Chapter 3: Matrices.

Exercise 3.1

1. In the matrix $A=\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$ write

   i. The order of the matrix

   ii. The number of elements

   iii. Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Solution: The given matrix is $A=\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$

i. In the matrix, the number of rows is 3 and the number of columns is 4. Hence the order of the matrix is 3×4

ii. If the order of the matrix is $m\times n$, then the number of elements in the matrix is product of m,n. So that the number of elements of the given matrix is $3\cdot 4=12$

iii. The element a_{ij} is i^{th} row and j^{th} column element.

   a. $a_{13}=19$

   b. $a_{21}=35$

   c. $a_{33}=-5$

   d. $a_{24}=12$

2. If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Solution: If a matrix is of the order $m\times n$, then it has $m\cdot n$ elements.

Given that the number of elements in the matrix is 24.

The possible pairs of factors of 24 are S $(1,24),(2,12),(3,8),(4,6)$

Therefore, the possible orders of the matrix having 24 elements are $1\times24, 2\times12, 3\times8$ $4\times6, 6\times4, 8\times3, 12\times2, 24\times1$.

If the matrix has 13 elements, then the possible orders of the matrix are $1\times13, 13\times1$

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution: If a matrix is of the order $m\times n$, then it has $m\cdot n$ elements.

Given that the number of elements in the matrix is 18

The possible pairs of factors of 18 are $(1,18),(2,9),(3,6)$

Therefore, the possible orders of the matrix having 18 elements are $1\times18, 2\times9, 3\times6$ and $18\times1, 9\times2, 6\times3$

If the matrix has 5 elements, then the possible orders of the matrix are $1\times5, 5\times1$

4. Construct a matrix of order 3×4, whose elements are given by

i. $a_{ij}=\frac{1}{2}|-3i+j|$

ii. $a_{ij}=2i-j$

Solution: The general 3×4 matrix is $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

i. Given $a_{ij}=\frac{1}{2}|-3i+j|$

Hence the matrix is $A=\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

ii. Given $a_{ij}=2i-j$

Hence, the matrix is $A=\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

5. Find the values of x, y, z from the following equations

i. $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix}=\begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

ii. $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

iii. $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Solution: Two matrices are said to be equal if the order of those two matrices are equal and each entry must be equal to the corresponding entry**.**

i. Given $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix}=\begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

Orders of both matrices are equal. Each entry equal to corresponding entries

Hence, $x=1, y=3$

ii. Given $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix}=\begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Orders of both matrices are equal. Each entry equal to corresponding entries

Hence, $x+y=6, xy=8, 5+z=5$

Therefore, $z=0$

Consider

$$\begin{aligned}(x-y)^2 &= (x+y)^2-4xy \\ &= 36-32 \\ &= 4 \\ x-y &= \pm 2\end{aligned}$$

Suppose that $x-y=2$

It gives $2x=8 \Rightarrow x=4, y=2$

Suppose that $x-y=-2$

It gives $2x=6 \Rightarrow x=3, y=3$

Therefore $x=4, y=2, z=0$ and $x=3, y=3, z=0$,

iii. Given $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Orders of both matrices are equal. Each entry equal to corresponding entries

Hence, $x+y+z=6, x+y=6, y+z=7$

Therefore, $z=0, y=7, x=-1$

6. Find the value of a, b, c and d from the equation $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Solution: Given, $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Comparing the corresponding elements, we get

$$a-b=-1, 2a+c=5, 2a-b=0, 3c+d=13$$

Solving the above equations

$$\begin{aligned} a-b-2a+b &= -1 \\ -a &= -1 \\ a &= 1 \end{aligned}$$

Hence, $1-b=-1 \Rightarrow b=2$

$$2(1)+c=5 \Rightarrow c=5-2 \Rightarrow c=3$$

$$3c+d=13 \Rightarrow 3(3)+d=13 \Rightarrow d=4$$

Therefore, $a=1, b=2, c=3, d=4$

7. $A=\left[a_{ij}\right]_{mxn}$ is a square matrix, if

A) $m<n$ B) $m>n$ C) $m=n$ D) None

Solution: A matrix having number of rows and columns are equal is called a square matrix

Hence, $m = n$

This is matching with the option (C)

8. Which of the given values of x and y when $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

A) $x = -\frac{1}{3}, y = 7$ B) Not possible to find

C) $y = 7, x = -\frac{2}{3}$ D) $x = -\frac{1}{3}, y = \frac{2}{3}$

Solution: Given, $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

When two matrices are equal the corresponding entries are also equal

$$3x + 7 = 0 \Rightarrow x = -\frac{7}{3}, y - 2 = 5 \Rightarrow y = 7$$

But by equating a_{22}, we get $2 - 3x = 4 \Rightarrow x = -\frac{2}{3}$

But there are two different values of x, which is contradiction.

Hence, the above two matrices are not equal

This is matching with the option (B)

9. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

(A) 27 (B) 18 (C) 81 (D) 512

Solution: Given matrix is of the order 3×3 has 9 elements and each of these elements can be either 0 or 1

Now, each of the 9 elements can be filled in two possible ways.

Therefore, the required number of possible matrices is $2^9 = 512$