

Chapter: 12. Linear Programming

Exercise 12.2

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs. 60/kg and Food Q costs Rs. 80/kg. Food P contains 3 units /kg of vitamin A and 5 units /kg of vitamin B while food Q contains 4 units /kg of vitamin A and 2 units /kg of vitamin B. Determine the minimum cost of the mixture?

Solution. Let the mixture contain x kg of food P and y kg of food Q. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be given in a table as follows.

	Vitamin A (units/kgs)	Vitamin B (units/kgs)	Vitamin C (Rs/kgs)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kgs)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B.

Thus, the constraints are

$$3x + 4y \geq 8$$

$$5x + 2y \geq 11$$

Total cost, Z , of purchasing food is, $Z = 60x + 80y$

The mathematical formulation of the given problem is

$$\text{Minimize } Z = 60x + 80y \dots (1)$$

subject to the constraints,

$$3x + 4y \geq 8 \dots (2)$$

$$5x + 2y \geq 11 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is given by

Since, the feasible region is unbounded. $A\left(\frac{8}{3}, 0\right)$, $B\left(2, \frac{1}{2}\right)$, and $C\left(0, \frac{11}{2}\right)$ The corner points of the feasible region are The values of Z at these corner points are as follows.

Corner point	$Z = 60x + 80y$	
$A\left(\frac{8}{3}, 0\right)$	160	Minimize
$B\left(2, \frac{1}{2}\right)$	160	
$C\left(0, \frac{11}{2}\right)$	440	

As the feasible region is unbounded, therefore, 160 may or may not be the minimum value of Z.

For this, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half plane has points in common with the feasible region or not.

Since, the feasible region has no common point with $3x + 4y < 8$

Thus, the minimum cost of the mixture will be Rs. 160 at the line segment joining the points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$

2. One kind of cake requires 200g flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes?

Solution. Let there be x cakes of first kind and y cakes of second kind. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be given in a table as follows.

Corner point	Flour (g)	Fat (g)
Cakes of the first kind, x	200	25

Cakes of the second kind, y	100	50
Availability	5000	1000

$$\therefore 200x + 100y \leq 5000$$

$$\Rightarrow 2x + y \leq 50$$

$$25x + 50y \leq 1000$$

$$\Rightarrow x + 2y \leq 40$$

Total numbers of cakes, Z, that can be made are, $Z = x + y$

The mathematical formulation of the given problem is

Maximize $Z = x + y \dots (1)$

subject to the constraints,

$$2x + y \leq 50 \text{-----}(2)$$

$$x + 2y \leq 40 \text{-----}(3)$$

$$x, y \geq 0 \text{-----}(4)$$

The feasible region determined by the system of constraints is given by

The corner points are A (25, 0), B (20, 10), O (0, 0), and C (0, 20).

The values of Z at these corner points are as follows.

Corner point	Z = x + y	
A (25,0)	25	
B (20,10)	30	Maximum
C (0,20)	20	
O (0,0)	0	

Thus, the maximum numbers of cakes that can be made are 30 (20 of one kind and 10 of the other kind).

3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

- (i) What number of rackets and bats must be made if the factory is to work at full capacity?
- (ii) If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the maximum profit of the factory when it works at full capacity.

Solution. (i) Let the number of rackets and the number of bats to be made be x and y respectively.

The machine time is not available for more than 42 hours.

$$\therefore 1.5x + 3y \leq 42 \text{-----(1)}$$

The craftsman's time is not available for more than 24 hours.

$$3x + y \leq 24 \text{-----(2)}$$

The factory is to work at full capacity. Therefore,

$$1.5x + 3y = 42$$

$$3x + y = 24$$

On solving these equations, we obtain

$$x = 4 \text{ and } y = 12$$

Thus, 4 rackets and 12 bats must be made.

(i) The given information can be given in a table as follows.

Corner point	Tennis Rocket	Cricket bat	Availability
Machine Time (h)	1.5	3	42
Craftsman's Time (h)	3	1	24

$$\therefore 1.5x + 3y \leq 42$$

$$3x + y \leq 24$$

$$x, y \geq 0$$

The profit on a racket is Rs. 20 and on a bat is Rs. 10.

$$\text{Maximize } Z = 20x + 10y \text{.....(1)}$$

subject to the constraints,

$$1.5x + 3y \leq 42 \text{ (2)}$$

$$3x + y \leq 24 \text{ (3)}$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is given by

The corner points are A (8, 0), B (4, 12), C (0, 14), and O (0, 0).

The values of Z at these corner points are as follows.

Corner point	Z= 20x + 10y	
A (8,0)	160	
B (4,12)	200	Maximum
C (0,14)	140	
O (0,0)	0	

Thus, the maximum profit of the factory when it works to its full capacity is Rs. 200.

4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit, of Rs. 17.50 per package on Nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?

Solution. Let the manufacturer produce x packages of nuts and y packages of bolts. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be given in a table as follows.

	Nuts	Bolts	Availability
Machine A (h)	1	3	12
Machine B (h)	3	1	12

The profit on a package of nuts is Rs. 17.50 and on a package of bolts is Rs. 7.

Therefore, the constraints are

$$x + 3y \leq 12$$

$$3x + y \leq 12$$

Total profit, $Z = 17.5x + 7y$

The mathematical formulation of the given problem is

Maximize $Z = 17.5x + 7y \dots (1)$

subject to the constraints,

$$x + 3y \leq 12 \dots (2)$$

$$3x + y \leq 12 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is given by

The corner points are A (4, 0), B (3, 3), and C (0, 4).

The values of Z at these corner points are as follows.

Corner point	$Z = 17.5x + 7y$	
O (0,0)	0	
A (4,0)	70	
B (3,3)	73.5	Maximum
C (0,4)	28	

The maximum value of Z is Rs. 73.50 at (3, 3).

5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs. 7 and screws B at a profit of Rs.10. Assuming that he can sell all the screws he

manufactures, how many packages of each type should the factory owner produces in a day in order to maximize his profit? Determine the maximum prof it.

Solution. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be given in a table as follows.

The profit on a package of screws A is Rs. 7 and on the package of screws B is Rs. 10

Therefore, the constraints are

	Screw A	Screw B	Availability
Automatic Machine (min)	4	6	$4 \times 60 = 240$
Hand Operated Machine (min)	6	3	$4 \times 60 = 240$

The profit on a package of screws A is Rs. 7 and on the package of screws B is Rs. 10.

Therefore, the constraints are

$$4x + 6y \leq 240$$

$$6x + 3y \leq 240$$

$$\text{Total profit, } Z = 7x + 10y$$

The mathematical formulation of the given problem is

$$\text{Maximize } Z = 7x + 10y \dots (1)$$

subject to the constraints,

$$4x + 6y \leq 240$$

$$6x + 3y \leq 240$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is given by

The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are as follows.

Corner point	$Z = 7x + 10y$	
A (4,0)	280	
B (3,3)	410	Maximum
C (0,4)	400	

The maximum value of Z is 410 at (3, 3).

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding /cutting machine and a sprayer. It takes 2 hours on grinding /cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding /cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs. 5 and that from a shade is Rs. 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit?

Solution. Let the cottage industry manufacture x pedestal lamps and y wooden shades.

Therefore,

$$x \geq 0 \text{ and } y \geq 0$$

The given information can be given in a table as follows.

	crew A	Screw B	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs. 5 and on the shades is Rs. 3.

Therefore, the constraints are

$$2x + y \leq 12$$

$$3x + 2y \leq 20$$

Total profit, $Z = 5x + 3y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 3y \dots (1)$

subject to the constraints,

$$2x + y \leq 12 \dots (2)$$

$$3x + 2y \leq 20 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	$Z = 5x + 3y$	
A (6,0)	30	
B (4,4)	32	Maximum
C (0,10)	30	

The maximum value of Z is 32 at (4, 4)

7. A company manufactures two types of novelty Souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours of assembling. The profit is Rs. 5 each for type A and Rs. 6 each for type B Souvenirs. How many Souvenirs of each type should the company manufacture in order to maximize the profit?

Solution. Let the company manufacture x Souvenirs of type A and y Souvenirs of type B.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be given in a table as follows.

	type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A Souvenirs is Rs. 5 and on type B Souvenirs is Rs. 6.

Therefore, the constraints are

$$5x + 8y \leq 200$$

$$10x + 8y \leq 240 \text{ i.e., } 5x + 4y \leq 120$$

Total profit, $Z = 5x + 6y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 6y \dots (1)$

subject to the constraints,

$$5x + 8y \leq 200 \text{ --- (2)}$$

$$5x + 4y \leq 120 \text{ --- (3)}$$

$$x, y \geq 0 \text{ --- (4)}$$

The feasible region determined by the system of constraints is given by

The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows.

Corner point	$Z = 5x + 6y$	
A (24,0)	120	
B (8,20)	160	Maximum
C (0,25)	150	

The maximum value of Z is 200 at (8, 20).

8. A merchant plans to sell two types of personal computers – a desktop model and a portable

model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and if h is profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000.

Solution. Let the merchant stock x desktop models and y portable models. Therefore, $x \geq 0$ and $y \geq 0$, The cost of a desktop model is Rs. 25000 and of a portable model is Rs. 4000. However, the merchant can invest a maximum of Rs. 70 lakhs.

$$\therefore 25000x + 40000y \leq 7000000$$

$$5x + 8y \leq 1400$$

The monthly demand of computers will not exceed 250 units.

$$\therefore x + y \leq 250$$

The profit on a desktop model is Rs. 4500 and the profit on a portable model is Rs. 5000.

$$\text{Total profit, } Z = 4500x + 5000y$$

Thus, the mathematical formulation of the given problem is

$$\text{Maximum } Z = 4500x + 5000y \dots (1)$$

Subject to the constraints,

$$5x + 8y \leq 1400 \dots (2)$$

$$x + y \leq 250 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is given by

The corner points are A (250, 0), B (200, 50), and C (0, 175).

The values of Z at these corner points are as follows.

Corner point	$Z = 4500x + 5000y$	
A (250,0)	1125000	
B (200,50)	1150000	Maximum
C (0,175)	875000	

The maximum value of Z is 1150000 at (200, 50).

9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. 4 per unit food and F_2 costs Rs. 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements?

Solution. Let the diet contain x units of food F_1 and y units of food F_2 .

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be given in a table as follows.

Corner point	Vitamin (A)	Mineral (Units)	Cost per unit (Rs)
Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Requirement	80	100	

The cost of food F_1 , is Rs. 4 per unit and of Food F_2 is Rs. 6 per unit.

Therefore, the constraints are

$$3x + 6y \geq 80$$

$$4x + 3y \geq 100$$

$$x, y \geq 0$$

Total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem is

$$\text{Minimize } Z = 4x + 6y \dots (1)$$

subject to the constraints,

$$3x + 6y \geq 80 \dots (2)$$

$$4x + 3y \geq 100 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the constraints is given by

Since, the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{80}{3}, 0\right)$, $B\left(24, \frac{4}{3}\right)$, and $C\left(0, \frac{100}{3}\right)$

The values of Z at these corner points are as follows.

Corner point	$Z = 4x + 6y$	
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.67$	
$B\left(24, \frac{4}{3}\right)$	104	Minimum
$C\left(0, \frac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of z . For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not. Since, the feasible region has no common point with $2x + 3y < 52$.

Therefore, the minimum cost of the mixture will be Rs. 104.

10. There are two types of fertilizers F_1 and F_2 . F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 cost Rs. 6/kg and F_2 costs Rs. 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Solution. Let the farmer buy x kg of fertilizer F_1 and y kg of fertilizer F_2 . Therefore, $x \geq 0$ and $y \geq 0$. The given information can be given in a table as follows

Corner point	Nitrogen (%)	Phosphoric Acid (%)	Cost (Rs/kg)
$F_1 (x)$	10	6	6
$F_2 (y)$	5	10	5
Requirement(kg)	14	14	

F_1 consists of 10% nitrogen and F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

$$\therefore 10\% \text{ of } x + 5\% \text{ of } y \geq 14$$

$$\frac{x}{10} + \frac{y}{20} \geq 14$$

$$2x + y \geq 280$$

F1 consists of 6% phosphoric acid and F2 consists of 10% phosphoric acid. However, the farmer

requires at least 14 kg of phosphoric acid.

$$\therefore 6\% \text{ of } x + 10\% \text{ of } y \geq 14$$

$$\frac{6x}{100} + \frac{10y}{100} \geq 14$$

$$3x + 5y \geq 700$$

Total cost of fertilizers, $Z = 6x + 5y$

The mathematical formulation of the given problem is

$$\text{Minimize } Z = 6x + 5y \dots (1)$$

subject to the constraints,

$$2x + y \geq 280 \dots (2)$$

$$3x + 5y \geq 700 \dots (3)$$

$$x, y \geq 0 \dots (4)$$

The feasible region determined by the system of constraints is given by

Since, the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3}, 0\right)$, $B(100, 80)$, and $C(0, 280)$.

The values of Z at these points are as follows.

Corner point	$Z = 6x + 5y$	
$A\left(\frac{700}{3}, 0\right)$	1400	
$B(100, 80)$	1000	Minimum
$C(0, 280)$	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z . For this, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half plane has points in common with the feasible region or not.

Since, the feasible region has no common point with $6x + 5y < 1000$

Thus, 100 kg of fertilizer F1 and 80 kg of fertilizer F 2 should be used to minimize the cost. The minimum cost is Rs. 1000.

11. The corner points of the feasible region determined by the following system of linear inequalities:

$$2x + y \leq 10, x + 3y \leq 15, xy \geq 0, \text{ and } (0,0), (5,0), (3,4), \text{ and } (0,5)$$
 Let $Z = px + qy$,

where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both $(3, 4)$ and $(0, 5)$ is

(A) $p = q$ (B) $p = 2q$ (C) $p = 3q$ (D) $q = 3p$

Solution. The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points, $(3, 4)$ and $(0, 5)$.

$$\therefore \text{Value of } Z \text{ at } (3, 4) = \text{Value of } Z \text{ at } (0, 5)$$

$$\Rightarrow p(3) + q(4) = p(0) + q(5)$$

$$\Rightarrow 3p + 4q = 5q$$

$$\Rightarrow q = 3p$$