

Chapter 4: Determinants

Exercise 4.1

1. Evaluate the determinants in Exercise 1 and 2. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Solution:

$$\begin{aligned} \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} &= 2(-1) - 4(-5) \\ &= -2 + 20 \\ &= 18 \end{aligned}$$

2. Evaluate the determinants in Exercise 1 and 2

i) $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$ ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Solution:

$$\begin{aligned} \text{i) } \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} &= (\cos \theta)(\cos \theta) - (-\sin \theta)(\sin \theta) \\ &= \cos^2 \theta + \sin^2 \theta = 1 \end{aligned}$$

$$\begin{aligned} \text{ii) } \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} &= (x^2 - x + 1) - (x - 1)(x + 1) \\ &= x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1) \\ &= x^3 + 1 - x^2 + 1 \\ &= x^3 - x^2 + 2 \end{aligned}$$

3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Solution:

$$\text{Given, } A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$\therefore 2A = 2 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\therefore \text{L.H.S} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$$

$$= 2 \times 4 - 4 \times 8$$

$$= 8 - 32$$

$$= -24$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$$

$$\text{Now, } = 1 \times 2 - 2 \times 4$$

$$= 2 - 8$$

$$= -6$$

$$\therefore \text{R.H.S} = 4|A| = 4 \times (-6) = -24$$

$$\therefore \text{L.H.S} = \therefore \text{R.H.S}$$

4. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$

Solution:

$$\text{Given, } A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

Expanding along the first column (C_1)

$$|A| = 1 \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 1(4 - 0) - 0 + 0 = 4$$

$$\therefore 27|A| = 27(4) = 108 \dots \dots \dots (i)$$

$$\text{Now, } 3A = 3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\therefore |3A| = 3 \begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0 \begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0 \begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$

$$= 3(36 - 0) = 3(36) = 108 \dots \dots \dots (ii)$$

From equations (i) and (ii), we have

$$|3A| = 27|A|$$

Hence proved

5. Evaluate the determinants

i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Solution:

i) Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

expanding along the second row

$$|A| = -0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0 \begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix}$$

$$= (-15 + 3) = -12$$

$$\text{ii) Let } A = \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

expanding along the first row

$$|A| = 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= 3(1 + 6) + 4(1 + 4) + 5(3 - 2)$$

$$= 3(7) + 4(5) + 5(1)$$

$$= 21 + 20 + 5 = 46$$

$$\text{iii) Let } A = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

expanding along the first row

$$|A| = 0 \begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$$

$$= 0 - 1(0 - 6) + 2(-3 - 0)$$

$$= -1(-6) + 2(-3)$$

$$= 6 - 6 = 0$$

$$\text{iv) let } \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

expanding along the first column

$$|A| = 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3 \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$$

$$= 2(0-5) - 0 + 3(1+4)$$

$$= -10 + 15 = 5$$

6. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$

Solution:

$$\text{Given, } A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$$

Expanding along the first row

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$$

$$|A| = 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$$= 1(-9+12) - 1(-18+15) - 2(8-5)$$

$$= 1(3) - 1(-3) - 2(3)$$

$$= 3 + 3 - 6 = 6 - 6 = 0$$

7. Find value of X, if

$$\text{i) } \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

$$\text{ii) } \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

Solution:

$$\text{i) } \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

$$\Rightarrow 2 \times 1 - 5 \times 4 = 2x \times x - 6 \times 4$$

$$\Rightarrow 2 - 20 = 2x^2 - 24$$

$$\Rightarrow 2x^2 = 6$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow x = \pm\sqrt{3}$$

$$\text{ii) } \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

$$\Rightarrow 2 \times 5 - 3 \times 4 = x \times 5 - 3 \times 2x$$

$$\Rightarrow 10 - 12 = 5x - 6x$$

$$\Rightarrow -2 = -x$$

$$\Rightarrow x = 2$$

8. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then X is equal to

- A) 6 B) ± 6 C) -3 D) 0

Solution:

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$\Rightarrow x^2 - 36 = 36 - 36$$

$$\Rightarrow x^2 - 36 = 0$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x = \pm 6$$

Hence, (B) is the correct answer