Example 1

Convert $40^{\circ}20'$ into radian measure.

Solution

Given that $40^{\circ}20' = 40\frac{1}{3}$ degree

We know that $180^{\circ} = \pi$ radian.

$= \frac{\pi}{180} \times \frac{121}{3}$ radian

$= \frac{121\pi}{540}$ radian.

Therefore $40^{\circ}20' = \frac{121\pi}{540}$ radian.

Example 2

Convert 6 radians into degree measure.

Solution

Given that π radian $= 180^{\circ}$.

Hence

6 radians $= \frac{180}{\pi} \times 6$ degree

$= \frac{1080 \times 7}{22}$ degree

Take $1^{\circ} = 60'$

$= 343\frac{7}{11}$ degree $= 343^{\circ} + \frac{7 \times 60}{11}$ minute

$= 343^{\circ} + 38' + \frac{2}{11}$ minute

$= 343^{\circ}38'11''$ The answer

Example 3

Find the radius of the circle in which a central angle of 60° intercepts an arc of length $37.4\,\text{cm}$ (use $\pi = \frac{22}{7}$).

Solution

Given that

$l = 37.4\,\text{cm}$ and $\theta = 60^\circ = \frac{60\pi}{180}$

radian $= \frac{\pi}{3}$

The value $r = \frac{l}{\theta}$,

Solving we get

$$r = \frac{37.4 \times 3}{\pi} = \frac{37.4 \times 3 \times 7}{22} = 35.7\,\text{cm}$$

Example 4

The minute hand of a watch is $1.5\,\text{cm}$ long. How far does its tip move in 40 minutes?

(Use $\pi = 3.14$).

Solution

Given that

Watch is 1.5cm long

It complete 60 revolution in one minute.

in 40 minute,

minute hand will turn $\frac{2}{3}$ of a revolution

ie, $\theta = \frac{2}{3} \times 360^\circ$

or $\frac{4\pi}{3}$ radian.

The distance $l = r\theta = 1.5 \times \frac{4\pi}{3}\,\text{cm} = 2\pi\,\text{cm} = 2 \times 3.14\,\text{cm} = 6.28\,\text{cm}$

Example 5

If the arcs of the same lengths in two circles subtend angles 65° and 110° at the centre, find the ratio of their radii.

Solution

Given that 65° and 110°

Let r_1 and r_2 be the radii of the two circles.

Given that $\theta_1 = 65^\circ = \frac{\pi}{180} \times 65 = \frac{13\pi}{36}$ radian and $\theta_2 = 110^\circ = \frac{\pi}{180} \times 110 = \frac{22\pi}{36}$ radian

Let l be the length of each of the arc. Then $l = r_1\theta_1 = r_2\theta_2$, which gives

$$\frac{13\pi}{36} \times r_1 = \frac{22\pi}{36} \times r_2, \text{ i.e., } \frac{r_1}{r_2} = \frac{22}{13}$$

Hence $r_1 : r_2 = 22 : 13$.