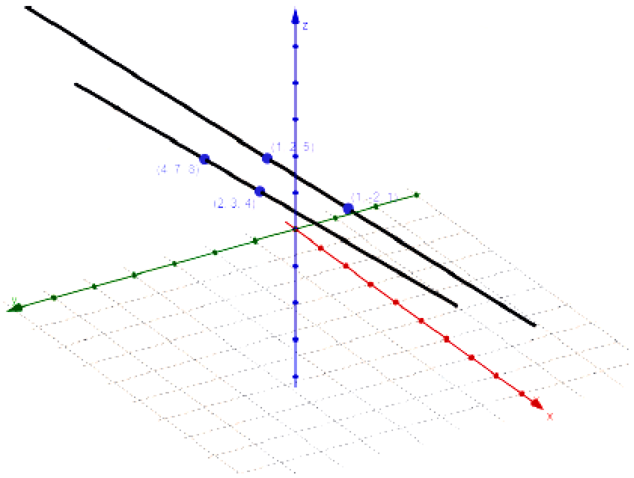
$\Rightarrow 2\times3 + 5\times2 - 4\times4 = 6 + 10 - 16$

$\Rightarrow 0$

Therefore, AB and CD are perpendicular to each other.

3. Show that the line through the points $(4,7,8)(2,3,4)$ is parallel to the line through the points $(1,-2,1)(1,2,5)$.

**Solution:** Let us consider the lines AB and CD that pass through points $(4,7,8),(2,3,4),$ and $(-1,-2,1),(1,2,5)$ respectively.



Now, we get

$a_1=(2-4), b_1=(3-7), c_1=(4-8)$ and $a_2=(1+1), b_2=(2+2), c_2=(5-1)$

$a_1=(-2), b_1=(-4), c_1=(-4)$ and $a_2=(2), b_2=(4), c_2=(4)$

Now, we know that if $AB \| CD$ then $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$,

Now,

$$\frac{a_1}{a_2}=\frac{-2}{2}\Rightarrow -1, \frac{b_1}{b_2}=\frac{-4}{4}\Rightarrow -1, \frac{c_1}{c_2}=\frac{-4}{4}=-1$$

We got $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore, AB is parallel to CD.

4. Find the equation of the line if it is parallel to vector $3i+2j-2k$ and which passes through point (1, 2, 3).

**Solution:** Now, let us consider the position vector A be $a=i+2j+3k$ and let $\vec{b}=3\hat{i}+2\hat{j}-2\hat{k}$

Now, we know that the line passes through A and is parallel to $\vec{b}$,

As we know $\vec{r}=\vec{a}+\lambda\vec{b}$ where $\lambda$ is a constant

$$\Rightarrow \vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$$

Therefore, the equation of the line is $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$

5. If the line passes through the point with positive vector $2\hat{i}-\hat{j}-4\hat{k}$ and is in the direction $\hat{i}+2\hat{j}-\hat{k}$. Find the equation of the line in vector and in Cartesian form.

**Solution:** We know that the line passes through the point with positive vector

Now, let us consider $\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$

Now, line passes through point A and parallel to $\vec{b}$, we get

$$\vec{r}=2\hat{i}-\hat{j}+4\hat{k}+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$$

Therefore, the equation of the line in vector form is $\vec{r}=2\hat{i}-\hat{j}+4\hat{k}+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

Now, we know

$$\vec{r}=x\hat{i}-y\hat{j}+z\hat{k}\Rightarrow x\hat{i}-y\hat{j}+z\hat{k}=(\lambda+2)\hat{i}+(2\lambda-1)\hat{j}+(-\lambda+4)\hat{k}$$

Therefore, the equation of the line in Cartesian form will be $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$.

6. If the line passes through the point (-2, 4, -5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$, find the Cartesian equation of the line.

**Solution:** We know that the line passes through point (-2, 4, -5) and also parallel to

$$\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$$

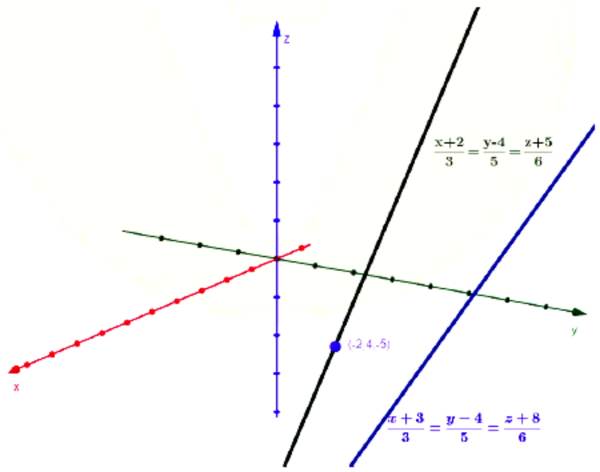
Now, as we can see the direction ratios of the line are 3,5 and 6.

As we know the required line is parallel to $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Therefore, the direction ratios will be 3k, 5k and 6k

As we know that the equation of the line through the point and with direction ratio is shown in form $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

Therefore, the equation of the line $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$.




7. Write the vector form of the line if the Cartesian equation of a line is

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}.$$

**Solution:** As we can see the Cartesian equation of the line, we can tell that the line is passing through $(5,4,-6)$, and he direction ratios are 3, 7 and 2.

Now, we got the position vector $\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$

From this we got the direction of the vector be $\vec{b}=3\hat{i}+7\hat{j}+2\hat{k}$

Therefore, the vector form of the line will be $\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda\left(3\hat{i}+7\hat{j}+2\hat{k}\right)$

8. If the line passes through the origin and $(5,-2,3)$, find the vector and the Cartesian equation of the line.

**Solution:** According to the question, line passes through the origin,

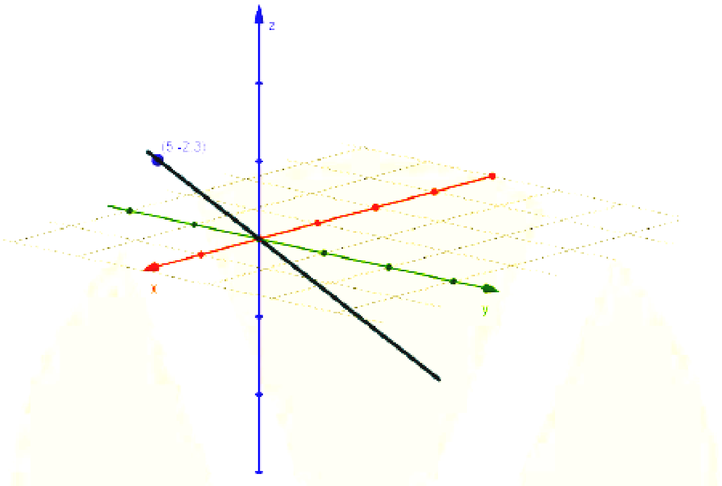
Now, the position vector will be $\vec{a}=0$

As the line pass through the point (5, 2, 3), the direction ratios of the line through origin will be 5, 2, 3

As the line is parallel to the vector $\vec{b}=5\hat{i}-2\hat{j}+3\hat{k}$

We can say that the equation of the line in vector form will be $\vec{r}=\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right)$

And, the equation of the line in the Cartesian form will be $\frac{x}{5}=\frac{y}{2}=\frac{z}{3}$.



9. If the line passes through the point $(3,-2,-5),(3,-2,6),$ find the vector and the Cartesian equation of the line.

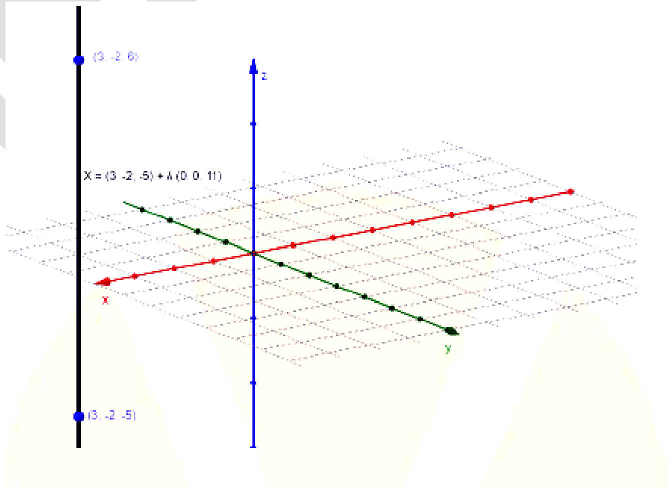
**Solution:** Let us consider the points be $P(3,-2,-5)$ and $Q(3,-2,6),$ so the line passing through the point will be PQ.

Therefore, the position vector will be $\vec{a}=3\hat{i}-2\hat{j}-5\hat{k}$ and the direction ratios will be $(3-3)=0,(-2+2)=0,(6+5)=11$

As the equation of the vector in the same direction as PQ, we get

$\vec{b}=0\hat{i}-0\hat{j}+11\hat{k}=11k$

Therefore, the equation of the line in vector form will be $\vec{r}=3\hat{i}-2\hat{j}-5\hat{k}+11k\lambda$ and in Cartesian form it will be $\dfrac{x-3}{11}=\dfrac{y+2}{11}=\dfrac{z+5}{11}$




10. Find the angle between the lines

(i) $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda\left(3\hat{i}-2\hat{j}+6\hat{k}\right)$ and $\vec{r}=7\hat{i}-6\hat{k}+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

**Solution:** Let us consider the angle be $\theta$,

As we know that the angle between the lines can be found by $\cos\theta=\left|\dfrac{\vec{b}_1\vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right|$

As the line are parallel to $\vec{b}_1=3\hat{i}+2\hat{j}+6\hat{k}$ and $\vec{b}_2=\hat{i}+2\hat{j}+2\hat{k}$, we got

$\left|\vec{b}_1\right|=\sqrt{3^2+2^2+6^2}=7,\left|\vec{b}_2\right|=\sqrt{1^2+2^2+2^2}=3$ and

$\vec{b}_1\vec{b}_2=\left(3\hat{i}+2\hat{j}+6\hat{k}\right)\left(\hat{i}+2\hat{j}+2\hat{k}\right)=19$

Therefore, the angle between the lines will be

$$\cos\theta=\frac{19}{7\times3}$$

$$\Rightarrow\theta=\cos^{-1}\frac{19}{21}$$

**(ii)** $\vec{r}=3\hat{i}+\hat{j}-2\hat{k}+\lambda\left(\hat{i}-\hat{j}-2\hat{k}\right)$ **and** $\vec{r}=3\hat{i}-\hat{j}-56\hat{k}+\mu\left(3\hat{i}-5\hat{j}-4\hat{k}\right)$

**Solution:** As the lines are parallel to the vectors $\vec{b}_1=\hat{i}-\hat{j}-2\hat{k}$ and $\vec{b}_2=3\hat{i}-5\hat{j}+-4\hat{k}$,

we get $\left|\vec{b}_1\right|=\sqrt{1^2+(-1)^2+(-2)^2}=\sqrt{6},\left|\vec{b}_2\right|=\sqrt{3^2+(-5)^2+(-2)^2}=5\sqrt{2}$ and

$\vec{b}_1\vec{b}_2=\left(\hat{i}-\hat{j}-2\hat{k}\right)\left(3\hat{i}-5\hat{j}+-4\hat{k}\right)=16$

Therefore, the angle between them will be,

$$\cos\theta=\frac{16}{10\sqrt{3}}$$

$$\Rightarrow\cos\theta=\frac{8}{5\sqrt{3}}$$

$$\Rightarrow\theta=\cos^{-1}\frac{8}{5\sqrt{3}}$$

11. Find the angle between the lines

(i) $\dfrac{x-2}{2}=\dfrac{y-1}{5}=\dfrac{z+3}{-3}$ and $\dfrac{x+2}{-1}=\dfrac{y-4}{8}=\dfrac{z-5}{4}$

**Solution:** Let us take $\vec{b}_1$ and $\vec{b}_2$ be the vectors parallel to the lines, we get

$\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\hat{k}$ and $\vec{b}_2 = \hat{i} + 8\hat{j} + 4\hat{k}$

Now $|\vec{b}_1| = \sqrt{2^2 + 5^2 + (-3)^2} = \sqrt{38}, |\vec{b}_2| = \sqrt{(-1)^2 + 8^2 + 4^2} = 9$

And,

$\vec{b}_1\vec{b}_2 = (2\hat{i} + 5\hat{j} - 3\hat{k})(-\hat{i} + 8\hat{j} + 4\hat{k})$

$= 2(-1) + 5(8) + 4(-3)$

=26

We can find the angle by using $\cos\theta = \left|\frac{\vec{b}_1\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

Therefore,

$\cos\theta = \frac{26}{9\sqrt{38}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

Therefore, the angle will be $\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$.

ii. $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

**Solution:** Similarly let us consider $\vec{b}_1$ and $\vec{b}_2$ be the vectors parallel to lines, we get

$\vec{b}_1 = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b}_2 = 4\hat{i} + \hat{j} + 8\hat{k}$

Now, $|\vec{b}_1| = \sqrt{2^2 + 2^2 + (1)^2} = 3, |\vec{b}_2| = \sqrt{4^2 + 1^2 + 8^2} = 9$ and

$\vec{b}_1\vec{b}_2 = (2\hat{i} + 2\hat{j} + 1\hat{k}).(4\hat{i} + \hat{j} + 8\hat{k})$

$= 2(4) + 2(1) + 1(8)$

$= 18$

As we know the angle can be found by $\cos\theta = \left|\frac{\vec{b}_1\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

Therefore,

$\cos\theta = \frac{18}{17} = \frac{2}{3}$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Therefore, the angle is $\cos^{-1}\left(\frac{2}{3}\right)$

12. We needed to find the values of p so the line $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{2}$ are at right angles.

**Solution:** As we know that the correct form of the equation is as follows,

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \text{ and } \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

From this we get the direction ratios as

$$a_1 = -3, b_1 = \frac{2p}{7}, c_1 = 2 \text{ and } a_2 = \frac{-3p}{7}, b_2 = 1, c_2 = -5$$

As we know the lines are perpendicular, we get

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \frac{9p}{7} + \frac{2p}{7} = 10$$

$$\Rightarrow 11p = 70$$

$$\Rightarrow p = \frac{70}{11}$$

Therefore, the value of p is $\frac{70}{11}$.

13. We needed to show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

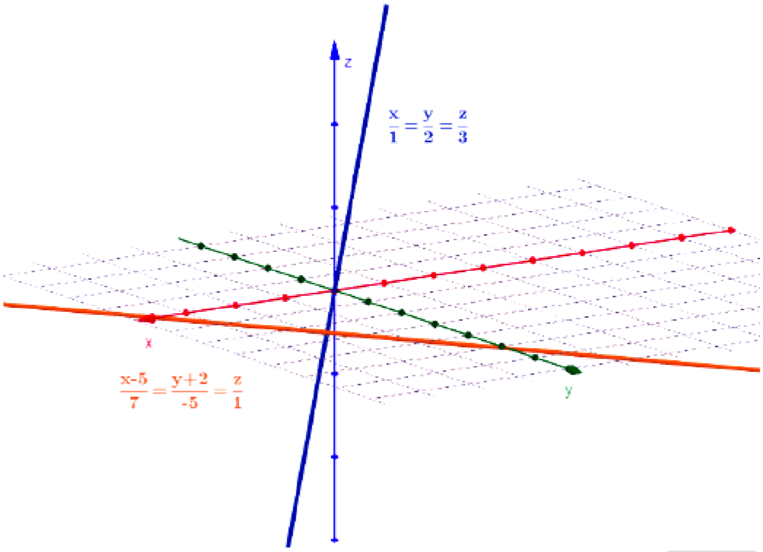
**Solution:** From the given equation, we get the direction ratios as,

$$a_1 = 7, b_1 = -5, c_1 = 1, a_2 = 1, b_2 = 2, c_2 = 3$$

As we know, if $a_1a_2 + b_1b_2 + c_1c_2 = 0,$ the lines are perpendicular to each other

Now,

$$7(1) + (-5)2 + 1(3) \Rightarrow 7 - 10 + 3 = 0$$




Therefore, the lines are perpendicular.

14. If the lines are $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$, find the shortest distance between them.

**Solution:** We have been given lines, $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$ and

$\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

As we know that the shortest distance can be found as $d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right)\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$

Now, from the given lines we get that

$\vec{a}_1=\hat{i}+2\hat{j}+\hat{k},$

$\vec{b}_1=\hat{i}-\hat{j}-\hat{k}$

$\vec{a}_2=2\hat{i}-\hat{j}-\hat{k},$

$\vec{b}_2=2\hat{i}+\hat{j}+2\hat{k}$

$\vec{a}_2-\vec{a}_1=\left(2\hat{i}-\hat{j}-k\right)-\left(\hat{i}+2\hat{j}+\hat{k}\right)$

$=\hat{i}-3\hat{j}-2\hat{k},$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & -1 & 2\end{vmatrix}$$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{k}$

Then, $\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-3)^2 + 3^2} = 3\sqrt{2}$

Now, if we put all the values in theirs places, we get

$$d = \left|\frac{\left(-3\hat{i}+3\hat{k}\right)\left(\hat{i}-3\hat{j}-2\hat{k}\right)}{3\sqrt{2}}\right| \Rightarrow d = \left|\frac{-3(1)+3(2)}{3\sqrt{2}}\right|$$

$$d = \left|\frac{-9}{3\sqrt{2}}\right| \Rightarrow d = \frac{3\sqrt{2}}{2}$$

Therefore, the shortest distance between the lines is $\frac{3\sqrt{2}}{2}$ units.

15. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

**Solution:** As we know that the shortest distance can be found by,

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

Now, from the given lines we got that

$x_1 = -1, y_1 = -1, z_1 = -1, a_1 = 7, b_1 = -6, c_1 = 1$

$x_2 = 3, y_2 = 5, z_2 = 7, a_2 = 1, b_2 = -2, c_2 = 1$.

And,

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$= 4(-6+2) - 6(1+7) + 8(-14+6)$

$= -16 - 36 - 64$

$= -116$

And,

$$\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}=\sqrt{(-6+2)^2+(1+7)^2+(-14+6)^2}$$

$$\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}=2\sqrt{29}$$

Putting all the values, we get

$$d=\frac{-116}{2\sqrt{29}}$$

$$d=\frac{-58}{\sqrt{29}}\Rightarrow\frac{-58\sqrt{29}}{29}$$

$$d=\frac{-58}{\sqrt{29}}\Rightarrow|d|=2\sqrt{29}$$

Therefore, the distance the distance between the lines is $2\sqrt{29}$ units.

16. Find the shortest distance between the lines whose vector equations are

$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+,\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

**Solution:** We have been given lines $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and

$$\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$$

As we know that the shortest distance between the lines can be found by,

$$d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right)(\vec{a}_2-\vec{a}_1)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$

Now, from the given lines, we got

$$\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k},\vec{b}_1=\hat{i}-3\hat{j}+2\hat{k}$$

$$\vec{a}_2=4\hat{i}+5\hat{j}+6\hat{k},\vec{b}_2=2\hat{i}+3\hat{j}+\hat{k}$$

$$\vec{a}_2-\vec{a}_1=\left(4\hat{i}+5\hat{j}+6\hat{k}\right),\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$=3\hat{i}+3\hat{j}+3\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}$$

$$\Rightarrow\vec{b}_1\times\vec{b}_2=-9\hat{i}+3\hat{j}+9\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-9)^2 + 3^2 + 9^2} = 3\sqrt{19}$$

Now, putting all the values, we get

$$d = \left|\frac{9}{3\sqrt{19}}\right| = \frac{3}{\sqrt{19}}$$

Therefore, the shortest distance between the lines is $\frac{3}{\sqrt{19}}$ units.

17. We needed to find the shortest distance between the lines whose vector equations are $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

**Solution:** We have been given lines $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

$\Rightarrow \vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ **and** $\vec{r} = \hat{i} - \hat{j} + \hat{k} + s\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

Now, the shortest distance can be found by,

$$d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

Now, from the given lines we got,

$$\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k},$$

$$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}, \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \left(\hat{i} - \hat{j} - \hat{k}\right), \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = \hat{j} - 4\hat{k},$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$\Rightarrow \vec{b}_1 \times \vec{b}_2 = 2\hat{i} - 4\hat{j} - 3\hat{k},$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2} = \sqrt{29}$$

$$\left(\vec{b}_1 \times \vec{b}_2\right) \times \left(\vec{a}_2 - \vec{a}_1\right) = \left(2\hat{i} - 4\hat{j} - 3\hat{k}\right)\left(\hat{j} - 4\hat{k}\right)$$

$$= -4 + 12$$

$$= 8$$

Putting all the values, we get

$$d = \left|\frac{8}{\sqrt{29}}\right| = \frac{8}{\sqrt{29}}$$

Therefore, the shortest distance between the lines is $\frac{8}{\sqrt{29}}$ units.