Chapter 4: Determinants

Exercise 4.3

1. Find area of the triangle with vertices at the point given in each of the following

i) $(1,0),(6,0),(4,3)$ ii) $(2,7),(1,1),(10,8)$ iii) $(-2,-3),(3,2),(-1,-8)$

Solution:

i) The area of the triangle with vertices $(1,0),(6,0),(4,3)$ is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ 6 & 0 & 1\\ 4 & 3 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[1(0-3)-0(6-4)+1(18-0)\right]$$

$$=\frac{1}{2}[-3+18]$$

$$=\frac{15}{2}\,square\,units$$

ii) The area of the triangle with vertices $(2,7),(1,1),(10,8)$ is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}2 & 7 & 1\\ 1 & 1 & 1\\ 10 & 8 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[2(1-8)-7(1-10)+1(8-10)\right]$$

$$=\frac{1}{2}\left[2(-7)-7(-9)+1(-2)\right]$$

$$=\frac{1}{2}[-14+63-2]$$

$=\frac{1}{2}[-16+63]$

$=\frac{47}{2}$ Square units

iii) The area of the triangle with vertices $(-2,-3),(3,2),(-1,-8)$ is given by the relation

$$\Delta=\frac{1}{2}\begin{vmatrix}-2 & -3 & 1\\ 3 & 2 & 1\\ -1 & -8 & 1\end{vmatrix}$$

$=\frac{1}{2}\left[-2(2+8)+3(3+1)+1(-24+2)\right]$

$=\frac{1}{2}\left[-2(10)+3(4)+1(-22)\right]$

$=\frac{1}{2}[-20+12-22]$

$=-\frac{30}{2}$

$=-15$

Hence, the area of the triangle is $|-15|=15$ square units

2. Show that points $A(a,b+c),B(b,c+a),C(c,a+b)$ are collinear

Solution:

Area of ΔABC is given by $\Delta=\frac{1}{2}\begin{vmatrix}a & b+c & 1\\ b & c+a & 1\\ c & a+b & 1\end{vmatrix}$

Applying $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$

$$=\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b-a & a-b & 0 \\ c-a & a-c & 0 \end{vmatrix}$$

$$=\frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

Applying $R_3 \to R_3 + R_2$

$$=\frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$=0$ (since, all elements of R_3 are 0)

Thus, the area of the triangle formed by points A, B and C is zero

Hence, the points are collinear

3. Find values of k if area of triangle is 4 square units and vertices are

i) $(k,0),(4,0),(0,2)$ ii) $(-2,0),(0,4),(0,k)$

Solution:

i) The area of the triangle with vertices $(k,0),(4,0),(0,2)$ is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[k(0-2)-0(4-0)+1(8-0)\right]$$

$$=\frac{1}{2}[-2k+8]=k+4$$

$\therefore -k+4=\pm 4$

When $-k+4=-4, k=8$

When $-k+4=+4, k=0$

Therefore, $k=0,8$

ii) The area of the triangle with vertices $(-2,0),(0,4),(0,k)$ is given by the relation

$$\Delta=\frac{1}{2}\begin{vmatrix}-2 & 0 & 1\\ 0 & 4 & 1\\ 0 & k & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[-2(4-k)\right]$$

$$=k-4$$

$$\therefore k-4=\pm 4$$

When $k-4=-4, k=0$

When $k-4=4, k=8$

Therefore, $k=0,8$

4. i) Find equation of line joining (1,2) and (3,6) using determinates

ii) Find equation of line joining (3,1) and (9,3) using determinants

Solution:

i) Let $P(x,y)$ be any point on the line joining points $A(1,2)$ and $B(3,6)$.

Then the points A, B and P are collinear

Therefore, the area of triangle ABP will be zero

$$\therefore \frac{1}{2}\begin{vmatrix}1 & 2 & 1\\ 3 & 6 & 1\\ x & y & 1\end{vmatrix}=0$$

$$\Rightarrow \frac{1}{2}\left[1(6-y)-2(3-x)+1(3y-6x)\right]=0$$

$\Rightarrow 6-y-6+2x+3y-6x=0$

$\Rightarrow 2y-4x=0$

$\Rightarrow y=2x$

Therefore, the equation of the line joining the given points is $y=2x$

ii) Let $P(x,y)$ be any point on the line joining points $A(3,1)$ and $B(9,3)$

then, the points are collinear

thus, the area of the triangle ABP will be zero

$$\therefore \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix}=0$$

$$\Rightarrow \frac{1}{2}\left[3(3-y)-1(9-x)+1(9y-3x)\right]=0$$

$\Rightarrow 9-3y-9+x+9y-3x=0$

$\Rightarrow 6y-2x=0$

$\Rightarrow x-3y=0$

Therefore, the equation of the line joining the given points is $x-3y=0$

5. If the area of triangle is 35 square units with vertices $(2,-6),(5,4)$ and $(k,4)$. Then k is

A) 12 B) -2 C) -12, -2 D) 12, -2

Solution:

The area of the triangle with vertices $(2,-6),(5,4)$ and $(k,4)$ is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$=\frac{1}{2}\left[2(4-4)+6(5-k)+1(20-4k)\right]$

$=\frac{1}{2}[30-6k+20-4k]$

$=\frac{1}{2}[50-10k]$

$=25-5k$

Given, the area of the triangle is ± 35

Thus, we have

$\Rightarrow 25-5k=\pm 35$

$\Rightarrow 5(5-k)=\pm 35$

$\Rightarrow (5-k)=\pm 7$

When $5-k=-7, k=5+7=12$

When $5-k=7, k=5-7=-2$

Therefore, $k=12,-2$