**Chapter: Three-dimensional Geometry.**

**Exercise: 11.3**

1. Determine the direction cosines of the normal to the plane and the distance from the origin.

(a) z=2

**Solution:** It is given that equation of the plane is z=2

Now we can tell that the direction ratios are 0,0,1.

Which means $\sqrt{0+0+1^2}=1$

Now we will divide both sides of equation by 1, we get

$$0+0+\frac{z}{1}=2$$

Therefore, the direction cosines and distance of the plane is $(0,0,1)$ and 2 units respectively.

(b) $x+y+z=1$

Ans : $x+y+z=1$ is the equation of the normal

Now, from the equation given we can say that the direction ratios of normal are 1, 1 and 1.

Which means $\sqrt{1^2+1^2+1^2}=\sqrt{3}$

Now we will divide the equation by $\sqrt{3},$ we get

$$\frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}}y+\frac{1}{\sqrt{3}}z=\frac{1}{\sqrt{3}}$$

Therefore, the direction cosines and distance from the origin $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

And $\frac{1}{\sqrt{3}}$ units respectively.

(c). $2x+3y-z=5$

**Solution:** $2x+3y-z=5$ is the equation of the normal

Now, from the given equation we get the direction ratios of normal as 2, 3,-1.

Which means $\sqrt{2^2+3^2+(-1)^2}=\sqrt{14}$

Now, we will divide the equation by $\sqrt{14}$, we get

$$\frac{2}{\sqrt{14}}x+\frac{3}{\sqrt{14}}y-\frac{1}{\sqrt{14}}z=\frac{5}{\sqrt{14}}$$

Therefore, the direction cosines and the distance from the origin of the normal are $\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}},\frac{-1}{\sqrt{14}}$ **and** $\frac{5}{\sqrt{14}}$ units respectively.

(d) $5y+8=0$.

**Solution**: $5y+8=0\Rightarrow 0x+5y+0z=-8$ is the given equation

Now, from the equation we can tell that the direction ratios of normal are 0,-5 and 0.

Which means $\sqrt{0^2+(-5)^2+0^2}=5$

Now, we will divide the equation by 5, we get

$$-y=\frac{8}{5}$$

Therefore, the direction cosines and the distance from the origin of the normal are 0,-1,0 and $\frac{8}{5}$ units respectively.

2. Find the vector equation of plane which is at the distance of 7 units from the origin and the normal vector $3\hat{i}+5\hat{j}-6\hat{k}$

**Solution:** Let us consider the normal vector be $\vec{a}=3\hat{i}+5\hat{j}-6\hat{k}$

We know that $\hat{n}=\frac{\vec{n}}{|\vec{n}|}=\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{3^2+5^2+6^2}}=\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}$

As we know the equation of the plane with position vector is shown in form $\vec{r}.\hat{n}=d$,

Therefore,

$$\Rightarrow \vec{r}.\left(\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}\right)=7$$

Therefore, the vector equation is in the form $\vec{r}.\left(\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}\right)=7$.

3. Find the Cartesian equation of planes

(a) $\vec{r}.\left(\hat{i}+\hat{j}-\hat{k}\right)=2$

**Solution**: We have been given equation of the plane as $\vec{r}.\left(\hat{i}+\hat{j}-\hat{k}\right)=2$

As we know, the position vector is $\vec{r}=x\hat{i}+y\hat{j}-z\hat{k}$

Putting the values of $\vec{r}$ in equation, we get

$$\left(x\hat{i}+y\hat{j}-z\hat{k}\right).\left(\hat{i}+\hat{j}-\hat{k}\right)=2$$

$$\Rightarrow x+y-z=2$$

Therefore, the cartesian equation will be $x+y-z=2$

(b) $\vec{r}.\left(2\hat{i}+3\hat{j}-4\hat{k}\right)=1$

**Solution**: We have been given equation of the plane as $\vec{r}.\left(2\hat{i}+3\hat{j}-4\hat{k}\right)=1$

As we know the position vector $\vec{r}$ is given by,

$$\vec{r}=x\hat{i}+y\hat{j}-z\hat{k}$$

Putting the values of $\vec{r}$ in equation, we get

$$(x\hat{i}+y\hat{j}-z\hat{k}).\left(2i+3\hat{j}-4\hat{k}\right)=1$$

$$\Rightarrow 2x+3y-4z=1$$

Therefore, the Cartesian equation will be $2x+3y-4z=1, x+y-z=2$

(c) $\vec{r}.\left((s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}\right)=15$

**Solution**: We have been given equation of the plane as

$$\vec{r}.\left((s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}\right)=15$$

As the position vector is in form as $\vec{r}=x\hat{i}+y\hat{j}-z\hat{k}$

Putting the values of $\vec{r}$ in equation, we get

$$(x\hat{i}+y\hat{j}-z\hat{k}).\left((s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}\right)=15$$

$$\Rightarrow(s-2t)x+(3-t)y+(2s+t)z=1$$

Therefore, the cartesian equation will be $(s-2t)x+(3-t)y+(2s+t)z=1$

4. Find the coordinates of the foot of the perpendicular drawn from the origin.

a) $2x+3y+4z-12=0$

**Solution:** Let us consider the coordinates of the foot be $(x_1,y_1,z_1)$

Now, we have been given equation as $2x+3y+4z-12=0$

As we can tell the direction ratios will be 2, 3 and 4.

Which means, $\sqrt{2^2+3^2+4^2}=\sqrt{29}$

Now, we will divide the equation by $\sqrt{29}$, we get

$$\frac{2}{\sqrt{29}}x+\frac{3}{\sqrt{29}}y+\frac{4}{\sqrt{29}}z=\frac{12}{\sqrt{29}}$$

The coordinates of the foot of the perpendicular will be,

$$\left(\frac{2}{\sqrt{29}}.\frac{12}{\sqrt{29}},\frac{3}{\sqrt{29}}.\frac{12}{\sqrt{29}},\frac{4}{\sqrt{29}}.\frac{12}{\sqrt{29}}\right)$$

$$\Rightarrow\left(\frac{24}{29},\frac{36}{49},\frac{48}{29}\right)$$

Therefore, the coordinates of the foot of the perpendicular will be $\left(\frac{24}{29},\frac{36}{49},\frac{48}{29}\right)$

(b) $3y+4z-6=0$

**Solution:** Let us take the coordinates of the foot of perpendicular be $(x_1,y_1,z_1)$

Now, we have been given equation as $3y+4z-6=0$

As we can tell the direction ratios will be 0,3 and 4.

Which means, $\sqrt{0+3^2+4^2}=5$

Now, we will divide the equation by 5, we get

$$0x+\frac{3}{5}y+\frac{4}{5}z=\frac{6}{5}$$

Now, the coordinates of the foot of the perpendicular will be,

$$\left(0,\frac{3}{5}.\frac{6}{5},\frac{4}{5}.\frac{6}{5}\right)$$

$$\Rightarrow\left(0,\frac{18}{25},\frac{24}{25}\right)$$

Therefore, the coordinates of the foot of the perpendicular will be $\left(0,\frac{18}{25},\frac{24}{25}\right)$

(c) $x+y+z=1$

**Solution:** Let us consider the coordinates of the foot of perpendicular be $(x_1,y_1,z_1)$

Now, we have been given equation $x+y+z=1$

As we can tell the direction ratios will be 1, 1 and 1.

Which means, $\sqrt{1^2+1^2+1^2}=\sqrt{3}$

Now, we will divide the equation by $\sqrt{3},$ we get

$$\frac{2}{\sqrt{3}}x+\frac{1}{\sqrt{3}}y+\frac{1}{\sqrt{3}}z=\frac{1}{\sqrt{3}}$$

Now, the foot of the perpendicular will be,

$$\left(\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$$

Therefore, the coordinates of the foot of the perpendicular will be $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

(d) $\Rightarrow 5y+8=0\backslash$

**Solution:** Let us consider the coordinates of the foot of perpendicular be $(x_1,y_1,z_1)$

Now, we have been given equation as $5y+8=0\Rightarrow 0x-5y+0z=8$

Now, we can tell that the direction ratios will be 0,5 and 0.

Which means, $\sqrt{0+5^2+0=5}$

Now, we will divide the equation by 5, we get

$$-y=\frac{8}{5}$$

Therefore, the foot of the perpendicular will be $\left(0,(-1).\frac{8}{5},0\right)$

Therefore, the coordinates of the foot of the perpendicular is $\left(0,-\left(\frac{8}{5}\right),0\right)$.

5. Find the vector and Cartesian equation of the planes

(a) That passes through the point $(1,0,-2)$ and the normal to the plane is $\hat{i}+\hat{j}-\hat{k}$

**Solution:** Now, according to the question, the position vector of point $(1,0,-2)$ be $\vec{a}=\hat{i}-2\hat{k}$

Now, the normal vector $\vec{N}$ perpendicular to the plane will be $\vec{N}=\hat{i}+\hat{j}-\hat{k}$

Now, the vector equation of the plane will be in form $(\vec{r}-\vec{a}).\vec{N}=0$

$$\left[\vec{r}-\left(\hat{i}-2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0$$

As, $\vec{r}$ is the positive vector of any point p(x,y,z), we get

$$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

Now,

$$\left[\left(x\hat{i}+y\hat{j}+z\hat{k}\right)-\left(\hat{i}-2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0\Rightarrow\left[(x-1)\hat{i}+y\hat{j}+(z+2)\hat{k}\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0$$

$$\Rightarrow(x-1)+y-(z-2)=0$$

$$\Rightarrow x+y-z=3$$

Therefore, the equation will be $x+y-z=3$.

(b) That passes through the point (1,4,6) and the normal to the plane is $\hat{i}-2\hat{j}+\hat{k}$

**Solution:** Now, according to question, the position vector of point vector of point (1, 4, 6) will be

$$\vec{a}=\hat{i}+4\hat{j}+6\hat{k}$$

As we know that the normal vector $\vec{N}$ perpendicular to the plane, $\vec{N}=\hat{i}-2\hat{j}+\hat{k}$

So, the vector equation of the plane is will be in form,

$$\left[\vec{r}-\left(\hat{i}+4\hat{j}+6\hat{k}\right)\right].\left(\hat{i}-2\hat{j}+\hat{k}\right)=0$$

As, $\vec{r}$ is the positive vector of any point p(x,y,z) in the plane,

Now,

$$\left[\left(x\hat{i}+y\hat{j}+z\hat{k}\right)-\left(\hat{i}+4\hat{j}+6\hat{k}\right)\right].\left(i-2\hat{j}+\hat{k}\right)=0\Rightarrow\left[(x-1)\hat{i}+(y-4)\hat{j}+(z+6)\hat{k}\right].\left(\hat{i}-2\hat{j}=\hat{k}\right)=0$$

$$\Rightarrow(x-1)-2(y-4)+(z-6)=0$$

$$\Rightarrow(x-1)-2(y-4)+(z-6)=0$$

$$\Rightarrow x-2y+z+1=0$$

Therefore, the equation of the plane will be x-2y+z+1=0.

6. If the plane passes through the given points, Find the equations of the plane.

(a) $(1,1,-1),(6,4,-5),(-4,-2,3)$

**Solution:** Now, let us consider the points be $A(1,1,-2),B(6,4,-5),C(-4,-2,3)$

Now, $\begin{vmatrix}1 & 1 & -1\\ 6 & 4 & -5\\ -4 & -2 & 3\end{vmatrix}=(12-10)-(18-20)-(12+16)=2+2-4=0$

Therefore, A,B,C are collinear points,

The number of planes passing through will be infinite

(b) (1,1,0), (1, 2, 1), (-2,2,-1)

**Solution:** Now, let consider the points be $A(1,1,0)), B(1,2,1), C(-2,2,1)$

Now, $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} = (-2-2)-(2+2) = -8 \neq 0$

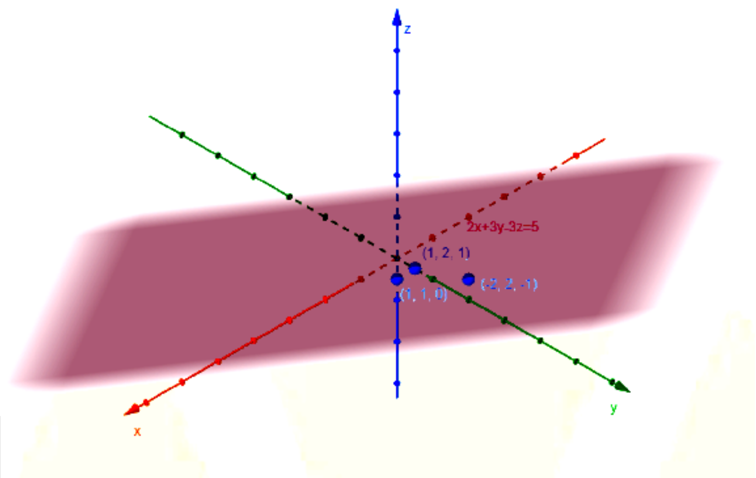
From this we get to know that a plane will pass through the points A, B, C

Now, the equation of the plane through the points will be,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (-2)(x-1)-3(y-1)+3z = 0 \Rightarrow -2x-3y+3z+2+3=0$$

$$\Rightarrow -2x-3y+3z=-5 \Rightarrow 2x+3y-3z=5$$




Therefore, the equation of the plane is $2x+3y-3z=5$.

7. Find the intercepts cut off by the plane $2x+y-z=5$

**Solution:** Now, the equation of the plane is given as $2x+y-z=5$

Now we will divide both sides by 5, we get intercepts,

$$\frac{2x}{5}+\frac{y}{5}-\frac{z}{5}=1 \Rightarrow \frac{x}{\frac{5}{2}}+\frac{y}{5}+\frac{z}{-5}=1$$

Now, as we know the equation of a plane in intercepts form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1,$ As

We can see that we got $a=\frac{5}{2}, b=5, c=-5$ as the intercepts of the plane.

Therefore, the intercepts of the plane will be $\frac{5}{2}, 5$ and 5.

8. Find the equation of the plane parallel to ZOX plane and having intercept 3 on the y-axis.

**Solution:** We have been given plane ZOX with intercept 3

As we know, if the plane is parallel to the equation, it will be in the form y=a
Since the y-intercept of the plane is 3, we get
A=3
Therefore, the equation of the required plane is y=3.

9. Find the equation of the plane through the point (2,2,1) and the intersection of the plane $3x-y+2z-4=0$ and $x+y+z-2=0$

**Solution:** As it's given that the equation of the plane pass through the intersection of the planes $3x-y+2z-4=0$ and $x+y+z-2=0$, and passes through the point (2, 2, 1)

We know that $(3x-y+2z-4)+\alpha(x+y+z-2)=0, \alpha \in R$

Therefore, $(3\times 2-2+2\times 1-4)+\alpha(2+2+1-2)=0$

$\Rightarrow 2+3\alpha=0 \Rightarrow \alpha=-\frac{2}{3}$

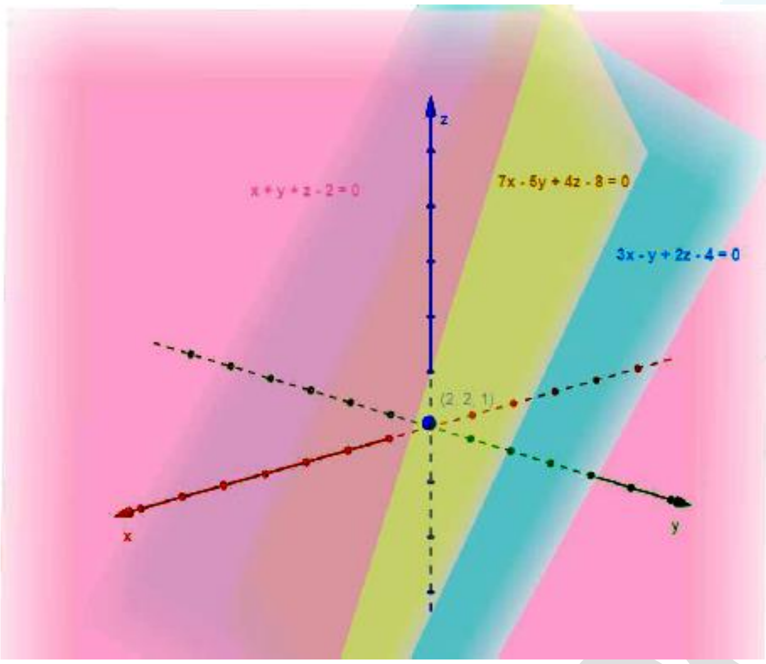
Now, putting $\alpha=-\frac{2}{3}$, we get

$(3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$

$\Rightarrow 3(3x-y+2z-4)-2(x+y+z-2)=0$

$\Rightarrow (6x-3y+6z-12)-2(x+y+z-2)=0$

$\Rightarrow 7x-5y+4z-8=0$




Therefore, the equation of the plane will be $7x-5y+4z-8=0$

10. Find the vector equation of the plane passing through the point (2, 1, 3) and the intersection of the planes $\vec{r}.\left(2\hat{i}+\hat{j}2-3\hat{k}\right)=7, \vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)=9$

**Solution:** It is given that the equations for planes passes through (2, 1, 3) and the intersection of the given planes,

$$\Rightarrow \vec{r}.\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7=0 \text{ and } \vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9=0$$

Now, the equation of the required plane will be

$$\left[\vec{r}.\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7\right]+\lambda\left[\vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9\right]=0, \lambda \in R$$

$$\vec{r}.\left[\left(2\hat{i}+2\hat{j}-3\hat{k}\right)+\lambda\left(2\hat{i}+5\hat{j}+3\hat{k}\right)\right]=9\lambda+7$$

$$\vec{r}.\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7$$

As plane passes through (2, 1, 3), the position vector will be,

$$\vec{r}=2\hat{i}+2\hat{j}+-3\hat{k}$$

Putting this in equation $\vec{r}.\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7$ we get,

$$\left(2\hat{i}+2\hat{j}+-3\hat{k}\right).\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7$$

$$\Rightarrow (2+2\lambda)+(2+5\lambda)+(3\lambda-3)=9\lambda+7$$

$$\Rightarrow 18\lambda-3=9\lambda+7$$

$\Rightarrow 9\lambda = 10 \Rightarrow \lambda = \frac{10}{9}$

After putting this value in equation $\vec{r}.\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7$ we

Will get,

$\vec{r}.\left[\frac{38}{9}\hat{i}+\frac{68}{9}\hat{j}+\frac{3}{9}\hat{k}\right]=17$

$\Rightarrow \vec{r}.\left[38\hat{i}+68\hat{j}+3\hat{k}\right]=153$

Therefore, the vector equation will be $\vec{r}.\left[38\hat{i}+68\hat{j}+3\hat{k}\right]=153$

11. Find the equation of the plane perpendicular to the plane $x-y+z=0$.

And through the line of intersection of the plane $x+y+z=1$ and $2x+3y+4z=5$

**Solution:** It is given that the equation of the plane is perpendicular to $x-y+z=0$ and pass through the intersection of planes, we got

$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$

$\Rightarrow (2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z-(5\lambda+1)=0$

From this we can tell that $a_1=(2\lambda+1), b_1(3\lambda+1), c_1=(4\lambda+1)$

Now, according to the question the plane is perpendicular to $x-y+z=0$

We got, $a_2=1, b_2=-1, c_2=1$

We know that if planes are perpendicular, then,

$a_1a_2+b_1b_2+c_1c_2=0$

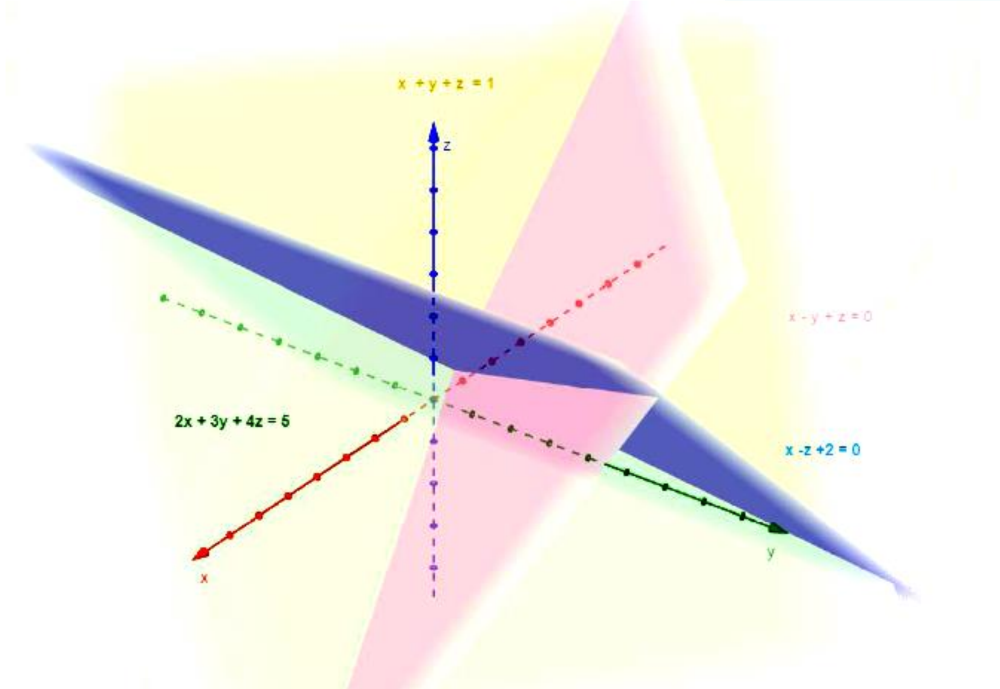
$\Rightarrow (2\lambda+1)-(3\lambda+1)+(4\lambda+1)=0$

$\Rightarrow 3\lambda+1=0 \Rightarrow \lambda=-\frac{1}{3}$

By putting the value in equation $(2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z-(5\lambda+1)=0,$ we got

$\frac{1}{3}x+\frac{1}{3}z+\frac{2}{3}=0$

$\Rightarrow x-z+2=0$




Therefore, the equation of the plane will be $x - z + 2 = 0$

12. For these vectors equations of planes $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3$, find the angle between them.

**Solution**: According to the question we have been given two equation of planes,

$$\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5 \text{ and } \vec{r}.\left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3$$

Now, we know that if $\vec{n}_1$ and $\vec{n}_2$ are normal to the planes, then,

$$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2,$$

As we know, we can find the angle by $\cos\theta = \left|\dfrac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$

We got,

$$\vec{n}_1 = 2\hat{i} + 2\hat{j} - 3\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} - 3\hat{j} + 5\hat{k}$$

$$\therefore \vec{n}_1.\vec{n}_2 = \left(2\hat{i} + 2\hat{j} - 3\hat{k}\right)\left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 2.3 + 2(-3) + (-3)5 = -15,$$

$$|\vec{n}_1| = \sqrt{2^2 + 2^2 + (-3)^2} = \sqrt{17}$$

$$|\vec{n}_2| = \sqrt{3^2 + (-3)^2 + (5)^2} = \sqrt{43}$$

Now, by putting all these values in equation $\cos\theta = \left|\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$

$$\cos\theta = \left|\frac{-15}{\sqrt{17}\sqrt{43}}\right|$$

$$\Rightarrow \cos\theta = \frac{15}{\sqrt{731}} \Rightarrow \theta = \cos^{-1}\left[\frac{15}{\sqrt{731}}\right]$$

Therefore, the angle between them is $\theta = \cos^{-1}\left[\frac{15}{\sqrt{731}}\right]$.

13. Determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

**Solution:** We know that the direction ratios of normal to the plane are $a_1, b_1, c_1$ and $a_2, b_2, c_2$,

We know that if lines are parallel then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and if lines are

Perpendicular then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

The angle between the planes can be found by, $\theta = \cos^{-1}\left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$

(a) $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$

**Solution:** The equations are given as $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$

From the equations we got $a_1 = 7, b_1 = 5, c_1 = 6$ and $a_2 = 3, b_2 = -1, c_2 = -10$

Now we will check whether the planes are perpendicular or parallel, then

Now, $a_1a_2 + b_1b_2 + c_1c_2 = 7\times3 + 5\times(-1) + 6\times(-10) = -44 \neq 0$

Therefore, the planes are not perpendicular.

Now, $\frac{a_1}{a_2}=\frac{7}{3}, \frac{b_1}{b_2}=\frac{5}{-1}, \frac{c_1}{c_2}=\frac{-3}{5}$

It can be seen that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the given planes are not parallel.

The angle between them is given by

$$\theta=\cos^{-1}\left|\frac{7\times3+5\times(-1)+6\times(-10)}{\sqrt{7^2+5^2+6^2}\sqrt{3^2+(-1)^2+(-10)^2}}\right|$$

$$\theta=\cos^{-1}\frac{44}{110}=\cos^{-1}=\frac{2}{5}$$

(b) 2x+y+3z-2=0 and x-2y+5=0

**Solution:** The equations are given as $2x+y+3z-2=0$ and $x-2y+5=0$

From this we got, $a_1=2, b_1=1, c_1=3$ and $a_2=1, b_2=2, c_2=0$

Now, $a_1a_2+b_1b_2+c_1c_2=2\times1+1\times(-2)+3\times0\neq0$

Therefore, the planes are not perpendicular to each other.

Now, $\frac{a_1}{a_2}=\frac{2}{1}, \frac{b_1}{b_2}=\frac{1}{2}, \frac{c_1}{c_2}=\frac{-3}{0}$

It can be seen that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Now, the angle between them will be

$$\theta=\cos^{-1}\left|\frac{2\times1+1\times(2)+3\times(0)}{\sqrt{2^2+1^2+3^2}\sqrt{1^2+(2)^2+0^2}}\right|$$

$$\theta=\cos^{-1}\frac{4}{\sqrt{70}}=\cos^{-1}=\frac{2}{5}$$

(c) $2x-2y+4z+5=0$ and $3x-3y+6z-1=0$

**Solution:** The equations are given as $2x-2y+4z+5=0$ and $3x-3y+6z-1=0$

From this we got, $a_1=2, b_1=-2, c_1=4$ and $a_2=3, b_2=-3, c_2=6$

Now, $a_1a_2+b_1b_2+c_1c_2=2\times3+(-2)\times(-3)+4\times(6)=36\neq0$

Therefore, the given planes are not perpendicular.

Now, $\frac{a_1}{a_2}=\frac{2}{3},\frac{b_1}{b_2}=\frac{2}{3},\frac{c_1}{c_2}=\frac{2}{3}$

$\therefore\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore, the given planes are parallel to each other.

(d) $2x-y+3z-1=0$ and $2x-y+3z+3=0$

**Solution:** The equations are given as $2x-y+3z-1=0$ and $2x-y+3x+3=0$

From this we got, $a_1=2,b_1=-1,c_1=3$ and $a_2=2,b_2=-1,c_2=3$

Now, as we can see $\frac{a_1}{a_2}=\frac{2}{2}=1,\frac{b_1}{b_2}=\frac{-1}{-1}=1,\frac{c_1}{c_2}=\frac{-1}{1-}=1$

$\therefore\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore, the given planes are parallel to each other.

(e) $4x+8y+z-8=0$ and $y+z-4=0$

**Solution:** The equations are given as $4x+8y+z-8=0$ and $y+z-4=0$

From this we get, $a_1=4,b_1=8,c_1=1$ and $a_2=0,b_2=1,c_2=1$

Now, $a_1a_2+b_1b_2+c_1c_2=4\times0+8\times(1)+1=9\neq0$

Therefore, the given planes are not perpendicular.

Now, $\frac{a_1}{a_2}=\frac{4}{0},\frac{b_1}{b_2}=\frac{8}{1}=8,\frac{c_1}{c_2}=\frac{1}{1}=1$

It can be seen that $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Therefore, the given planes are not parallel.

Therefore, the angle between them will be,

$$\theta=\cos^{-1}\left|\frac{4\times0+8\times(1)+1\times1}{\sqrt{4^2+8^2+1^2}\sqrt{0+(1)^2+(1)^2}}\right|$$

$$\theta=\cos^{-1}\frac{9}{9\sqrt{2}}=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=45^0$$

Therefore, the angle between them is $45°$.

14. In the following cases, find the distance of each of the given points from the corresponding given plane.

**Solution:** The distance between a point and a plane is given by,

$$d=\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

**(a)** $(0,0,0)3x-4y+12z=3$

**Ans:** The given point is (0, 0, 0) and the plane is $3x-4y+12z=3$

Now the distance will be,

$$d=\left|\frac{3\times0-4\times0+12\times0-3}{\sqrt{(-3)^2+(-4)^2+12^2}}\right|=\frac{3}{\sqrt{169}}=\frac{3}{13}$$

Therefore, the distance will be $\frac{3}{13}$.

(b) $(3,-2,1)2x-y+2z+3=0$

**Ans**: The given point is $(3,-2,1)$ and the plane is $2x-y+2z+3=0$

Now the distance will be $d=\left|\frac{2\times3-(-2)2\times1+3}{\sqrt{(2)^2+(-1)^2+2^2}}\right|=\left|\frac{13}{3}\right|=\frac{13}{3}$

Therefore, the distance will be $\frac{3}{13}$.

(c) $(2,3,-5)x+2y-2z=9$

**Ans:** The given point is (2, 3, -5) and the plane is x+2y-2z=9

Now the distance will be,

$$d=\left|\frac{2+2\times3-2\times(-5)-3}{\sqrt{(1)^2+(2)^2+(-2)^2}}\right|=\frac{9}{3}=3$$

Therefore, the distance will be 3.

(d) (-6,0,0) 2x-3y+6z-2=0)

**Ans:** The given point is (-6, 0, 0) and the plane is 2x-3y+6z-2=0

Now, the distance will be,

$$d=\left|\frac{2\times(-6)-3\times0+6\times0-2}{\sqrt{(2)^2+(-3)^2+6^2}}\right|=\left|\frac{-14}{\sqrt{49}}\right|=\frac{14}{7}=2$$

Therefore, the distance will be $\frac{3}{13}$.