Chapter 2: Relations and Functions

Exercise 2.3

Question 1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Solution :

(i) The relation is $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$.

This relation is a function. Because, $2,5,8,11,14,$ and 17 are the elements of the domain of the given relation having their unique images.

Here, Domain $=\{2,5,8,11,14,17\}$ and Range $=\{1\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

This relation is a function. Because, $2,4,6,8,10,12,$ and 14 are the elements of the domain of the given relation having their unique images.

Here, Domain $=\{2,4,6,8,10,12,14\}$ and Range $=\{1,2,3,4,5,6,7\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

This relation is not a function. Because, the same first element that is, 1 corresponds to two different images 3 and 5.

Question 2. Find the domain and range of the following real function:

(i) $f(x)=-|x|$

(ii) $f(x)=\sqrt{9-x^2}$

Solution :

(i) $f(x)=-|x|, x\in \mathrm{R}$

It is known that, $|x|=\begin{cases} x, \text{if } x\geq 0 \\ -x, \text{if } x<0 \end{cases}$

Thus, $f(x)=-|x|=\begin{cases} -x, \text{if } x\geq 0 \\ x, \text{if } x<0 \end{cases}$

Since $f(x)$ is defined for $x \in \mathbf{R}$, the domain of f is $\mathbf{R}$.

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

Therefore, the range of f is $(-\infty, 0]$.

(ii) $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \leq x \leq 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \leq x \leq 3$, the value of $f(x)$ will lie between 0 and 3. The range of $f(x)$ is $\{x : 0 \leq x \leq 3\}$ or $[0, 3]$.

Question 3. A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$,

(ii) $f(7)$,

(iii) $f(-3)$

Solution : The given function is $f(x) = 2x - 5$.

(i) Substitute $x = 0$,

$f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) Substitute $x = 7$,

$f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) Substitute $x = -3$,

$f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question 4. The function $'t'$ which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find

(i) $t(0)$

(ii) $t(28)$

(iii) $t(-10)$

(iv) The value of C, when $t(C) = 212$

Solution : The given function is $t(\mathrm{C})=\frac{9\mathrm{C}}{5}+32$.

(i) Substitute $\mathrm{C}=0$,

$$t(0)=\frac{9\times 0}{5}+32=0+32=32$$

(ii) Substitute $\mathrm{C}=28$,

$$t(28)=\frac{9\times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}$$

(iii) Substitute $\mathrm{C}=-10$,

$$t(-10)=\frac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$$

(iv) It is given that $t(\mathrm{C})=212$

$$212=\frac{9\mathrm{C}}{5}+32$$

Subtract 32 from both sides,

$$\Rightarrow \frac{9\mathrm{C}}{5}=212-32$$

Subtract the numbers,

$$\Rightarrow \frac{9\mathrm{C}}{5}=180$$

Multiply both sides by 5,

$$\Rightarrow 9\mathrm{C}=180\times 5$$

$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Therefore, the value of t, when $t(\mathrm{C})=212$ is 100.

Question 5. Find the range of each of the following functions.

(i) $f(x)=2-3x, x\in\mathbf{R}, x>0$.

(ii) $f(x)=x^2+2, x$, is a real number.

(iii) $f(x)=x, x$ is a real number

Solution :

(i) $f(x)=2-3x, x\in\mathbf{R}, x>0$

The values of $f(x)$ for various values of real numbers $x>0$ can be written in the tabular form as,

x	0.01	0.1	0.9	1	2	2.5	4	5	…..
$f(x)$	1.97	1.7	–0.7	–1	–4	–5.5	–10	–13	…..

Therefore, it can be clearly observed that the range of f is the set of all real numbers less than 2 .

Therefore, range of $f=(-\infty,2)$.

(ii) $f(x)=x^2+2, x$, is a real number

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as,

x	0	±0.3	±0.8	±1	±2	±3	…..
$f(x)$	2	2.09	2.64	3	6	11	…..

Therefore, it can be clearly observed that the range of f is the set of all real numbers greater than 2 .

Therefore, range of $f=(2,\infty)$.

(iii) $f(x)=x, x$ is a real number

It is clear that the range of f is the set of all real numbers.

Therefore, Range of $f=\mathbf{R}$.