Chapter 3: Trigonometric Functions

Exercise 3.3

Question 1:

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Answer 1:

Given that

$$\text{L.H.S.} \quad =\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$$

Put the values

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

$$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$$

$$= \text{ R.H.S.}$$

Question 2:

Prove that $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Answer 2:

Given that

$$\text{L.H.S.} \quad =2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

$$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$$

$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$

$=$ R.H.S.

Question 3:

Prove that $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$

Answer 3:

Given that

$$\text{L.H.S.} = \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6}$$

$$= (\sqrt{3})^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3}$$

$$= 3 + 2 + 1 = 6$$

$=$ R.H.S

Question 4:

Prove that $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3} = 10$

Answer 4:

Given that

$$\text{L.H.S} = 2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3}$$

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\left\{\sin \frac{\pi}{4}\right\}^2 + 2 \times \frac{1}{2} + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$=10$

$=$ R.H.S

Question 5:

Find the value of:

(i) $\sin 75^{\circ}$

(ii) $\tan 15^{\circ}$

Answer 5:

(i) $\sin 75^{\circ}=\sin\left(45^{\circ}+30^{\circ}\right)$

$=\sin 45^{\circ}\cos 30^{\circ}+\cos 45^{\circ}\sin 30^{\circ}$

$[\sin(x+y)=\sin x\cos y+\cos x\sin y]$

$$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$$ (ii) $\tan 15^{\circ}=\tan\left(45^{\circ}-30^{\circ}\right)$

$$=\frac{\tan 45^{\circ}-\tan 30^{\circ}}{1+\tan 45^{\circ}\tan 30^{\circ}}\left[\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right]$$

$$=\frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)}=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2}$$

$$=\frac{4-2\sqrt{3}}{3-1}=2-\sqrt{3}$$

Question 6:

Prove that: $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Answer 6:

Given that

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$=\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$=\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$[\because 2\cos A\cos B=\cos(A+B)+\cos(A-B)$$

$$-2\sin A\sin B=\cos(A+B)-\cos(A-B)]$$

$$=2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$=\cos\left[\frac{\pi}{2}-(x+y)\right]$$

$$=\sin(x+y)$$

$$=R\cdot H\cdot S$$

Question 7:

Prove that: $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

Answer 7:

It is known that

$$t\,\mathrm{an}(\mathrm{A}+\mathrm{B})=\frac{\tan \mathrm{A}+\tan \mathrm{B}}{1-\tan \mathrm{A}\tan \mathrm{B}}$$

$$\tan(\mathrm{A}-\mathrm{B})=\frac{\tan \mathrm{A}-\tan \mathrm{B}}{1+\tan \mathrm{A}\tan \mathrm{B}}$$

Then just apply the formula

We get: $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

proved

Question 8:

$\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}=\cot^2 x$ Prove that $\dfrac{x}{2}$

Answer 8:

Given that

$$\text{L.H.S.} \quad =\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}$$

$$=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$=\frac{-\cos^2 x}{-\sin^2 x}$$

$$=\cot^2 x$$

$$= \text{R.H.S.}$$

Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Answer 9:

Given that

$$\text{L.H.S.} \quad =\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$$=\sin x\cos x[\tan x+\cot x]$$

$$= \sin x \cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$= (\sin x \cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x \cos x}\right]$$

$$= 1 = \text{ R.H.S.}$$

Question 10:

Prove that $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

Answer 10:

Given that

$$\text{L.H.S.} = \sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$$

$$=\frac{1}{2}[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x]$$

$$=\frac{1}{2}[\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}]$$

$$\because -2\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}]$$

$$2\cos A\cos B=\cos(A+B)+\cos(A-B)$$

$$=\frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$$

$$=\cos(-x)=\cos x= \text{ R.H.S.}$$

Question 11:

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Answer 11:

Given that

$$\cos \mathrm{A}-\cos \mathrm{B}=-2\sin\left(\frac{\mathrm{A}+\mathrm{B}}{2}\right)\cdot\sin\left(\frac{\mathrm{A}-\mathrm{B}}{2}\right)$$

$$\therefore \text{ L.H.S.} = \cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}\cdot\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

simplify

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

$=$ R.H.S.

Question 12:

Prove that $\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x$

Answer 12:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

L.H.S. $=\sin^2 6x-\sin^2 4x$

$$=(\sin 6x+\sin 4x)(\sin 6x-\sin 4x)$$

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right)\cdot\sin\left(\frac{6x-4x}{2}\right)\right]$$

$$=(2\sin 5x\cos x)(2\cos 5x$$

$$\sin x)=(2\sin 5x\cos 5x)(2$$

$$\sin x\cos x)$$

$$=\sin 10x\sin 2x$$

$=$ R.H.S.

Question 13:

Prove that $\cos^2 2x-\cos^2 6x=\sin 4x\sin 8x$

Answer 13:

It is known that

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$ L.H.S. $=\cos^2 2x-\cos^2 6x$

$=(\cos 2x+\cos 6x)(\cos 2x-6x)$

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$=[2\cos 4x\cos(-2x)][-2\sin 4x\sin(-2x)]$

$=[2\cos 4x\cos 2x][-2\sin 4x(-\sin 2x)]$

$=(2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

$=\sin 8x\sin 4x=$ R.H.S.

Question 14:

Prove that $\sin 2x+2\sin 4x+\sin 6x=4\cos^2 x\sin 4x$

Answer 14:

L.H.S. $=\sin 2x+2\sin 4x+\sin 6x$

$=[\sin 2x+\sin 6x]+2\sin 4x$

$$=\left[2\sin\left(\frac{2x+6x}{2}\right)\left(\frac{2x-6x}{2}\right)\right]+2\sin 4x$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$=2\sin 4x\cos(-2x)+2\sin 4x$

$=2\sin 4x\cos 2x+2\sin 4x$

$=2\sin 4x(\cos 2x+1)$

$=2\sin 4x\left(2\cos^2 x-1+1\right)$

$= 2\sin 4x\left(2\cos^2 x\right)$

$= 4\cos^2 x\sin$

$4x =$ R.H.S.

Question 15:

Prove that $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

Answer 15:

Given that

L.H.S $= \cot 4x(\sin 5x + \sin 3x)$

$$= \frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x\cos x]$$

$= 2\cos 4x\cos x$

R.H.S. $= \cot x(\sin 5x - \sin 3x)$

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\cos x}{\sin x}[2\cos 4x\sin x]$$

$= 2\cos 4x \cdot \cos x$

L.H.S. = R.H.S.

Question 16:

Prove that $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

Answer 16:

It is known that

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{ L.H.S } = \frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right)\cdot\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\cdot\sin\left(\frac{17x-3x}{2}\right)}$$

$$=\frac{-2\sin 7x\cdot\sin 2x}{2\cos 10x\cdot\sin 7x}$$

$$=-\frac{\sin 2x}{\cos 10x}$$

$$=R\cdot H\cdot S$$

Question 17:

Prove that: $\dfrac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

Answer 17:

It is known that

$$\sin \mathrm{A}+\sin \mathrm{B}=2\sin\left(\frac{\mathrm{A}+\mathrm{B}}{2}\right)\cos\left(\frac{\mathrm{A}-\mathrm{B}}{2}\right), \cos \mathrm{A}+\cos \mathrm{B}=2\cos\left(\frac{\mathrm{A}+\mathrm{B}}{2}\right)\cos\left(\frac{\mathrm{A}-\mathrm{B}}{2}\right)$$

$$\therefore \text{ L.H.S. } = \frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin 4x\cdot\cos x}{2\cos 4x\cdot\cos x}$$

$$=\frac{\sin 4x}{\cos 4x}$$

$$=\tan 4x= \text{ R.H.S.}$$

Question 18:

Prove that $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\frac{x-y}{2}$

Answer 18:

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore\ \text{L.H.S.}\ =\frac{\sin x-\sin y}{\cos x+\cos y}$$

$$=\frac{2\cos\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}$$

$$=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$=\tan\left(\frac{x-y}{2}\right)=R.H.S$$

Question 19:

Prove that $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Answer 19:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore\ \text{L.H.S.}\ =\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$= \frac{\sin 2x}{\cos 2x}$

$= \tan 2x$

$=$ R.H.S

Question 20:

Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Answer 20:

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \cos^2 A - \sin^2 A = \cos 2A$$

$\therefore$ L.H.S. $= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$= -2 \times (-\sin x)$

$= 2\sin x =$ R.H.S.

Question 21:

Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Solution 21:

Given that

L.H.S. $= \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$$

$$\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

Apply the formula

$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$

$$=\cot 3x=R\cdot H\cdot S$$

Question 22:

Prove that $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

Answer 22:

Given that

$$\text{L.H.S.}=\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x$$

$$=\cot x\cot 2x-\cot 3x(\cot 2x+\cot x)$$

$$=\cot x\cot 2x-\cot(2x+x)(\cot 2x+\cot x)$$

$$=\cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot x+\cot 2x}\right](\cot 2x+\cot x)$$

$$\left[\because \cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}\right]$$

$$=\cot x\cot 2x-(\cot 2x\cot x-1)=1=\text{ R.H.S.}$$

Question 23:

Prove that $\tan 4x=\frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$

Answer 23:

It is known that. $\tan 2\mathrm{A} = \dfrac{2\tan \mathrm{A}}{1-\tan^2 \mathrm{A}}$

$\therefore$ L.H.S. $= \tan 4x = \tan 2(2x)$

$$= \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{4\tan x}{1-\tan^2 x})$$

$$= \frac{4-\dfrac{1\tan^2 x}{\left(1-\tan^2 x\right)^2}]}{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}$$

$$= \frac{\left(1-\tan^2 x\right)^2 - 4\tan^2 x]}{\left(1-\tan^2 x\right)^2}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-\tan^2 x)^2 - 4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x + \tan^4 x} = \text{R.H.S.}$$

Question 24:

Prove that: $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

Answer 24:

L.H.S. $= \cos 4x$

$= \cos 2(2x)$

$= 1 - 2\sin^2 2x\left[\cos 2A = 1 - 2\sin^2 A\right]$

$= 1 - 2(2\sin x \cos x)^2[\sin 2A = 2\sin A \cos A]$

$=1-8\sin^2 x$

$\cos^2 x =$ R.H.S.

Question 25:

Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Answer 25:

L.H.S. $= \cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x \left[\cos 3A = 4\cos^3 A - 3\cos A\right]$

$= 4\left[\left(2\cos^2 x - 1\right)^3 - 3\left(2\cos^2 x - 1\right)\left[\cos 2x = 2\cos^2 x - 1\right]\right.$

$= 4\left[\left(2\cos^2 x\right)^3 - (1)^3 - 3\left(2\cos^2 x\right)^2 + 3\left(2\cos^2 x\right)\right] - 6\cos^2 x + 3$

$= 4\left[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x\right] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18$

$\cos^2 x - 1 =$ R.H.S.

Example 18

Find the principal solutions of the equation $\sin x = \frac{\sqrt{3}}{2}$

Solution

Given that

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and

$\sin \frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right)$

simplify

$= \sin \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$.

Value of $x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$.

Example 19

Find the principal solutions of the equation $\tan x = -\frac{1}{\sqrt{3}}$.

Solution

We know that, $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$. Thus, $\tan\left(\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$ and

$\tan\left(2\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

Thus $\tan\frac{5\pi}{6} = \tan\frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$.

Therefore, $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Already we seen that

$\sin x = 0$ gives $x = n\pi$, where $n \in \mathbf{Z}$

$\cos x = 0$ gives $x = (2n+1)\frac{\pi}{2}$, where $n \in \mathbf{Z}$

Example 20.

Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$.

Solution

Given that

$\sin x = -\frac{\sqrt{3}}{2}$

$= -\sin\frac{\pi}{3}$

$$= \sin\left(\pi + \frac{\pi}{3}\right)$$

$$= \sin\frac{4\pi}{3}$$

Hence $\sin x = \sin\frac{4\pi}{3}$,

which gives $x = n\pi + (-1)^n \frac{4\pi}{3}$,

where $n \in \mathbf{Z}$

Example 21

Solve $\cos x = \frac{1}{2}$.

Solution

Given that

$$\cos x = \frac{1}{2}$$

$$= \cos\frac{\pi}{3}$$

Therefore

$$x = 2n\pi \pm \frac{\pi}{3},$$

where $n \in \mathbf{Z}$.

Example 22

Solve $\tan 2x = -\cot\left(x + \frac{\pi}{3}\right)$.

Solution

Given that

$$\tan 2x = -\cot\left(x + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{2} + x + \frac{\pi}{3}\right)$$

or

$$\tan 2x = \tan\left(x + \frac{5\pi}{6}\right)$$

Hence

$$2x = n\pi + x + \frac{5\pi}{6},$$

where $n \in \mathbf{Z}$

or $x = n\pi + \frac{5\pi}{6}$,

where $n \in \mathbf{Z}$

Example 23

Solve $\sin 2x - \sin 4x + \sin 6x = 0$

Solution

Given that

$$\sin 2x - \sin 4x + \sin 6x = 0$$

Rewrite

$$\sin 6x + \sin 2x - \sin 4x = 0$$

$$2\sin 4x \cos 2x - \sin 4x = 0$$

$$\sin 4x(2\cos 2x - 1) = 0$$

i.e.

Therefore $\sin 4x = 0$ or $\cos 2x = \frac{1}{2}$

i.e. $\sin 4x = 0$ or $\cos 2x = \cos\frac{\pi}{3}$

Hence $4x = n\pi$ or $2x = 2n\pi \pm \frac{\pi}{3}$, where $n \in \mathbf{Z}$

i.e. $x = \frac{n\pi}{4}$ or $x = n\pi \pm \frac{\pi}{6}$, where $n \in \mathbf{Z}$.

Example 24

Solve $2\cos^2 x + 3\sin x = 0$

Solution

Given that

$$2\cos^2 x + 3\sin x = 0$$

$$2\left(1 - \sin^2 x\right) + 3\sin x = 0$$

or

$$2\sin^2 x - 3\sin x - 2 = 0$$

or

$$(2\sin x + 1)(\sin x - 2) = 0$$

Hence $\sin x = -\frac{1}{2}$ or $\sin x = 2$

But $\sin x = 2$ is not possible

Then

$$\sin x = -\frac{1}{2} = \sin\frac{7\pi}{6}$$

Hence,

$$x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbf{Z}.$$