Chapter 13: Probability.

Exercise 13.4

1. State which of the following are not the probability distribution of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P(X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	- 0.1	0.3

(iii)

Y	- 1	0	1
P(Y)	0.6	0.1	0.2

(iv)

Z	3	2	1	0	-1
P(Z)	0.3	0.2	0.4	0.1	0.05

Solution:

Since the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities = 0.4 + 0.4 +0.2 =1

Thus, the given table is a probability distribution of random variable.

(ii) For X = 3, P (X) = - 0.1

Since probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities = 0.6 + 0.1 + 0.2 = $0.9 \neq 1$

Thus, the given table is not a probability distribution of random variables

(iv) Sum of the probabilities = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = $1.05 \neq 1$

Thus, the given table is not a probability distribution of random variable.

2. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X? Is X a random variable?

Solution:

Let B represents a black ball and R represents a red ball.

The two balls selected can be represented as BB, BR, RB, RR

X represents the number of black balls.

Thus, the possible values of X are 0, 1 and 2.

Yes, X is a random variable

3. Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Solution:

A coin is tossed six times and X represents the difference between the number of heads and the number of tails.

$\therefore X(6H, 0T) = |6-0| = 6$

$X(5H, 1T) = |5-1| = 4$

$X(4H, 2T) = |4-2| = 2$

$X(3H, 3T) = |3-3| = 0$

$X(2H, 4T) = |2-4| = 2$

$X(1H, 5T) = |1-5| = 4$

$X(0H, 6T) = |0-6| = 6$

Therefore, the possible values of X are 6, 4, 2 and 0

4. Find the probability distribution of

(i) Number of heads in two tosses of a coin

(ii) Number of tails in the simultaneous tosses of three coins

(iii) Number of heads in four tosses of a coin

Solution:

i) The sample space is $\{HH, HT, TH, TT\}$

Let X represent the number of heads.

$\therefore X(HH) = 2, X(HT) = 1, X(TH) = 1, X(TT) = 0$

Thus, X can take the value of 0, 1 or 2

Since,

$$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$$

$$P(X=0) = P(TT) = \frac{1}{4}$$

$$P(X=1) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(X=2) = P(HH) = \frac{1}{4}$$

Therefore, the required probability distribution is as follows

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) The sample space is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Let X represents the number of tails.

Since, X can take the value of 0, 1, 2 or 3

$$P(X=0)=P(HHH)=\frac{1}{8}$$

$$P(X=1)=P(HHT)+P(HTH)+P(THH)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

$$P(X=2)=P(HTT)+P(THT)+P(TTH)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

$$P(X=3)=P(TTT)=\frac{1}{8}$$

Therefore, the probability distribution is as follows..

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) The sample space is

$$S=\left\{\begin{matrix} HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT, \\ THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT \end{matrix}\right\}$$

Let X be the random variable, which represents the number of heads.

Since, X can take the value of 0, 1, 2, 3 or 4

$$P(X=0)=P(TTTT)=\frac{1}{16}$$

$$P(X=1)=P(TTTH)+P(TTHT)+P(THTT)+P(HTTT)=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$$

$$P(X=2)=P(HHTT)+P(THHT)+P(TTHH)+P(HTTH)+P(HTHT)+P(THTH)$$

$$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{6}{16}=\frac{3}{8}$$

$$P(X=4)=P(HHHH)=\frac{1}{16}$$

Therefore, the probability distribution is as follows.

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. Find the probability distribution of the number of success in two tosses of die, where a success is defined as

(i) Number greater than 4

(ii) Six appears on at least one die

Solution:

Let X be the random variable, which represents the number of success

(i) Here, success refers to the number greater than 4

P(X=0) = P (number less than or equal to 4 on both the tosses) = $\frac{4}{6}\times\frac{4}{6}=\frac{4}{9}$

P (X= 1) = P (number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$=\frac{4}{6}\times\frac{2}{6}+\frac{4}{6}\times\frac{2}{6}=\frac{4}{9}$

P(X=2) = P (number greater than 4 on both the tosses)

$=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$

Therefore, the probability distribution is as follows.

X	1	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success means six appears on at least one die

P (Y=0) = P (six does not appear on any of the dice) = $\frac{5}{6}\times\frac{5}{6}=\frac{25}{36}$

P(Y=1) = P (six appears on at least one of the dice) $=\frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}=\frac{5}{36}+\frac{5}{36}=\frac{10}{36}$

Therefore, the required probability is as follows.

Y	0	1
P(Y)	$\frac{25}{36}$	$\frac{10}{36}$

6. From a lot of 30 bulbs which includes 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Solution:

Given, out of 30 bulbs, 6 are defective.

$\Rightarrow$ Number of non – defective bulbs = 30 -6 =24

4 bulbs are drawn from the lot with replacement

Let X be the random variable that denotes the number of defective bulbs in the selected

Bulbs

$\therefore P(X=0)=P$(4 non – defective and 0 defective) $={}^4C_0.\frac{4}{5}.\frac{4}{5}.\frac{4}{5}.\frac{4}{5}=\frac{256}{625}$

P (X = 1) = P (3 non – defective and 1 defective) $={}^4C_1.\left(\frac{1}{5}\right).\left(\frac{4}{5}\right)^3=\frac{256}{625}$

P (X = 2) = P (2 non – defective and 2 defective) $= {}^4C_2.\left(\frac{1}{5}\right)^2.\left(\frac{4}{5}\right)^2=\frac{96}{625}$

P (X = 3) = P (1 non – defective and 3 defective) $={}^4C_3.\left(\frac{1}{5}\right)^3.\left(\frac{4}{5}\right)=\frac{16}{625}$

P (X = 4) = P (0 non – defective and 4 defective) $={}^4C_4.\left(\frac{1}{5}\right)^4.\left(\frac{4}{5}\right)^0=\frac{1}{625}$

Thus, the required probability distribution is as follows

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Solution:

Let the probability of getting a tail in the biased coin be x.

$\therefore P(T)=x$

$\Rightarrow P(H)=3x$

For a biased coin, $P(T)+P(H)=1$

$\Rightarrow x+3x=1$

$\Rightarrow 4x=1$

$\Rightarrow x=\frac{1}{4}$

$\therefore P(T)=\frac{1}{4}$ and $P(H)=\frac{3}{4}$

When the coin is tossed twice, the sample space is $\{HH, TT, HT, TH\}$.

Let X be the random variable representing the number of tails.

$\therefore P(X=0)=P(\text{no tail}) = P(H)\times P(H)=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$

P (X=1) = P (one tail) = P(HT) + P(TH)

$=\frac{3}{4}.\frac{1}{4}+\frac{1}{4}.\frac{3}{4}$

$=\frac{3}{16}+\frac{3}{16}$

$= \frac{3}{8}$

P (X=2) = P (two tails) = $P(TT) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Thus, the required probability distribution is as follows

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

8. A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine

(i) k

(ii) $P(X < 3)$

(iii) $P(X > 6)$

(iv) $P(0 < X < 3)$

Solution:

(i) Since, the sum of probabilities of a probability distribution of random variable is one

$\therefore 0 + k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow (10k - 1)(k + 1) = 0$

$\Rightarrow k = -1, \frac{1}{10}$

(ii) P (X< 3) = P (X = 0) + P (X = 1) + P (X = 2)

= 0 + k + 2k

= 3k

$= 3 \times \frac{1}{10}$

$= \frac{3}{10}$

(iii) P (X > 6) = P (X = 7)

$= 7k^2 + k$

$= 7 \times \left(\frac{1}{10}\right)^2 + \frac{1}{10}$

$= \frac{7}{100} + \frac{1}{10}$

$= \frac{17}{100}$

(iv) $P(0 < x < 3) = P(x = 1) + P(x = 2)$

$= k + 2k$

$= 3k$

$= 3 \times \frac{1}{10}$

$= \frac{3}{10}$

9. The random variable X has probability P(X) of the following form, where k is some number $P(X) = \begin{cases} k, & if\ \ x = 0 \\ 2k, & if\ \ x = 1 \\ 3k, & if\ \ x = 2 \\ 0 & otherwise \end{cases}$

(a) Determine the value of k

(b) Find $P(X < 2), P(X \geq 2), P(X \geq 2)$

Solution:

(a) Since, the sum of probabilities of a probability distribution of random variable is one.

$\therefore k + 2k + 3k + 0 = 1$

$\Rightarrow 6k = 1$

$\Rightarrow k = \frac{1}{6}$

$(b)\ P(X < 2) = P(X = 0) + P(X = 1)$

$\therefore k + 2k$

$= 3k = \frac{3}{6} = \frac{1}{2}$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= k + 2k + 3k$

$= 6k = \frac{6}{6} = 1$

$P(X \ge 2) = P(X = 2) + P(X > 2)$

$= 3k + 0$

$= 3k = \frac{3}{6} = \frac{1}{2}$

10. Find the mean number of heads in three tosses of a fair coin

Solution:

Let X denote the success of getting heads.

Thus, the sample space is $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Here, X can take the value of 0, 1, 2 or 3

$\therefore P(X = 0) = P(TTT)$

$= P(T).P(T).P(T)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

$\therefore P(X = 1) = P(HHT) + P(HTH) + P(THH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

$\therefore P(X = 2) = P(HHT) + P(HTH) + P(THH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

$\therefore P(X = 2) = P(HHT) + P(HTH) + P(THH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

$\therefore P(X = 3) = P(HHH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Thus, the required probability is as follows

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean $E(X) = \sum X_i P(X_i)$

$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= \frac{3}{8} + \frac{3}{4} + \frac{3}{8}$

$= \frac{3}{2} = 1.5$

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Solution:

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Thus, X can be take the value of 0, 1 or 2

$\therefore P(X = 0) = P$ (not getting six on any of the dice) $= \frac{25}{26}$

$P(X = 1) = P$ (six on first die and no six on second die) + P (no six on first die and six on second die)

$= 2 \times \left(\frac{1}{6} \times \frac{5}{6}\right) = \frac{10}{36}$

$P(X = 2) = P$ (six on both the dice) $= \frac{1}{36}$

Thus, the required probability distribution is as follows

X	0	1	2

P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of $X = E(X) = \sum X_i P(X_i)$

$$= 0\times\frac{25}{36} + 1\times\frac{10}{36} + 2\times\frac{1}{36}$$

$$= \frac{1}{3}$$

12. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of two numbers obtained. Find E(X)

Solution:

The two positive integers can be selected from the first six positive integers without replacement in $6\times5 = 30\ ways$

X represents the larger of the two numbers obtained

Thus, X can take the value of 2, 3, 4, 5 or 6

For X = 2, the possible observations are (1, 2) and (2, 1)

$$\therefore P(x=2) = \frac{2}{30} = \frac{1}{15}$$

For X = 3, the possible observations are $(1,3),(2,3),(3,1)\ and\ (3,2)$

$$\therefore P(X=3) = \frac{4}{30} = \frac{2}{15}$$

For X = 4, the possible observations are $(1,4),(2,4),(3,4),(4,3),(4,2)\ and\ (4,1)$

$$\therefore P(X=4) = \frac{6}{36} = \frac{1}{5}$$

For X = 5, the possible observations are

$(1,5),(2,5),(3,5),(4,5),(5,4),(5,3),(5,2)\ and\ (5,1)$

For X = 6, the possible observations are

$(1,6),(2,6),(3,6),(4,6),(5,6),(6,4),(6,3),(6,2)\ and\ (6,1)$

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Thus, the required probability distribution is as follows

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

Then, $E(x)=\sum X_i P(X_i)$

$$=2.\frac{1}{15}+3.\frac{2}{15}+4.\frac{1}{5}+5.\frac{4}{15}+6.\frac{1}{3}$$

$$=\frac{2}{15}+\frac{2}{5}+\frac{4}{5}+\frac{4}{3}+2$$

$$=\frac{70}{15}=\frac{14}{3}$$

13. Let X denotes the sum of the number obtained when two fair dice are rolled. Find the variance and standard deviation of X.

Solution:

Here, X can take values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12

$$P(X=2)=P(1,1)=\frac{1}{36}$$

$$P(X=3)=P(1,2)+P(2,1)=\frac{2}{36}=\frac{1}{18}$$

$$P(X=4)=P(1,3)+P(2,2)+P(3,1)=\frac{3}{36}=\frac{1}{12}$$

$$P(X=5)=P(1,4)+P(2,3)+P(3,2)+P(4,1)=\frac{4}{36}=\frac{1}{9}$$

$$P(X=6)=P(1,5)+P(2,4)+P(3,3)+P(4,2)+P(5,1)=\frac{5}{36}$$

$$P(X=7)=P(1,6)+P(2,5)+P(3,4)+P(4,3)+P(5,2)+P(6,1)=\frac{6}{36}=\frac{1}{6}$$

$$P(X=8)=P(2,6)+P(3,5)+P(4,4)+P(5,3)+P(6,2)=\frac{5}{36}$$

$$P(X=9)=P(3,6)+P(4,5)+P(5,4)+P(6,3)=\frac{4}{36}=\frac{1}{9}$$

$$P(X=10)=P(4,6)+P(5,5)+P(6,4)=\frac{3}{36}=\frac{1}{12}$$

$$P(X=11)=P(5,6)+P(6,5)=\frac{2}{36}=\frac{1}{18}$$

$$P(X=12)=P(6,6)=\frac{1}{36}$$

Thus, the required probability distribution is as follows.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Then, $E(X)=\sum X_i.P(X_i)$

$$=2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}$$

$$=\frac{1}{18}+\frac{1}{6}+\frac{1}{3}+\frac{5}{9}+\frac{7}{6}+\frac{10}{9}+1+\frac{5}{6}+\frac{11}{18}+\frac{1}{3}$$

= 7

$$E(X^2)=\sum X_i^2.P(X_i)$$

$$=4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{36}$$

$$=\frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{9}+9+\frac{25}{3}+\frac{121}{18}+4$$

$$=\frac{987}{18}=\frac{329}{6}=54.833$$

Then, Var $(X)=E(X)^2-[E(X)]^2$

$$=54.833-(7)^2$$

$$=54.833-49$$

$$=5.833$$

$\therefore$ Standard deviation = $\sqrt{Var(X)}$

$$=\sqrt{5.833}$$

$$=2.415$$

14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One students is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X

Solution:

There are 15 students in the class. Each student has the same chance to be chosen

Thus, the probability of each student to be selected is $\frac{1}{15}$

The given information can be shown in the frequency table as follows

x	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$$P(X=14)=\frac{2}{15}, P(X=15)=\frac{1}{15}, P(X=16)=\frac{2}{15}, P(X=16)=\frac{3}{15},$$

$$P(X=18)=\frac{1}{15}, P(X=19)=\frac{2}{15}, P(X=20)=\frac{3}{15}, P(X=21)=\frac{1}{15}$$

Thus, the probability distribution of random variable X is as follows.

x	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean X= E(X)

$$\sum X_i P(X_i)$$

$$=14\times\frac{2}{15}+15\times\frac{1}{15}+16\times\frac{2}{15}+17\times\frac{3}{15}+18\times\frac{1}{15}+19\times\frac{2}{15}+20\times\frac{3}{15}+21\times\frac{1}{15}$$

$$=\frac{1}{15}(28+15+32+51+18+38+60+21)$$

$$=\frac{263}{15}=17.53$$

$$E(X^2)=\sum X_i^2 P(X_i)$$

$$=(14)^2.\frac{2}{15}+(15)^2.\frac{1}{15}+(16)^2.\frac{2}{15}+(17)^2.\frac{3}{15}+(18)^2.\frac{1}{15}+(19)^2.\frac{2}{15}+(20)^2.\frac{3}{15}+(21)^2\frac{1}{15}$$

$$=\frac{1}{15}(392+225+512+867+324+722+1200+441)$$

$$=\frac{4683}{15}=312.2$$

$$\therefore Variance(X)=E(X)^2-[E(X)]^2$$

$$=312.2-\left(\frac{263}{15}\right)^2$$

$$=312.2-307.4177$$

$= 4.7823 = 4.78$

Standard deviation = $\sqrt{Variance(X)}$

$= \sqrt{4.78}$

$= 2.816 \approx 2.19$

15. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and var (X)

Solution:

Given, $P(X = 0) = 30\% = \frac{30}{100} = 0.3$

$P(X = 1) = 70\% = \frac{70}{100} = 0.7$

Thus, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

Then, $E(X) = \sum X_i P(X_i)$

$= 0 \times 0.3 + 1 \times 0.7$

$= 0.7$

$E(X^2) = \sum X_i^2 P(X_i)$

$= 0^2 \times 0.3 + (1)^2 \times 0.7 = 0.7$

Since, $Var(X) = E(X^2) - [E(X)]^2$

$= 0.7 - (0.7)^2$

$= 0.7 - 0.49 = 0.21$

16. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

A) 1 B) 2 C) 5 D) $\frac{8}{3}$

Solution:

Let X be the random variable representing a number on the die.

The total number of observations is six.

$\therefore P(X=1)=\frac{3}{6}=\frac{1}{2}$

$P(X=2)=\frac{2}{6}=\frac{1}{3}$

$P(X=5)=\frac{1}{6}$

Thus, the probability distribution is as follows.

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Mean $=E(X)=\sum X_i P(X_i)$

$=\frac{1}{2}\times 1+\frac{1}{3}\times 2+\frac{1}{6}.5$

$=\frac{1}{2}+\frac{2}{3}+\frac{5}{6}$

$=\frac{3+4+5}{6}=\frac{12}{6}=2$

17. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

A) $\frac{37}{221}$ B) $\frac{5}{13}$ C) $\frac{1}{13}$ D) $\frac{2}{13}$

Solution:

Let X denote the number of aces obtained.

Thus, X can be take any of the value of 0, 1 or 2.

Since, in a deck of 52 cards, 4 cards are aces. Thus, there are 48 non – ace cards.

$\therefore P(X=0)=P$ (0 ace and 2 non – ace cards) = $\frac{{}^{4}C_0 \times {}^{48}C_2}{{}^{52}C_2}=\frac{1128}{1326}$

P(X= 1) = P (1 ace and 1 non – ace cards) = $\frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2}=\frac{192}{1326}$

P (X = 2) = P (2 ace and 0 non – ace cards) = $\frac{{}^{4}C_2 \times {}^{48}C_0}{{}^{52}C_2}=\frac{6}{1326}$

Thus, the probability distribution is as follows

X	0	1	2
P(X)	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

Then, $E(X)=\sum X_i P(X_i)$

$$=0\times\frac{1128}{1326}+1\times\frac{192}{1326}+2\times\frac{6}{1326}$$

$$=\frac{204}{1326}=\frac{2}{13}$$