Infinity Learn by Sri Chaitanya Educational Institutions

Chapter 5: Continuity and differentiability.

Exercise 5.4

1. Differentiating the following w.r.t $x : \dfrac{e^x}{\sin x}$

Solution:

Let $y = \dfrac{e^x}{\sin x}$

Differentiating wrt x, we obtain

$$\frac{dy}{dx} = \frac{\sin x \frac{d}{dx}\left(e^x\right) - e^x \frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{\sin x.\left(e^x\right) - e^x.(\cos x)}{\sin^2 x}$$

$$= \frac{e^x(\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in Z$$

2. Differentiating the following $e^{\sin^{-1} x}$

Solution:

Let $y = e^{\sin^{-1} x}$

Differential wrt x, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} x}\right)$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1} x}.\frac{d}{dx}\left(\sin^{-1} x\right)$$

$$\Rightarrow e^{\sin^{-1} x}.\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}, x \in (-1,1)$$

3. Differentiating the following wrt $x : e^{x^3}$

Solution:

Let $y = e^{x^3}$

By using the quotient rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx} = \left(e^{x^3}\right) = e^{x^3} . \frac{d}{dx}\left(x^3\right) = e^{x^3} .3x^2 = 3x^2 e^{x^3}$$

4. Differentiating the following w.r.t $x : \sin\left(\tan^{-1} e^{-x}\right)$

Solution:

Let $y = \sin\left(\tan^{-1} e^{-x}\right)$

By using the chain rule, we obtain

$$\frac{dy}{dx} : \frac{d}{dx}\left[\sin\left(\tan^{-1} e^{-x}\right)\right]$$

$$= \cos\left(\tan^{-1} e^{-x}\right) . \frac{d}{dx}\left(\tan^{-1} e^{-x}\right)$$

$$= \cos\left(\tan^{-1} e^{-x}\right) . \frac{1}{1+\left(e^{-x}\right)^2} . \frac{d}{dx}\left(e^{-x}\right)$$

$$= \frac{\cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}} . \mathrm{e}^{-x} . \frac{d}{dx}(-x)$$

$$= \frac{e^{-x} \cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}} (-1)$$

$$= \frac{-e^{-x} \cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}}$$

5. Differentiating the following w.r.t $x : \log\left(\cos e^x\right)$

Solution:

Let $y = \log\left(\cos e^x\right)$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\cos e^x\right)\right]$$

$$= \frac{1}{\cos e^x} \cdot \frac{d}{dx}\left(\cos e^x\right)$$

$$= \frac{1}{\cos e^x} \cdot \left(-\sin e^x\right)\frac{d}{dx}\left(e^x\right)$$

$$= \frac{-\sin e^x}{\cos e^x} . e^x$$

$$= -e^x \tan e^x, e^x \neq (2n+1)\frac{\pi}{2}, n \in N$$

6. Differentiating the following w.r.t $x : e^x + e^{x^2} + \ldots\ldots\ldots\ldots + e^{x^5}$

Solution:

$$\frac{d}{dx}\left(e^x + e^{x^2} + \ldots\ldots\ldots\ldots + e^{x^5}\right)$$

$$= \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(e^{x^2}\right) + \frac{d}{dx}\left(e^{x^3}\right) + \frac{d}{dx}\left(e^{x^4}\right) + \frac{d}{dx}\left(e^{x^5}\right)$$

$$= e^x + \left[e^{x^2}\frac{d}{dx}\left(x^2\right)\right] + \left[e^{x^3}\frac{d}{dx}\left(x^3\right)\right] + \left[e^{x^4}\frac{d}{dx}\left(x^4\right)\right] + \left[e^{x^5}\frac{d}{dx}\left(x^5\right)\right]$$

$$= e^x + \left(e^{x^2} 2x\right) + \left(e^{x^3} 3x^2\right) + \left(e^{x^4} 4x^3\right) + \left(e^{x^5} 5x^4\right)$$

$$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 5x^4e^{x^5}$$

7. Differentiating the following w.r.t $x:\sqrt{e^{\sqrt{x}}}, x>0$

Solution:

Let $y=\sqrt{e^{\sqrt{x}}}$

Then, $y^2=e^{\sqrt{x}}$

By differentiating this relationship with respect to x, we obtain

$y^2=e^{\sqrt{x}}$ [by applying the chain rule]

$$\Rightarrow 2y\frac{dy}{dx}=e^{\sqrt{x}}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$\Rightarrow 2y\frac{dy}{dx}=e^{\sqrt{x}}\frac{1}{2}\frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{\sqrt{x}}}{4y\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}, x>0$$

8. Differentiating the following w.r.t $x:\log(\log x), x>1$

Solution:

Let $y=\log(\log x)$

By using the chain rule, we obtain

$$\frac{dy}{dx}=\frac{d}{dx}\left[\log(\log x)\right]$$

$$=\frac{1}{\log x}.\frac{d}{dx}(\log x)$$

$$=\frac{1}{\log x}.\frac{1}{x}$$

$$\frac{1}{x\log x},x>1$$

9. Differentiating the following w.r.t $x:\dfrac{\cos x}{\log x},x>0$

Solution:

Let $y=\dfrac{\cos x}{\log x}$

By using the quotient rule, we obtain

$$\frac{dy}{dx}=\frac{\frac{d}{dx}(\cos x)\log x-\cos x\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$\frac{-\sin x\log x-\cos x\frac{1}{x}}{(\log x)^2}$$

$$=\frac{-[x\log x.\sin x+\cos x]}{x(\log x)^2},x>0$$

10. Differentiating the following w.r.t $x:\cos\left(\log x+e^x\right),x>0$

Solution:

Let $y=\cos\left(\log x+e^x\right)$

By using the chain rule, we obtain

$$y=\cos\left(\log x+e^x\right)$$

$$\frac{dy}{dx}=-\sin\left[\log x+e^x\right]\frac{d}{dx}\left(\log x+e^x\right)$$

$$=\sin\left(\log x+e^x\right)\left[\frac{d}{dx}(\log x)+\frac{d}{dx}\left(e^x\right)\right]$$

$$= -\sin\left(\log x + e^x\right)\left(\frac{1}{x} + e^x\right)$$

$$= \left(\frac{1}{x} + e^x\right)\sin\left(\log x + e^x\right), x > 0$$