Exercise 3.2

Question 1:

Find the values of other five trigonometric functions if $\cos x = -\frac{1}{2}, x$ lies in third quadrant.

Answer 1:

Given that

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$$

Since x lies in the 3^{rd} quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\text{cosec}\, x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x=\frac{1}{\tan x}=\frac{1}{\sqrt{3}}$$

Question 2:

Find the values of other five trigonometric functions if $\sin x=\frac{3}{5}, x$ lies in second quadrant.

Answer 2:

Given that

$$\sin x=\frac{3}{5}$$

$$\operatorname{cosec} x=\frac{1}{\sin x}=\frac{1}{\left(\frac{3}{5}\right)}=\frac{5}{3}$$

$$\sin^2 x+\cos^2 x=1$$

$$\Rightarrow \cos^2 x=1-\sin^2 x$$

$$\Rightarrow \cos^2 x=1-\left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 x=1-\frac{9}{25}$$

$$\Rightarrow \cos^2 x=\frac{16}{25}$$

$$\Rightarrow \cos x=\pm\frac{4}{5}$$

Since x lies in the 2^{nd} quadrant, the value of $\cos x$ will be negative

$$\therefore \cos x=-\frac{4}{5}$$

$$\sec x=\frac{1}{\cos x}=\frac{1}{\left(-\frac{4}{5}\right)}=-\frac{5}{4}$$

$$\tan x=\frac{\sin x}{\cos x}=\frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)}=-\frac{3}{4}$$

$$\cot x=\frac{1}{\tan x}=-\frac{4}{3}$$

Question 3:

Find the values of other five trigonometric functions if $\cot x=\frac{3}{4}, x$ lies in third quadrant.

Answer 3:

Given that

$$\cot x=\frac{3}{4}$$

$$\tan x=\frac{1}{\cot x}=\frac{1}{\left(\frac{3}{4}\right)}=\frac{4}{3}$$

$$1+\tan^2 x=\sec^2 x$$

$$\Rightarrow 1+\left(\frac{4}{3}\right)^2=\sec^2 x$$

$$\Rightarrow 1+\frac{16}{9}=\sec^2 x$$

$$\Rightarrow \frac{25}{9}=\sec^2 x$$

$$\Rightarrow \sec x=\pm\frac{5}{3}$$

Since x lies in the 3^{rd} quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x=-\frac{5}{3}$$

$$\cos x=\frac{1}{\sec x}=\frac{1}{\left(-\frac{5}{3}\right)}=-\frac{3}{5}$$

$$\tan x=\frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3}=\frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\text{cosec}\, x = \frac{1}{\sin x} = -\frac{5}{4}$$

Question 4:

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Answer 4:

Given that

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since x lies in the 4^{th} quadrant, the value of $\sin x$ will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\text{cosec}\, x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

Question 5:

Find the values of other five trigonometric functions if $\tan x = -\frac{5}{12}, x$ lies in second quadrant.

Answer 5:

Given that

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{13}{12}$$

Since x lies in the 2^{nd} quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x = -\frac{13}{12}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Question 6:

Find the value of the trigonometric function $\sin 765^{\circ}$

Answer 6:

It is known that the values of $\sin x$ repeat after an interval of 2π or 360°.

$$\therefore \sin 765^{\circ} = \sin\left(2\times 360^{\circ} + 45^{\circ}\right)$$

$$= \sin 45^{\circ} = \frac{1}{\sqrt{2}}$$

Question 7:

Find the value of the trigonometric function $\operatorname{cosec}\left(-1410^{\circ}\right)$

Answer 7:

It is known that the values of $\operatorname{cosec} x$ repeat after an interval of 2π or 360°.

$$\therefore \operatorname{cosec}\left(-1410^{\circ}\right) = \operatorname{cosec}\left(-1410^{\circ} + 4\times 360^{\circ}\right)$$

$$= \operatorname{cosec}\left(-1410^{\circ} + 1440^{\circ}\right)$$

$$= \operatorname{cosec} 30^{\circ} = 2$$

Question 8:

Find the value of the trigonometric function $\tan \frac{19\pi}{3}$

Answer 8:

It is known that the values of $\tan x$ repeat after an interval of π or 180°.

$\therefore \tan\frac{19\pi}{3}$

$= \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right)$

$= \tan\frac{\pi}{3} = \tan 60^{\circ} = \sqrt{3}$

Question 9:

Find the value of the trigonometric function $\sin\left(-\frac{11\pi}{3}\right)$

Answer 9:

It is known that the values of $\sin x$ repeat after an interval of 2Π or 360°

$\therefore \sin\left(-\frac{11\pi}{3}\right)$

Substitute the values

$= \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right)$

Simplify the term

$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Question 10:

Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

Answer 10:

It is known that the values of $\cot x$ repeat after an interval of Π or 180°.

Given that

$\therefore \cot\left(-\frac{15\pi}{4}\right)$

Substitute the values

$$= \cot\left(-\frac{15\pi}{4}+4\pi\right)$$

Simplify the term

$$= \cot\frac{\pi}{4} = 1$$

Example 10

Prove that

$$3\sin\frac{\pi}{6}\sec\frac{\pi}{3}-4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}=1$$

Solution

Given that

$$\text{L.H.S.} \quad = 3\sin\frac{\pi}{6}\sec\frac{\pi}{3}-4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$$

Substituting the terms

$$= 3\times\frac{1}{2}\times 2-4\sin\left(\pi-\frac{\pi}{6}\right)\times 1$$

$$= 3-4\sin\frac{\pi}{6}$$

$$= 3-4\times\frac{1}{2} = 1 = \text{ R.H.S.}$$

Hence proved

Example 11

Find the value of $\sin 15^{\circ}$.

Solution

We have

$$\sin 15^{\circ} \quad = \sin\left(45^{\circ}-30^{\circ}\right)$$

Expand terms

$$= \sin 45^{\circ}\cos 30^{\circ}-\cos 45^{\circ}\sin 30^{\circ}$$

Substitute value

$$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}$$

$$=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

Example 12

Find the value of $\tan\frac{13\pi}{12}$.

Solution

We have

$$\tan\frac{13\pi}{12}\ =\tan\left(\pi+\frac{\pi}{12}\right)$$

Expand terms

$$=\tan\frac{\pi}{12}=\tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$

simplify

$$=\frac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}}$$

$$=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=2-\sqrt{3}$$

Example 13

Prove that

$$\frac{\sin(x+y)}{\sin(x-y)}=\frac{\tan x+\tan y}{\tan x-\tan y}$$

Solution

We have

$$\text{L.H.S.} = \frac{\sin(x+y)}{\sin(x-y)} = \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y}$$

Dividing the numerator and denominator by $\cos x \cos y$, we get

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$$

Example 14

Show that

$\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

Solution

We know that $3x = 2x + x$

Therefore, $\tan 3x = \tan(2x + x)$

or $\quad \tan 3x = \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

or $\quad \tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

or $\quad \tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$ or

$\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$.

Example 15

Prove that

$$\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x$$

Solution 15

Given that

$$\text{L.H.S.} = \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right)$$

Expand terms

$$= 2\cos\left(\frac{\frac{\pi}{4}+x+\frac{\pi}{4}-x}{2}\right)\cos\left(\frac{\frac{\pi}{4}+x-\left(\frac{\pi}{4}-x\right)}{2}\right)$$

simplify

$$= 2\cos\frac{\pi}{4}\cos x = 2\times\frac{1}{\sqrt{2}}\cos x = \sqrt{2}\cos x = \text{ R.H.S.}$$

Example 16

Prove that $\dfrac{\cos 7x+\cos 5x}{\sin 7x-\sin 5x} = \cot x$

Solution

Given that

$$\text{L.H.S.} = \frac{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}}{2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}}$$

simplify

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

$$= \text{ R.H.S.}$$

Example 17

Prove that $= \dfrac{\sin 5x-2\sin 3x+\sin x}{\cos 5x-\cos x} = \tan x$

Solution

Given that

$$\text{L.H.S.} = \frac{\sin 5x-2\sin 3x+\sin x}{\cos 5x-\cos x}$$

$$= \frac{\sin 5x+\sin x-2\sin 3x}{\cos 5x-\cos x}$$

simplify

$$= \frac{2\sin 3x \cos 2x - 2\sin 3x}{-2\sin 3x \sin 2x} = -\frac{\sin 3x(\cos 2x - 1)}{\sin 3x \sin 2x}$$

$$= \frac{1 - \cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x \cos x} = \tan x = \text{ R.H.S.}$$