Exercise 13.2

1. If $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$, find $P(A\cap B)$ if A and B are independent events

Solution: Given, $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$

Given A and B are independent events. It implies that $P(A\cap B)=P(A)\cdot P(B)$

Hence,

$$\begin{aligned}P(A\cap B)&=P(A)\cdot P(B)\\&=\frac{3}{5}\cdot\frac{1}{5}\\&=\frac{3}{25}\end{aligned}$$

Therefore, $P(A\cap B)=\frac{3}{25}$

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black

Solution: Since there are 26 black cards in the deck of 52 cards

Let A be the event of the getting two black cards without replacement

It implies that

$$\begin{aligned}P(A)&=\frac{C(26,2)}{C(52,2)}\\&=\frac{26\cdot 25}{52\cdot 51}\\&=\frac{25}{102}\end{aligned}$$

Therefore, the probability that two cards drawn are to be black is $\frac{25}{102}$

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale

Solution: Given that there are 15 oranges in box out of 12 are good and 3 are bad ones.

Suppose that three randomly selected oranges drawn without replacement

Suppose that A be the event of getting all three oranges are good

$$\begin{aligned} P(A) &= \frac{C(12,3)}{C(15,3)} \\ &= \frac{12\cdot 11\cdot 10}{15\cdot 14\cdot 13} \\ &= \frac{44}{91} \end{aligned}$$

Therefore, the probability that the box approved for sale is $\frac{44}{91}$

4. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not

Solution: The Sample space S is given by

$$S = \begin{Bmatrix} (H,1),(H,2),(H,3),(H,4),(H,5),(H,6) \\ (T,1),(T,2),(T,3),(T,4),(T,5),(T,6) \end{Bmatrix}$$

Let A be the event which shows Head on the coin

$$A = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$$

It implies that $P(A) = \frac{6}{12} = \frac{1}{2}$

Let B be the event which shows 3 on the die

$$B = \{(H,3),(T,3)\}$$

It implies that $P(B)=\frac{2}{12}=\frac{1}{6}$

$$\therefore A\cap B=\{(H,3)\}$$

$$P(A\cap B)=\frac{1}{12}$$

$$P(A).P(B)=\frac{1}{2}\times\frac{1}{6}=P(A\cap B)$$

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the events, ‘the number is even,’ and B be the event, ‘the number is red’. Are A and B independent?

Solution:

The sample space (S) is $S=\{1,2,3,4,5,6\}$

Let A: the number is even $=\{2,4,6\}$

$$\Rightarrow P(A)=\frac{3}{6}=\frac{1}{2}$$

B: the number is red $=\{1,2,3\}$

$$\Rightarrow P(B)=\frac{3}{6}=\frac{1}{2}$$

$$\therefore A\cap B=\{2\}$$

$$P(AB)=P(A\cap B)=\frac{1}{6}$$

$$P(A)P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}\neq\frac{1}{6}$$

$$\Rightarrow P(A).P(B)\neq P(AB)$$

Therefore, A and B are not independent

6. Let E and F be the events with $P(E)=\frac{3}{5}, P(F)=\frac{3}{10}$ and $P(E\cap F)=\frac{1}{5}$. Are E and F independent?

Solution:

Given, $P(E)=\frac{3}{5}, P(F)=\frac{3}{10}$ and $P(EF)=P(E\cap F)=\frac{1}{5}$

$P(E).P(F)=\frac{3}{5}\times\frac{3}{10}=\frac{9}{50}\neq\frac{1}{5}$

$\Rightarrow P(E).P(F)\neq P(EF)$

Thus, E and F are not independent

7. Given that the events A and B are such that $P(A)=\frac{1}{2}, P(A\cap B)=\frac{3}{5}$ and $P(B)=p$

Find p if they are (i) mutually exclusive (ii) independent.

Solution:

Given, $P(A)=\frac{1}{2}, P(A\cap B)=\frac{3}{5}$ and $P(B)=p$

(i) When A and B are mutually exclusive, $A\cap B=\phi$

$\therefore P(A\cap B)=0$

Since, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-0$

$\Rightarrow p=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$

(ii) when A and B are independent, $P(A\cap B)=P(A).P(B)=\frac{1}{2}p$

Since, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p$$

$$\Rightarrow \frac{3}{5} = \frac{1}{2} + \frac{p}{2}$$

$$\Rightarrow \frac{p}{2} = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

$$\Rightarrow p = \frac{2}{10} = \frac{1}{5}$$

8. Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$. Find

i) $P(A \cap B)$ ii) $P(A \cup B)$ iii) $P(A/B)$ iv) $P(B/A)$

Solution:

Given, $P(A) = 0.3$ and $P(B) = 0.4$

i) If A and B are independent events, then

$$P(A \cap B) = P(A).P(B) = 0.3 \times 0.4 = 0.12$$

ii) Since, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = 0.3 + 0.4 - 0.12 = 0.58$$

iii) Since, $\Rightarrow P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$\Rightarrow P(A/B) = \frac{0.12}{0.4} = 0.3$$

iv) Since, $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$$\Rightarrow P(B/A) = \frac{0.12}{0.3} = 0.4$$

9. If A and B are two events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P (not A and not B)

Solution:

Given, $P(A)=\frac{1}{4}$ and $P(A\cap B)=\frac{1}{8}$

P (not on A and not on B) $=P(A'\cap B')$

P (not on A and not on B) $=P((A\cup B))'$

$=1-P(A\cup B)$

$=1-[P(A)+P(B)-P(A\cap B)]$

$=1-\frac{5}{8}$

$=\frac{3}{8}$

10. Events A and B are such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and P (not A or not B) $=\frac{1}{4}$. State whether A and B are independent?

Solution:

Given, $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and P (not A or not B) $=\frac{1}{4}$

$\Rightarrow P(A'\cup B')=\frac{1}{4}$

$\Rightarrow P((A\cap B))'=\frac{1}{4}$

$\Rightarrow 1-P(A\cap B)=\frac{1}{4}$

$\Rightarrow P(A\cap B)=\frac{3}{4}$..........(1)

But, $P(A).P(B)=\frac{1}{2}.\frac{7}{12}=\frac{7}{24}$.........(2)

Here, $\frac{3}{4} \neq \frac{7}{24}$

$\therefore P(A \cap B) = P(A).P(B)$

Thus, A and B are independent events

11. Given two independent events A and B such P(A) = 0.3, P(B) = 0.6. Find (i) P (A and B) (ii) P (A and not B) (iii) P (A or B) (iv) P (neither A or B)

Solution:

Given, $P(A) = 0.3, P(B) = 0.6$

i) $\therefore P(A and\, B) = P(A).P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.6 = 0.18$

ii) P (A and not B) = $P(A \cap B')$

$= P(A) - P(A \cap B)$

$= 0.3 - 0.18$

$= 0.12$

iii) P (A or B) = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18$

$= 0.72$

iv) P (neither A nor B) $= P(A' \cap B')$

$= P((A \cup B))'$

$= 1 - P(A \cup B)$

$=1-0.72$

$=0.28$

12. A die tossed thrice. Find the probability of getting an odd number at least once

Solution:

Probability of getting an odd number in a single throw of a die $=\frac{3}{6}=\frac{1}{2}$

Similarly, probability of getting an even number $=\frac{3}{6}=\frac{1}{2}$

Probability of getting an even number three times $=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

Therefore, probability of getting an odd number at least once

=1-probability of getting an odd number in none of the throws

=1-probability of getting an even number thrice

$=1-\frac{1}{8}$

$=\frac{7}{8}$

13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red

(ii) first ball is black and second is red.

(iii) one of them is black and other is red

Solution:

Given,

Total number of balls = 18

Number of red balls = 8

Number of black balls = 10

(i) Probability of getting a red ball in the first draw = $\frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw.

$\therefore$ Probability of getting a red ball in the second draw = $\frac{8}{18} = \frac{4}{9}$

Thus, probability of getting both the balls red = $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

(ii) Probability of getting first ball black = $\frac{10}{18} = \frac{5}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as red = $\frac{8}{18} = \frac{4}{9}$

Thus, probability of getting first ball as black and second ball as red = $\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

(iii) Probability of getting first ball as red = $\frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as black = $\frac{10}{18} = \frac{5}{9}$

Thus, probability of getting first ball as black and second ball as red = $\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$

Therefore, probability that one of them is black and other is red

= Probability of getting first ball black and second as red + Probability of getting first ball red and second ball black = $\frac{20}{81} + \frac{20}{81}$

$$= \frac{40}{81}$$

14. Probability of solving specific problem independently by A and B are respectively. $\frac{1}{2}$ and $\frac{1}{3}$ If both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one of them solves the problem.

Solution:

Probability of solving the problem by A, $P(A)=\frac{1}{2}$

Probability of solving the problem by B, $P(B)=\frac{1}{3}$

Since the problem is solved independently by A and B

$$\therefore P(A\cap B)=P(A).P(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$$

$$P(A')=1-P(A)=1-\frac{1}{2}=\frac{1}{2}$$

$$P(B')=1-P(B)=1-\frac{1}{3}=\frac{2}{3}$$

i) Probability that the problem is solved $=P(A\cup B)$

$$=P(A)+P(B)-P(A\cap B)$$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$$

$$=\frac{4}{6}$$

$$=\frac{2}{3}$$

ii) Probability that exactly one of them solves the problem is given by

$$P(A).P(B')+P(B).P(A')$$

$$=\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}$$

$= \frac{1}{3} + \frac{1}{6}$

$= \frac{1}{2}$

15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E: 'the card drawn is a spade'

F: 'the card drawn is an ace'

(ii) E: 'the card drawn is black'

F: 'the card drawn is a king'

(iii) E: 'the card drawn is a king and queen'

F: 'the card drawn is a queen or jack'

Solution:

i) Since, in a deck of 52 cards, 13 cards are spades and 4 cards are aces.

$\therefore P(E) = P$ (the card drawn is a spade) = $\frac{13}{52} = \frac{1}{4}$

$\therefore P(F) = P$ (the cards drawn in an ace) $= \frac{4}{52} = \frac{1}{13}$

In the deck of cards, only 1 card is an ace of spades.

P(EF) = P (the card drawn is spade and an ace) = $\frac{1}{52}$

$$P(E) \times P(F) = \frac{1}{4} . \frac{1}{13} = \frac{1}{52} = P(EF)$$

$$\Rightarrow P(E) \times P(F) = P(EF)$$

Thus, the events E and F are independent

ii) Since, in a deck of 52 cards, 26 cards are black and 4 cards are kings

$\therefore P(E)$= P(the card drawn is a black) $= \frac{26}{52} = \frac{1}{2}$

$\therefore P(F) =$ P(the card drawn in an king) $= \frac{4}{52} = \frac{1}{13}$

In the pack of 52 cards, 2 cards are black as well as kings

$\therefore P(EF) =$P (the card drawn is black king) $= \frac{2}{52} = \frac{1}{26}$

$$P(E) \times P(F) = \frac{1}{2} . \frac{1}{13} = \frac{1}{26} = P(EF)$$

Thus, the given events E and F are independent

iii) Since, in a deck of 52 cards, 4 cards are kings, 4 cards are queens and 4 cards are jacks

$\therefore P(E) =$P (the card drawn is a king or a queen) $= \frac{8}{52} = \frac{2}{13}$

$\therefore P(F) =$P (the card drawn in a queen or a jack) $= \frac{8}{52} = \frac{2}{13}$

There are 4 cards which are king and queen or jack

$\therefore P(EF) =$P(the card drawn is king or a queen, or queen or a jack)

$$= \frac{4}{52} = \frac{1}{13}$$

$$P(E) \times P(F) = \frac{2}{13} . \frac{2}{13} = \frac{4}{169} \neq \frac{1}{3}$$

$$\Rightarrow P(E).P(F) \neq P(EF)$$

Thus, the given events E and F are not independent

16. In a hotel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English news papers. A student is selected at random. (a) Find the probability that she reads neither Hindi and English news papers.

(b) If she reads Hindi news paper, find the probability that she reads English news paper.

(c) If she reads English news paper, find the probability that she reads Hindi news paper.

Solution:

Let H denote the students who read Hindi newspaper and E denote the students who read English newspaper.

Given,

$$P(H) = 60\% = \frac{60}{100} = \frac{3}{5}$$

$$P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(H \cap E) = 20\% = \frac{20}{100} = \frac{1}{5}$$

i) Probability that a student reads Hindi and English newspaper is,

$$P(H \cup E') = 1 - P(H \cup E)$$

$$= 1 - \{P(H) + P(E) - P(H \cap E)\}$$

$$= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right)$$

$$= 1 - \frac{4}{5} = \frac{1}{5}$$

ii) Probability that a randomly chosen student reads English newspapers, if she reads Hindi news paper, is given by P (E/H).

$$P(E/H) = \frac{P(E \cap H)}{P(H)} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$$

iii) Probability that a random chosen student reads Hindi newspaper, if she reads English newspaper, is given by $P(H/E)$

$$P(H/E) = \frac{P(H \cap E)}{P(E)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$$

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{12}$ D) $\frac{1}{36}$

Solution:

The only even prime number is 2

Let E be the event of getting an even prime number on each die

$\therefore E = \{(2,2)\}$

$\Rightarrow P(E) = \frac{1}{36}$

18. Two events A and B will be independently, if

A) A and B are mutually exclusive

B) $P(A'B') = [1-P(A)][1-P(B)]$

C) $P(A) = P(B)$

D) $P(A)+P(B) = 1$

Solution:

Two events A and B are said to be independent, if $P(AB) = P(A) \times P(B)$

Let take option B

$P(A'B') = [1-P(A)][1-P(B)]$

$\Rightarrow P(A'\cap B') = 1 - P(A) + P(A).P(B)$

$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A).P(B)$

$\Rightarrow P(A) + P(B) - P(AB) = P(A) + P(B) - P(A).P(B)$

$\Rightarrow P(AB) = P(A).P(B)$

This implies that A and B are independent, if $P(A'B') = [1-P(A)][1-P(B)]$

A. Let $P(A) = m, P(B) = n, 0 < m, n < 1$

A and B are mutually exclusive

$\therefore A\cap B = \phi$

$\Rightarrow P(AB) = 0$

However, $P(A).P(B) = mn \neq 0$

$\therefore P(A).P(B) \neq P(AB)$

C. Let A: Event of getting an odd number on throw of a die $= \{1,3,5\}$

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B: Event of getting an even number on throw of a die $= \{2,4,6\}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

Here, $A\cap B = \phi$

$\therefore P(AB) = 0$

$P(A).P(B) = \frac{1}{4} \neq 0$

$P(A).P(B) \neq P(AB)$

D: $P(A)+P(B)=\frac{1}{2}+\frac{1}{2}=1$

But, it cannot be inferred that A and B are independently

Thus, the correct answer is B.