Chapter 4: Determinants

Exercise 4.6

1. Examine the consistency of the system of equations $x+2y=2$, $2x+3y=3$

Solution:

Given,

$x+2y=2$

$2x+3y=3$

Let $A=\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}, X=\begin{bmatrix}2\\3\end{bmatrix}$ and $B=\begin{bmatrix}2\\3\end{bmatrix}$

Such that, the given system of equations can be written in the form of $AX=B$

Now,

$|A|=1(3)-2(2)=3-4=-1\neq 0$

$\therefore A$ is non-singular

Thus, A^{-1} exists

Therefore, the given system of equations is consistent

2. Examine the consistency of the system of equations $2x-y=5$, $x+y=4$

Solution:

Given,

$2x-y=5$

$x+y=4$

Let $A=\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\z\end{bmatrix}$ and $B=\begin{bmatrix}5\\4\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX = B$

Now,

$|A| = 2(1) - (-1)(1) = 2 + 1 = 3 \neq 0$

$\therefore A$ is non-singular

Thus, A^{-1} exists

Therefore, the given system of equations is consistent

3. Examine the consistency of the system of equations $x + 3y = 5, 2x + 6y = 8$

Solution:

Given,

$x + 3y = 5$
$2x + 6y = 8$

Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX = B$

Now,

$|A| = 1(6) - 3(2) = 6 - 6 = 0$

$\therefore A$ is a singular matrix

$(adjA) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$(adjA)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$

Thus, the solution of the given system of equations does not exists

Therefore, the given system of equation is inconsistent

4. Examine the consistency of the system of equations

$x+y+z=1$

$2x+3y+2z=2$

$ax+ay+2az=4$

Solution:

Given,

$x+y+z=1$

$2x+3y+2z=2$

$ax+ay+2az=4$

Let $A=\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ a & a & 2a\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}1\\ 2\\ 4\end{bmatrix}$

Such that, the system of equation can be written in the form of $AX=B$

Now,

$|A|=1(6a-2a)-1(4a-2a)+1(2a-3a)$

$=4a-2a-a=4a-3a=a\neq 0$

$\therefore A$ is a non-singular matrix

Thus, A^{-1} exists

Therefore, the given system of equation is consistent

5. Examine the consistency of the system of equations

$3x-y-2z=2$

$2y-z=-1$

$3x-5y=3$

Solution:

Given,

$$3x-y-2z=2$$

$$2y-z=-1$$

$$3x-5y=3$$

Let $A=\begin{bmatrix}3 & -1 & -2\\0 & 2 & -1\\3 & -5 & 0\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}2\\-1\\3\end{bmatrix}$

Such that, this system of equations can be written in the form of $AX=B$

Now,

$$|A|=3(-5)-0+3(1+4)=-15+15=0$$

$\therefore A$ is a singular matrix

Now,

$$(adjA)=\begin{bmatrix}-5 & 10 & 5\\-3 & 6 & 3\\-6 & 12 & 6\end{bmatrix}$$

$$\therefore (adjA)B=\begin{bmatrix}-5 & 10 & 5\\-3 & 6 & 3\\-6 & 12 & 6\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}-10-10+15\\-6-6+9\end{bmatrix}=\begin{bmatrix}-5\\-3\end{bmatrix}\neq 0$$

Thus, the solution of the given system of equation does not exist

Therefore, the system of equation is inconsistent

6. Examine the consistency of the system of equations

$$5x-y+4z=5$$

$$2x+3y+5z=2$$

$$5x-2y+6z=-1$$

Solution:

Given,

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

Let $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 3 & -2 & 6 \end{bmatrix}, X \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

Such that, the system of equation can be written in the form of $AX = B$

Now,

$$|A| = 5(18+10) + 1(12-25) + 4(-4-15)$$

$$= 5(28) + 1(-13) + 4(-19)$$

$$= 140 - 13 - 76$$

$$= 51 \neq 0$$

$\therefore A$ is non-singular

Thus, A^{-1} exists

Therefore, the given system of equations is consistent

7. Solve system of linear equations, using matrix method

$$5x + 2y = 4$$

$$7x + 3y = 5$$

Solution:

Given,

$$5x + 2y = 4$$

$$7x + 3y = 5$$

Let $A=\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}4\\ 5\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX=B$

Now,

$|A|=15-14=1\neq 0$

Thus, A is non-singular

Therefore, its inverse exists

Now,

$$A^{-1}=\frac{1}{|A|}(adjA)$$

$$\therefore A^{-1}=\begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}$$

$$\therefore X=A^{-1}B=\begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}\begin{bmatrix}4\\ 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}12-10\\ -28+25\end{bmatrix}=\begin{bmatrix}2\\ -3\end{bmatrix}$$

Thus, $x=2$ and $y=-3$

8. Solve system of linear equations, using matrix method

$2x-y=-2$

$3x+4y=3$

Solution:

Given,

$2x-y=-2$

$3x+4y=3$

Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}-2\\ 3\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX=B$

Now,

$|A|=8+3=11\neq 0$

Thus, A is non-singular

Therefore, its inverse exists

Now,

$$A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}\begin{bmatrix}-2\\ 3\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-8+3\\ 6+6\end{bmatrix}$$

$$=\frac{1}{11}\begin{bmatrix}-5\\ 12\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-5}{11}\\ \frac{12}{11}\end{bmatrix}$$

Thus, $x=\frac{-5}{11}$ and $y=\frac{12}{11}$

9. Solve system of linear equations, using matrix method

$4x-3y=3$

$3x-5y=7$

Solution:

Given,

$$4x-3y=3$$

$$3x-5y=7$$

Let $A=\begin{bmatrix}4 & -3\\ 3 & -5\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 7\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX=B$

Now,

$$|A|=-20+9=-11\neq 0$$

Thus, A is non-singular

Therefore, its inverse exist

Now,

$$A^{-1}=\frac{1}{|A|}(adjA)=-\frac{1}{11}\begin{bmatrix}-5 & 3\\ -3 & 4\end{bmatrix}=\frac{1}{11}\begin{bmatrix}5 & -3\\ 3 & -4\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{11}\begin{bmatrix}5 & -3\\ 3 & -4\end{bmatrix}\begin{bmatrix}3\\ 7\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{11}\begin{bmatrix}5 & -3\\ 3 & -4\end{bmatrix}\begin{bmatrix}3\\ 7\end{bmatrix}$$

$$=\frac{1}{11}\begin{bmatrix}15-21\\ 9-28\end{bmatrix}$$

$$=\frac{1}{11}\begin{bmatrix}-6\\ -19\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{6}{11}\\ -\frac{19}{11}\end{bmatrix}$$

Thus, $x=\frac{-6}{11}$ and $y=\frac{-19}{11}$

10. Solve system of linear equations, using matrix method

$5x+2y=3$

$3x+2y=5$

Solution:

Given,

$5x+2y=3$

$3x+2y=5$

Let $A=\begin{bmatrix}5 & 2\\ 3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 5\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX=B$

Now,

$|A|=10-6=4\neq 0$

Thus A is non-singular

Therefore, its inverse exists

11. Solve system of linear equations, using matrix method

$2x+y+z=1$

$x-2y-z=\frac{3}{2}$

$3y-5z=9$

Solution:

Given,

$2x+y+z=1$

$x-2y-z=\frac{3}{2}$

$3y-5z=9$

Let $A=\begin{bmatrix}2 & 1 & 1\\ 1 & -2 & -1\\ 0 & 3 & -5\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}1\\ \frac{3}{2}\\ 9\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX=B$

Now,

$|A|=2(10+3)-1(-5-3)+0=2(13)-1(-8)=26+8=34\neq 0$

Thus A is non-singular

Therefore, its inverse exists

Now,

$A_{11}=13, A_{12}=5, A_{13}=3$

$A_{21}=8, A_{22}=-10, A_{23}=-6$

$A_{31}=1, A_{32}=3, A_{33}=-5$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -16 & -5\end{bmatrix}$$

$$\therefore X=A^{-1}B=\frac{1}{34}\begin{bmatrix}13 & 8 & 1\\ 5 & -10 & 3\\ 3 & -16 & -5\end{bmatrix}\begin{bmatrix}1\\ \frac{3}{2}\\ 9\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{34}\begin{bmatrix}13+12+9\\ 5-15+27\\ 3-9-45\end{bmatrix}$$

$$=\frac{1}{34}\begin{bmatrix}34\\ 17\\ -51\end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$$

Thus, $x = 1, y = \frac{1}{2}$ and $z = -\frac{3}{2}$

12. Solve system of linear equations, using matrix method

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Solution:

Given,

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX = B$

Now,

$|A| = 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0$

Thus A is non-singular

Therefore, its inverse exists

Now,

$A_{11} = 4, A_{12} = -5, A_{13} = 1$

$A_{21}=2, A_{22}=0, A_{23}=-2$

$A_{31}=2, A_{32}=5, A_{33}=3$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)$$

$$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$$

$$\therefore X=A^{-1}B$$

$$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}16+0+4\\ -20+0+10\\ 4+0+6\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}20\\ -10\\ 10\end{bmatrix}$$

$$=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$$

Thus, $x=2, y=-1$ and $z=1$

13. Solve system of linear equation, using matrix method

$2x+3y+3z=5$

$x-2y+z=-4$

$3x-y-2z=3$

Solution:

Given,

$2x+3y+3z=5$

$x-2y+z=-4$

$3x-y-2z=3$

Let $A=\begin{bmatrix}2 & 3 & 3\\1 & -2 & 1\\3 & -1 & -2\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}5\\-4\\3\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX=B$

$|A|=2(4+1)-3(2-3)+3(-1+6)=2(5)-3(-5)=10+15+15=40\neq 0$

Thus, A is non-singular

Therefore, its inverse exists

Now,

$A_{11}=5, A_{12}=5, A_{13}=5$

$A_{21}=3, A_{22}=-13, A_{23}=11$

$A_{31}=9, A_{32}=1, A_{33}=-7$

$\therefore A^{-1}=\frac{1}{|A|}(adjA)$

$=\frac{1}{40}\begin{bmatrix}5 & 3 & 9\\5 & -13 & 1\\5 & 11 & -7\end{bmatrix}$

$\therefore X=A^{-1}B$

$=\frac{1}{40}\begin{bmatrix}5 & 3 & 9\\5 & -13 & 1\\5 & 11 & -7\end{bmatrix}\begin{bmatrix}5\\-4\\3\end{bmatrix}$

$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{40}\begin{bmatrix}25-12+27\\25+52+3\\25-44-21\end{bmatrix}$

$$=\frac{1}{40}\begin{bmatrix}40\\80\\-40\end{bmatrix}$$

$$=\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

Thus, $x=1, y=2$ and $z=-1$

14. Solve system of linear equations, using matrix method

$x-y+2z=7$

$3x+4y-5z=-5$

$2x-y+3z=12$

Solution:

Given,

$x-y+2z=7$

$3x+4y-5z=-5$

$2x-y+3z=12$

Let $A=\begin{bmatrix}1 & -1 & 2\\3 & 4 & -5\\2 & -1 & 3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}7\\-5\\12\end{bmatrix}$

Such that, the given system of equation can be written in the form of $AX=B$

Now,

$|A|=1(12-5)+1(9+10)+2(-3-8)=7+19-22=4\neq 0$

Thus, A is non-singular

Therefore, its inverse exists

Now,

$A_{11}=7, A_{12}=-19, A_{13}=11$

$A_{21}=1, A_{22}=-1, A_{23}=-1$

$A_{31}=-3, A_{32}=11, A_{33}=7$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)$$

$$=\frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore X=A^{-1}B$$

$$=\frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$$

$$=\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Thus, $x=2, y=1$ and $z=3$

15. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs60. The cost of 2 kg onion, 4kg wheat and 6kg rice is Rs90. The cost of 6kg onion 2kg wheat and 3kg rice is Rs 70. Find cost of each item per kg by matrix method

Solution:

Let the cost of onions, wheat and rice per kg be Rs x, Rs y and Rs z respectively.

Then the given situation can be represented by a system of equations as

$$4x+3y+2z=60$$

$$2x+4y+6z=90$$

$$6x+2y+3z=70$$

Let $A=\begin{bmatrix}4&3&2\\2&4&6\\6&2&3\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}60\\90\\70\end{bmatrix}$

Such that, this system of equation can be written in the form of $AX=B$

$$|A|=4(12-12)-3(6-36)+2(4-24)=0+90-40=50\neq 0$$

Now,

$$A_{11}=0, A_{12}=30, A_{13}=-20$$

$$A_{21}=-5, A_{22}=0, A_{23}=10$$

$$A_{31}=10, A_{32}=-20, A_{33}=10$$

$$\therefore adjA=\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}adjA$$

$$=\frac{1}{50}\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}$$

Now,

$$X=A^{-1}B$$

$$\Rightarrow X = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0-450+700 \\ 1800+0-1400 \\ -1200+900+700 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore x = 5, y = 8$, and $z = 8$

Hence, the cost of onion is Rs 5 per kg, the cost of wheat is Rs 8 per kg and the cost of rice is Rs 8 per kg