Chapter: 2. Inverse trigonometric functions.

Exercise 2.1

1. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Solution: Let $\sin^{-1}\left(-\frac{1}{2}\right)=y$, Apply sine function on both sides, we get

$$\sin y=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right)$$

Range of the principal value of $sin^{-1}x$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Therefore, the principal value of $sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$.

2. Find the principal value of $cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Solution: Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y$, Apply cosine function on both sides

We get $\cos y=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right)$

Range of the principal value of $\cos^{-1}x$ is $[0,\pi]$

Therefore, the principal value of $cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$

3. Find the principal value of $\operatorname{cosec}^{-1}(2)$

Solution: Let $\operatorname{cosec}^{-1}(2)=y$

Apply cosecant function on both sides

$$\csc y = 2 = \csc\left(\frac{\pi}{6}\right)$$

Range of the principal value of $\csc^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Thus, the principal value of $csc^{-1}(2)$ is $\frac{\pi}{6}$

4. Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Solution: Let $\tan^{-1}\left(-\sqrt{3}\right) = y$

Apply tangent function on both sides

It gives $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$

Range of the principal value of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Thus, known that the principal value of $tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$

5. Find the principal value of $cos^{-1}\left(-\frac{1}{2}\right)$

Solution: Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$,

Apply cosine function on both sides

We get

$$\cos y = -\frac{1}{2}$$

$$= -\cos\left(\frac{\pi}{3}\right)$$

$$= \cos\left(\pi - \frac{\pi}{3}\right)$$

$$= \cos\left(\frac{2\pi}{3}\right)$$

Range of the principal value of $\cos^{-1} x$ is $[0, \pi]$

Thus, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$

6. Find the principal value of $\tan^{-1}(-1)$

Solution: Let $\tan^{-1}(-1) = y$, Apply tangent function on both sides

It gives

$$\Rightarrow \tan y = -1 = -\tan\left(\frac{\pi}{4}\right)$$

$$= \tan\left(-\frac{\pi}{4}\right)$$

Range of the principal value of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1$

Thus, the principal value of $\tan(-1)$ *is* $-\frac{\pi}{4}$

7. Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Solution: Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

$$\Rightarrow \sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$$

Range of the principal value of $\sec^{-1} x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ *and* $\sec\left(\frac{\pi}{6}\right)=\frac{2}{\sqrt{3}}$

Thus, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ *is* $\frac{\pi}{6}$

8. Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Solution: Let $\cot^{-1}\left(\sqrt{3}\right)=y$

$\Rightarrow \cot y=\sqrt{3}=\cot\left(\frac{\pi}{6}\right)$

Range of the principal value of $\cot^{-1} x$ is (0, n) and $\cot\left(\frac{\pi}{6}\right)=\sqrt{3}$

Thus, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ *is* $\frac{\pi}{6}$

9. Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Solution: Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=y$

$\Rightarrow \cos y=-\frac{1}{\sqrt{2}}$

$=-\cos\left(\frac{\pi}{4}\right)$

$=\cos\left(\pi-\frac{\pi}{4}\right)$

$=\cos\left(\frac{3\pi}{4}\right)$

Range of the principal value of $\cos^{-1} x$ is $[0,n]$ and $\cos\left(\frac{3\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

Thus, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ *is* $\frac{3\pi}{4}$

10. Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Solution:

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=y$

$\Rightarrow \operatorname{cosec} y=-\sqrt{2}$

$=-\operatorname{cosec}\left(\frac{\pi}{4}\right)$

$=\operatorname{cosec}\left(-\frac{\pi}{4}\right)$

Range of the principal value of $\operatorname{cosec}^{-1} x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$, $\operatorname{cosec}\left(-\frac{\pi}{4}\right)=-\sqrt{2}$ c

Thus, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ *is* $-\frac{\pi}{4}$

11. Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

Solution: Let $\tan^{-1}(1)=x \Rightarrow \tan x=1=\tan\frac{\pi}{4} \Rightarrow \tan^{-1}(1)=\frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y \Rightarrow \cos y=-\frac{1}{2}=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right) \Rightarrow \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

Suppose that $\sin^{-1}\left(-\frac{1}{2}\right)=z \Rightarrow \sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)$

Hence, $\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

Therefore, $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}=\frac{3\pi}{4}$

12. Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Solution: Let $\cos^{-1}\left(\frac{1}{2}\right)=x \Rightarrow \cos x=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$

It implies that $\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

Suppose that $\sin^{-1}\left(\frac{1}{2}\right)=y \Rightarrow \sin y=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$

It implies that $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

Therefore, $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+\frac{2\pi}{6}=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

13. Find the value of if $\sin^{-1}x=y$, then

(A) $0\le y\le\pi$ (B) $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$ (C) $0<y<\pi$ (D) $-\frac{\pi}{2}<y<\frac{\pi}{2}$

Solution: It is given that $\sin^{-1}x=y$

Range of the principal value of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Thus, $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

14. Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ is equal to

(A) 0 (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Solution: Let $\tan^{-1}\sqrt{3}=x \Rightarrow \tan x=\sqrt{3}=\tan\frac{\pi}{3}$

Range of the principal value of $\tan^{-1}x$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Therefore, $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y \Rightarrow \sec y = -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\left(\frac{2\pi}{3}\right)$

Range of the principal value of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Therefore, $\sec^{-1}(-2) = \frac{2\pi}{3}$

Thus, $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$