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Chapter: 12. Linear Programming

Exercise 12. Miscellaneous

1. How many packets of each food should be used to maximize the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

Solution. Let the diet contain x and y packets of foods P and Q respectively.

Thus, $x \geq 0$ and $y \geq 0$

The mathematical formulation of the given problem is as follows.

Maximize $z = 6x + 3y$ … (1)

subject to the constraints,

$$\begin{aligned} 4x + y &\geq 80 ---(2) \\ x + 5y &\geq 115 ----(3) \\ 3x + 2y &\leq 150 ---(4) \\ x, y &\geq 0 ---(5) \end{aligned}$$

The feasible region determined by the system of constraints is given by The corner points of the feasible region are A (15, 20), B (40, 15), and C (2,72).

The values of z at these corner points are as follows.

Corner point	Z = 6x + 3y	
A (15,20)	150	
B (40,15)	285	Maximum
C (2,72)	228	

Thus, the maximum value of z is 285 at (40, 15).

2. A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs. 250 per bag contains units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs. 200 per bag contains 1.5 units of nutritional elements A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Solution. Let the farmer mix x bags of brand P and y bags of brand Q.

The given information can be given in a table as follows.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Cost (Rs/kg)
Food P	3	5	60
Food Q	4	2	80
Requirements (units/kg)	8	11	

The given problem can be formulated as follows.

Minimize z = 250x + 200y … (1)

subject to the constraints,

$$3x + 1.5y \geq 18 ---(2)$$
$$2.5x + 11.25y \geq 45 ---(3)$$
$$2x + 3y \geq 24 ---(4)$$
$$x, y \geq 0 ----(5)$$

The feasible region determined by the system of constraints is given by
The corner points of the feasible region are A (18, 0), B (9, 2), C (3, 6), and D (0, 12).

The values of z at these corner points are as follows.

Corner point	Z = 6x + 3y	
A (18,0)	4500	
B (9,2)	2650	
C (3,6)	1950	Minimum
D (0,12)	2400	

As the feasible region is unbounded, therefore, 1950 may or may not be the minimum value of z.

For this, we draw a graph of the inequality, 250x + 200y < 1950 or 5x + 4y < 39, and check

whether the resulting half plane has points in common with the feasible region or not.

Since, the feasible region has no common point with 5x + 4y < 39

Thus, the minimum value of z is 2000 at (3, 6).

3. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture

contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin content of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs. 16 and one kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet?

Solution. Let the mixture contain x kg of food X and y kg of food Y.

The mathematical formulation of the given problem is as follows.

Minimize z = 16x + 20y … (1)

subject to the constraints,

$$\begin{aligned} x+2y &\geq 10 \quad \ldots(2) \\ x+y &\geq 6 \quad \ldots(3) \\ 3x+y &\geq 8 \quad \ldots(4) \\ x, y &\geq 0 \quad \ldots(5) \end{aligned}$$

The feasible region determined by the system of constraints is given by

The corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5), and D (0, 8).

The values of z at these corner points are as follows.

Corner point	Z = 16x + 20y	
A (10,0)	160	
B (2,4)	112	Minimum
C (1,5)	116	
D (0,8)	160	

As the feasible region is unbounded, therefore, 112 may or may not be the minimum value of z. For this, we draw a graph of the inequality, 16x + 20y < 112 or 4x + 5y < 28, and check whether the resulting half plane has points in common with the feasible region or not. Since, the feasible region has no common point with 4x + 5y < 28 Thus, the minimum value of z is 112 at (2, 4).

4. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Types of toys	Machines		
	I	II	III
B (2,4)	12	18	6
C (1,5)	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type is Rs. 7.50 and that on each toy of type B is Rs. 5, show that 15 toys of type A and 30 of type should be manufactured in a day to get maximum profit.

Solution. Let x and y toys of type A and type B respectively be manufactured in a day.

The given problem can be formulated as follows.

Maximize z = 7.5x + 5y … (1)

subject to the constraints,

$$2x + y \le 60 \ldots (2)$$
$$x \le 20 \ldots (3)$$
$$2x + 3y \le 120 \ldots (4)$$
$$x, y \ge 0 \ldots (5)$$

The feasible region determined by the constraints is given by

The corner points of the feasible region are A (20, 0), B (20, 20), C (15, 30), and D (0, 40).

The values of z at these corner points are as follows.

Corner point	Z = 7.5x + 5y	
A (20,0)	150	
B (20,20)	250	
C (15,30)	262.5	Maximum
D (0,40)	200	

Thus, The maximum value of z is 262.5 at (15, 30).

5. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Solution. Let the airline sell x tickets of executive class and y tickets of economy class.

The mathematical formulation of the given problem is as follows.

Maximize z = 1000x + 600y … (1)

subject to the constraints,

$$x + y \le 200 \ldots (2)$$
$$x \ge 20 \ldots (3)$$
$$y - 4x \ge 0 \ldots (4)$$
$$x, y \ge 0 \ldots (5)$$

The feasible region determined by the constraints is given by

The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180).

The values of z at these corner points are as follows.

Corner point	Z = 1000x + 600y	
A (20,80)	68000	
B (40,160)	136000	Maximum
C (20,180)	128000	

Thus, the maximum value of z is 136000 at (40, 160).

6. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation Cost per (in Rs)		
From/To	A	B
D	6	4
E	3	2

F	2.5	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Solution. Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, (100 − x − y) will be supplied to shop F. The requirement at shop D is 60 quintals since x quintals are transported from godown A. Therefore, the remaining (60 −x) quintals will be transported from godown B. Similarly, (50 − y) quintals and 40 − (100 − x − y) = (x + y − 60) quintals will be transported from godown B to shop E and F respectively. The given problem can be represented diagrammatically as follows.

$$x \geq 0, y \geq 0, \text{ and} 100 - x - y \geq 0$$
$$\Rightarrow x \geq 0, y \geq 0, \text{ and } x + y \leq 100$$
$$60 - x \geq 0, 50 - y \geq 0 \text{ and } x + y - 60 \geq 0$$
$$\Rightarrow x \leq 60, y \leq 50, \text{ and } x + y \geq 60$$

Total transportation cost z is given by,

$$z = 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$$
$$= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180$$
$$= 2.5x + 1.5y + 410$$

The given problem can be formulated as

Minimize z = 2.5x + 1.5y + 410 … (1)

subject to the constraints,

$$x + y \leq 100 \ldots (2)$$
$$x \leq 60 \ldots (3)$$
$$y \leq 50 \ldots (4)$$
$$x + y \geq 60 \ldots (5)$$
$$x, y \geq 0 \ldots (6)$$

The feasible region determined by the system of constraints is given by

The corner points are A (60, 0), B (60, 40), C (50, 50), and D (10, 50).

The values of z at these corner points are as follows.

Corner point	Z = 2.5x + 1.5y+410	
A (60,0)	560	

B (60,40)	620	
C (50,50)	610	
D (10,50)	510	Minimum

Thus, the minimum value of z is 510 at (10, 50).

7. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respective l y. The distance (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km)		
From / To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Rs. 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

Solution. Let x and y litres of oil be supplied from A to the petrol pumps, D and E. Then, (7000 − x − y) will be supplied from A to petrol pump F. The requirement at petrol pump D is 4500 L. Since x L are transported from depot A, the remaining (4500 −x) L will be transported from petrol pump B. Similarly, (3000 − y) L and 3500 − (7000 − x − y) = (x + y − 3500) L will be transported from depot B to petrol pump E and F respectively. The given problem can be represented diagrammatically as follows.

$$x \geq 0, y \geq 0, \text{ and} (700 - \mathrm{x} - \mathrm{y}) \geq 0$$

$$\Rightarrow \mathrm{x} \geq 0, \mathrm{y} \geq 0, \text{ and } \mathrm{x} + \mathrm{y} \leq 7000$$

$$4500 - x \geq 0, 3000 - y \geq 0, \text{ and } x + y - 3500 \geq 0$$

$$\Rightarrow x \leq 4500, y \leq 30000, \text{ and } x + y \geq 3500$$

Cost of transporting 10 L of petrol = Rs. 1

Cost of transporting 1 L of petrol = 1/10

Therefore, total transportation cost is given by,

$$\mathrm{z} = \frac{7}{10} \times x + \frac{6}{10} y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$

$$= 0.3x + 0.1y + 3950$$

The problem can be formulated as follows.

Minimize z = 0.3x + 0.1y + 3950 … (1)

subject to the constraints,

$$x + y \le 7000 \ldots (2)$$
$$x \le 4500 \ldots (3)$$
$$y \le 3000 \ldots (4)$$
$$x + y \ge 3500 \ldots (5)$$
$$x, y \ge 0 \ldots (6)$$

The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000,3000), and E (500, 3000).

The values of z at these corner points are as follows. The feasible region determined by the constraints is given by

Corner point	Z = 0.3x + 0.1y+3950	
A (35000,0)	5000	
B (4500,0)	5300	
C (4500,25000)	5550	
D (4000,3000)	5450	
E (500,3000)	4400	Minimum

Thus, the minimum value of z is 4400 at (500, 3000).

8. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

Kg per bag		
From / To	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric Acid	1	2

Potash	3	1.5
Chlorine	1.5	2

Let the fruit grower use x bags of brand P and y bags of brand Q.

The problem can be formulated as follows.

Minimize z = 3x + 3.5y … (1)

subject to the constraints,

The feasible region determined by the system of constraints is given by

$$x+2y \geq 240 ----(2)$$
$$x+0.5y \geq 90 ----(3)$$
$$1.5x+2y \leq 310 ----(4)$$
$$x, y \geq 0 -----(5)$$

The feasible region determined by the system of constraints is given by

The corner points are A (240, 0), B (140, 50), and C (20, 140).

The values of z at these corner points are as follows.

Corner point	Z = 3x + 3.5y	
A (140,50)	595	
B (20,0)	550	
C (40,100)	470	Minimum

Thus, the maximum value of z is 470 at (40, 100).

9. Refer to quest ion 8. If the grower wants to maximize the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen?

Solution. Let x and y be the number of dolls of type A and B respectively that are produced per week.

The given problem can be formulated as follows.

Maximize z = 12x + 16y … (1)

subject to the constraints,

$$x + y \leq 1200 - - - (2)$$

$$y \leq \frac{x}{2} - \Rightarrow x \geq 2y - - - - (3)$$

$$x - 3y \leq 600 - - - - (4)$$

$$x, y \geq 0 - - - - (5)$$

The feasible region determined by the system of constraints is given by

The corner points are A (140, 50), B (20, 140), and C (40, 100).

The values of z at these corner points are as follows.

Corner point	Z = 3x + 3.5y	
A (140,50)	595	
B (20,0)	550	
C (40,100)	470	Minimum

The feasible region determined by the system of constraints is given by

The corner points are A (600, 0), B (1050, 150), and C (800, 400).

The values of z at these corner points are as follows.

Corner point	Z = 12x + 16y	
A (600,0)	7200	
B (1050,150)	15000	
C (800,400)	16000	Maximum

Thus, the maximum value of z is 16000 at (800, 400).