Chapter: 12. Linear Programming

Exercise 12.1

1. Maximize Z = 3x + 4y

Subject to the constraints: $x+y\le 4, x\ge 0, y\ge 0$

Solution. The feasible region determined by the constraints, $x + y \le 4$, $x \ge 0$, $y \ge 0$, is given by

Since, the corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of

Z at these points are as follows.

Corner point	Z = 3x+ 4y	
O (0,0)	0	
A (4,0)	12	
B (0,4)	16	Maximum

Thus, the maximum value of Z is 16 at the point B (0, 4).

2. Minimize Z = −3x + 4y

subject to $x+2y\le 8, 3x+2y\le 12, x\ge 0, y\ge 0.$

Solution. The feasible region determined by the system of constraints,

$x+2y\le 8, 3x+2y\le 12, x\ge 0, y\ge 0.$ is given by

Since, the corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4).

The values of Z at these corner points are as follows.

Corner point	Z = - 3x+ 4y	
O (0,0)	0	
A (4,0)	-12	Minimize
B (0,4)	6	
C (0,4)	16	

Thus, the minimum value of Z is -12 at the point (4, 0).

3. Maximize Z = 5x + 3y

subject to $3x+5y\le 15, 5x+2y\le 10, x\ge 0, y\ge 0$

Solution. The feasible region determined by the system of constraints, $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$,

and $y \ge 0$, is given by

Since, the corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3), and $C\left(\frac{20}{19},\frac{45}{19}\right)$

The values of Z at these corner points are as follows.

Corner point	Z = 5x+ 3y	
O (0,0)	0	
A (2,0)	10	
B (0,3)	9	
$C\left(\frac{20}{19},\frac{45}{19}\right)$	$\frac{235}{19}$	Maximum

Thus, the maximum value of Z is at the point

4. Minimize Z = 3x + 5y

such that $x+3y\geq 3, x+y\geq 2, x, y\geq 0$

Solution. The feasible region determined by the system of constraints, $x+3y\geq 3, x+y\geq 2,$ and x, y ≥ 0,

is given by

Since, the feasible region is unbounded. The corner points of the feasible region are A (3, 0), $B\left(\frac{3}{2},\frac{1}{2}\right)$, and C (0, 2).

The values of Z at these corner points are as follows.

Corner point	Z = 3x+ 5y	
A (3,0)	9	
$B\left(\frac{3}{2},\frac{1}{2}\right)$	7	Smallest
C (0,2)	10	

As the feasible region is unbounded, therefore, 7 may or may not be the minimum value of Z.

For this, we draw the graph of the inequality, 3x + 5y < 7, and check whether the resulting half

plane has points in common with the feasible region or not.

Since, feasible region has no common point with 3x + 5y < 7

Thus, the minimum value of Z is 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$

5. Maximize Z = 3x + 2y subject to $x+2y \le 10, 3x+y \le 15, x, y \ge 0$

Solution. The feasible region determined by the constraints, x + 2y ≤ 10, 3x + y ≤ 15, x ≥ 0, and y ≥ 0, is given by

Since, the corner points of the feasible region are A (5, 0), B (4, 3), and C (0, 5).

The values of Z at these corner points are as follows.

Corner point	Z = 3x+ 2y	
A (3,0)	15	
B (4,3)	18	Maximum
C (0,2)	10	

Thus, the maximum value of Z is 18 at the point (4, 3).

6. Minimize Z = x + 2y

subject to $2x+y \ge 3, x+2y \ge 6, x, y \ge 0$

Solution. The feasible region determined by the constraints, 2x + y ≥ 3, x + 2y ≥ 6, x ≥ 0, and y ≥ 0, is

given by

Since, the corner points of the feasible region are A (6, 0) and B (0, 3).

The values of Z at these corner points are as follows.

Corner point	Z = x+ 2y
A (6,0)	6
B (0,3)	6

Since, the value of Z at points A and B is same. If we take any other point such as (2, 2) on line

x + 2y = 6, then Z = 6

Thus, the minimum value of Z occurs for more than 2 points.

Thus, the value of Z is minimum at every point on the line, x + 2y = 6

7. Minimize and Maximize Z = 5x + 10y

subject to $x+2y\le 120, x+y\ge 60, x-2y\ge 0, x, y\ge 0$

Solution. The feasible region determined by the constraints, $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, and $y \ge 0$, is given by

Since, the corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40,20).

The values of Z at these corner points are as follows.

Corner point	Z = 5x+ 10y	
A (60,0)	300	Minimum
B (120,0)	600	Maximum
C (60,30)	600	Maximum
D (40,20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

8. Minimize and Maximize Z = x + 2y
subject to $x+2y\ge 100, 2x-y\le 0, 2x+y\le 200, x, y\ge 0.$

Solution. The feasible region determined by the constraints, $x + 2y \ge 100$, $2x - y \le 0$, $2x + y \le 200$, $x \ge 0$, and $y \ge 0$, is given by

The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100), and D(0, 200).

The values of Z at these corner points are as follows.

Corner point	Z = x+ 2y	
A (0,50)	100	Minimum
B (20,40)	100	Minimum
C (50,100)	250	
D (0,200)	400	Maximum

The maximum value of Z is 400 at (0, 200) and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

9. Maximize Z = − x + 2y, subject to the constraints: $x\ge 3, x+y\ge 5, x+2y\ge 6, y\ge 0$

Solution. The feasible region determined by the constraints, $x \geq 3, x+y \geq 5, x+2y \geq 6,$ and $y \geq 0$ is given by

Since, the feasible region is unbounded.

The values of Z at corner points A (6, 0), B (4, 1), and C (3, 2) are as follows.

Corner point	Z = - x+ 2y
A (6,0)	Z= -6
B (4,1)	Z= -2
C (3,2)	Z=1

As the feasible region is unbounded, therefore, Z = 1 may or may not be the maximum value.

For this, we graph the inequality, $-x + 2y > 1$, and check whether the resulting half lane has points in common with the feasible region or not.

The resulting feasible region has points in common with the feasible region.

Thus, Z = 1 is not the maximum value. Z has no maximum value.

10. Maximize Z = x + y, subject to $x-y \leq -1, -x+y \leq 0, x, y \geq 0$

Solution. The region determined by the constraints, $x-y \leq -1, -x+y \leq 0, x, y \geq 0$ is given by

There is no feasible region and thus, Z has no maximum value.