Chapter 2: Relations and Functions

Exercise 2.2

Question 1. Let $A=\{1,2,3...14\}$. Define a relation R from A to A by $R=\{(x,y):3x-y=0$, where $x,y\in A\}$. Write down its domain, codomain and range.

Solution : The relation R from A to A is given as $R=\{(x,y):3x-y=0$, where $x,y\in A\}$

That is, $R=\{(x,y):3x=y$, where $x,y\in A\}$

Thus, $R=\{(1,3),(2,6),(3,9),(4,12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

Therefore, Domain of $R=\{1,2,3,4\}$

The complete set A is the codomain of the relation R.

Therefore, Codomain of $R=A=\{1,2,3...14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of $R=\{3,6,9,12\}$

Question 2. Define a relation R on the set $\mathbf{N}$ of natural numbers by $R=\{(x,y):y=x+5,x$ is a natural number less than $4;x,y\in\mathbf{N}\}$. Depict this relationship using roster form. Write down the domain and the range.

Solution : $R=\{(x,y):y=x+5,x$ is a natural number less than $4,x,y\in\mathbf{N}\}$ The natural numbers less than 4 are $1,2$, and 3.

Thus, the relation is $R=\{(1,6),(2,7),(3,8)\}$.

The domain of R is the set of all first elements of the ordered pairs in the relation.

Therefore, Domain of $R=\{1,2,3\}$

The range of R is the set of all second elements of the ordered pairs in the relation. Therefore, Range of $R=\{6,7,8\}$

Question 3. $A=\{1,2,3,5\}$ and $B=\{4,6,9\}$. Define a relation R from A to B by

$R=\{(x,y):$ the difference between x and y is odd; $x\in A,y\in B\}$. Write R in roster form.

Solution : Given that, $A=\{1,2,3,5\}$ and $B=\{4,6,9\}$.

$R=\{(x,y):$ the difference between x and y is odd; $x\in A,y\in B\}$

Therefore, $\mathrm{R}=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$

Question 4. The given figure shows a relationship between the sets P and Q. Write this relation (i) in set-builder form (ii) in roster form. What is its domain and range?

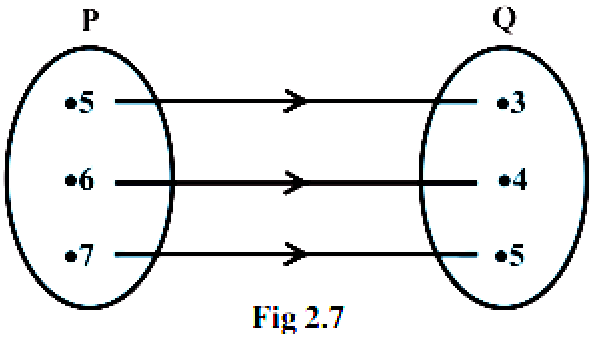

Fig 2.7

Solution : From the figure, $\mathrm{P}=\{5,6,7\}$ and $\mathrm{Q}=\{3,4,5\}$.

(i) The relation in set-builder form is,

$\mathrm{R}=\{(x, y): y=x-2; x \in \mathrm{P}\}$ or $\mathrm{R}=\{(x, y): y=x-2$ for $x=5,6,7\}$

(ii) The relation in roster form is,

$\mathrm{R}=\{(5,3),(6,4),(7,5)\}$

Domain of $\mathrm{R}=\{5,6,7\}$

Range of $\mathrm{R}=\{3,4,5\}$

Question 5. Let $\mathrm{A}=\{1,2,3,4,6\}$. Let R be the relation on A defined by $\{(a, b): a, b \in \mathrm{A}, b$ is exactly divisible by $a\}$.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Solution : Given that, $\mathrm{A}=\{1,2,3,4,6\}$ and $\mathrm{R}=\{(a, b): a, b \in \mathrm{A}, b$ is exactly divisible by $a\}$.

(i) R in roster form is,

$\mathrm{R}=\{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3)(3,6),(4,4),(6,6)\}$,

(ii) The domain of R is the set of all first elements of the ordered pairs in the relation.

Therefore, Domain of $\mathrm{R}=\{1,2,3,4,6\}$

(iii) The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of $R = \{1, 2, 3, 4, 6\}$

Question 6. Determine the domain and range of the relation R defined by $R = \{(x, x+5) : x \in \{0,1,2,3,4,5\}\}$.

Solution : Given that, $\mathrm{R} = \{(x, x+5) : x \in \{0,1,2,3,4,5\}\}$

Thus, $\mathrm{R}=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation. Therefore, Domain of $\mathrm{R}=\{0,1,2,3,4,5\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of $\mathrm{R}=\{5,6,7,8,9,10\}$

Question 7. Write the relation $\mathrm{R} = \{(x, x^3) : x$ is a prime number less than $10\}$ in roster form.

Solution : The relation is $\mathrm{R} = \{(x, x^3) : x$ is a prime number less than $10\}$.

The prime numbers less than 10 are $2, 3, 5,$ and 7.

Therefore, the relation in roster form is $\mathrm{R}=\{(2,8),(3,27),(5,125),(7,343)\}$.

Question 8. Let $\mathrm{A}=\{x, y, z\}$ and $\mathrm{B}=\{1,2\}$. Find the number of relations from A to B.

Solution : Given that $\mathrm{A}=\{x, y, z\}$ and $\mathrm{B}=\{1,2\}$.

Now, $\mathrm{A} \times \mathrm{B} = \{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$

Here, $n(\mathrm{A} \times \mathrm{B}) = 6$.

So, the number of subsets of $\mathrm{A} \times \mathrm{B}$ is 2^6.

Therefore, the number of relations from A to is 2^6.

Question 9. Let R be the relation on $\mathbf{Z}$ defined by $\mathrm{R} = \{(a,b) : a, b \in \mathbf{Z}, a-b$ is an integer $\}$. Find the domain and range of R.

Solution : Given that, $\mathrm{R} = \{(a,b) : a, b \in \mathbf{Z}, a-b$ is an integer $\}$

It is known that the difference between any two integers will be an integer.

Therefore, Domain of $\mathrm{R} = \mathbf{Z}$ and Range of $\mathrm{R} = \mathbf{Z}$.