Infinity Learn by Sri Chaitanya Educational Institutions

Chapter 2: Relations and Functions

Exercise 2.1

Question 1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

Solution : It is given that,

$$\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$$

Since the ordered pairs are equal, the corresponding elements will also be equal.

Thus, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

Consider $\frac{x}{3}+1=\frac{5}{3}$,

Subtract 1 from both sides,

$$\frac{x}{3}=\frac{5}{3}-1$$

Subtract the numbers,

$$\frac{x}{3}=\frac{2}{3}$$

Multiple both sides by 3,

$$x=2$$

Now, $y-\frac{2}{3}=\frac{1}{3}$

Add $\frac{2}{3}$ to both sides,

$$y=\frac{1}{3}+\frac{2}{3}$$

Add the numbers,

$$y=1$$

Therefore, $x=2$ and $y=1$.

Question 2. If the set A has 3 elements and the set $B=\{3,4,5\}$, then find the number of elements in $(A\times B)$?

Solution : It is given that set A has 3 elements and $B=\{3,4,5\}$

Number of elements in set $B=3$

Number of elements in $(A\times B)=($ Number of elements in $A)\times($ Number of elements in $B)$

$=3\times 3=9$

Therefore, the number of elements in $(A\times B)$ is 9 .

Question 3. If $G=\{7,8\}$ and $H=\{5,4,2\}$, find $G\times H$ and $H\times G$.

Solution : Given that, $G=\{7,8\}$ and $H=\{5,4,2\}$

It is known that the Cartesian product $P\times Q$ of two non-empty sets P and Q is defined as $P\times Q=\{(p,q): p\in P, q\in Q\}$.

Therefore, $G\times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

$H\times G=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$

Question 4. State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If $P=\{m,n\}$ and $Q=\{n,m\}$, then $P\times Q=\{(m,n),(n,m)\}$.

(ii) If A and B are non-empty sets, then $A\times B$ is a non-empty set of ordered pairs (x,y) such that $x\in A$ and $y\in B$.

(iii) If $A=\{1,2\},B=\{3,4\}$, then $A\times(B\cap\Phi)=\Phi$.

Solution :

(i) The Cartesian product of two sets X and Y, denoted $X\times Y$, is the set of all ordered pairs where x is in X and y is in Y.

If $P=\{m,n\}$ and $Q=\{n,m\}$, then $P\times Q=\{(m,m),(m,n),(n,m),(n,n)\}$.

So, the statement is false.

(ii) The Cartesian product of two sets X and Y, denoted $X\times Y$, is the set of all ordered pairs where x is in X and y is in Y.

If A and B are non-empty sets, then $A\times B$ is a non-empty set of ordered pairs (x,y) such that $x\in A$ and $y\in B$.

So, the statement is true.

(iii) $A=\{1,2\}, B=\{3,4\}$

Then, $A\times(B\cap\Phi)=\Phi$.

So, the statement is true.

Question 5. If $A=\{-1,1\}$, find $A\times A\times A$.

Solution : It is given that $A=\{-1,1\}$.

It is known that for any non-empty set A, $A\times A\times A$ is defined as $A\times A\times A=\{(a,b,c): a,b,c\in A\}$

Therefore,
$A\times A\times A=\{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$

Question 6. If $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$. Find A and B.

Solution : It is given that $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$.

It is known that the Cartesian product of two non-empty sets P and Q is defined as
$P\times Q=\{(p,q): p\in P, q\in Q\}$

Thus, A is the set of all first elements and B is the set of all second elements.

Therefore, $A=\{a,b\}$ and $B=\{x,y\}$.

Question 7. Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify that

(i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) $A\times C$ is a subset of $B\times D$

Solution :

(i) To verify: $A\times(B\cap C)=(A\times B)\cap(A\times C)$

It is given that, $B\cap C=\{1,2,3,4\}\cap\{5,6\}=\Phi$

Thus, L.H.S $= A\times(B\cap C)=A\times\Phi=\Phi$

$A\times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

Therefore, R.H.S $=(A\times B)\cap(A\times C)=\Phi$

Thus, L.H.S = R.H.S

Hence, $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) To verify : $A\times C$ is a subset of $B\times D$

Given that, $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$.

Then, $A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$A\times D=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\}$

It is seen that all the elements of set $A\times C$ are the elements of set $B\times D$.

Therefore, $A\times C$ is a subset of $B\times D$.

Question 8. Let $A=\{1,2\}$ and $B=\{3,4\}$. Write $A\times B$. How many subsets will $A\times B$ have? List them.

Solution : $A=\{1,2\}$ and $B=\{3,4\}$

Thus, $A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$

$\Rightarrow n(A\times B)=4$

It is known that if C is a set with $n(C)=m$, then $n[P(C)]=2^m$

Therefore, the set $A\times B$ has $2^4=16$ subsets.

These are,

$\Phi,\{(1,3)\},\{(1,4)\},\{(2,3)\},\{(2,4)\},\{(1,3),(1,4)\},\{(1,3),(2,3)\},\{(1,3),(2,4)\},,$
$\{(1,4),(2,3)\},\{(1,4),(2,4)\},\{(2,3),(2,4)\},\{(1,3),(1,4),(2,3)\},\{(1,3),(1,4),(2,4)\},,$
$\{(1,3),(2,3),(2,4)\},\{(1,4),(2,3),(2,4)\},\{(1,3),(1,4),(2,3),(2,4)\}$

Question 9. Let A and B be two sets such that $n(A)=3$ and $n(B)=2$. If $(x,1),(y,2)$, $(z,1)$ are in $A\times B$, find A and B, where x, y and z are distinct elements.

Solution : Given that, $n(A)=3$ and $n(B)=2$.

It is also given that, $(x,1),(y,2),(z,1)$ are in $A\times B$.

It is known that, A is the set of first elements of the ordered pair elements of $A\times B$.

B is the set of second elements of the ordered pair elements of $A\times B$.

Thus, x, y, and z are the elements of A and $1, 2$ are the elements of B.

Since $n(A)=3$ and $n(B)=2$, it is clear that $A=\{x,y,z\}$ and $B=\{1,2\}$.

Question 10. The Cartesian product $A\times A$ has 9 elements among which are found $(-1, 0)$ and $(0,1)$. Find the set A and the remaining elements of $A\times A$.

Solution : It is known that if $n(A)=p$ and $n(B)=q$, then $n(A\times B)=pq$

$n(A\times A)=n(A)\times n(A)$

It is given that $n(A\times A)=9$.

Thus, $n(A)\times n(A)=9$

$\Rightarrow n(A)=3$

Thus, the ordered pairs $(-1,0)$ and $(0,1)$ are two of the nine elements of $A\times A$.

It is also known that $A\times A=\{(a,a):a\in A\}$.

So, $-1,0$, and 1 are elements of A.

Since $n(A)=3$, it is clear that $A=\{-1,0,1\}$.

Therefore, the remaining elements of set $A\times A$ are $(-1,-1),(-1,1),(0,-1),(0,0)$, $(1,-1),(1,0)$, and $(1,1)$.