Chapter 6: Applications of Derivatives.

Exercise 6.3

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4

Solution:

$$\left.\frac{dy}{dx}\right]_{x=4} = \frac{d}{dx}\left(3x^4 - 4x\right) = 12x^3 - 4\Big]_{x=4} = 12(4)^3 - 4 = 12(64) - 4 = 764$$

2. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}, x \neq 2$ at x =10

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x-1}{x-2}\right) = \frac{(x-2)(1)-(x-1)(1)}{(x-2)^2} = \frac{x-2-x+1}{(x-2)^2} = \frac{-1}{(x-2)^2}$$

$$\therefore \left.\frac{dy}{dx}\right]_{x=10} = \left.\frac{-1}{(x-2)^2}\right]_{x=10} = \frac{-1}{(10-2)^2} = \frac{-1}{64}$$

3. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose X- coordinate is 2

Solution:

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(x^3 - x + 1\right) = 3x^2 - 1$$

$$\therefore \left.\frac{dy}{dx}\right]_{x=2} = 3x^2 - 1\Big]_{x=2} = 3(2)^2 - 1 = 12 - 1 = 11$$

4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose X- coordinate is 3.

Solution:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 - 3x + 2\right) = 3x^2 - 3$$

$$\therefore \left.\frac{dy}{dx}\right]_{x=3} = 3x^2 - 3\Big]_{x=3} = 3(3)^2 - 3 = 27 - 3 = 24$$

5. Find the slope of the normal to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

Solution:

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(a\cos^3\theta\right) = -3a\cos^2\theta\sin\theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(a\sin^3\theta\right) = 3a\sin^2\theta(\cos\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

$$\therefore \left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = -\tan\theta\Big]_{\theta=\frac{\pi}{4}} = -\tan\frac{\pi}{4} = -1$$

Slope of normal at $\theta = \frac{\pi}{4} = \frac{-1}{-1} = 1$

6. Find the slope of the normal to the curve $x = 1 - a\sin\theta$ *and* $y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$

Solution:

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(1 - a\sin\theta) = -a\cos\theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(b\cos^2\theta\right) = -2b\sin\theta\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-2b\sin\theta\cos\theta}{-a\cos\theta} = \frac{2b}{a}\sin\theta$$

$$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{2}} = \frac{2b}{a}\sin\theta\Big]_{\theta=\frac{\pi}{2}} = \frac{2b}{a}\sin\frac{\pi}{2} = \frac{2b}{a}$$

Slope of normal at $\theta = \frac{\pi}{2} = \frac{-1}{\left(\frac{2b}{a}\right)} = -\frac{a}{2b}$

7. Find the points at which tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the X-axis

Solution:

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(x^3 - 3x^2 - 9x + 7\right) = 3x^2 - 6x - 9$$

Since tangent is parallel to the X- axis, slope = 0

$$\therefore 3x^2 - 6x - 9 = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x-3)(x+1) = 0$$

$$\Rightarrow x = 3 \; or \; x = -1$$

$$x = 3, y = (3)^3 - 9(3) + 7 = 27 - 27 - 27 - 7 = -20$$

$$x = 1, y = (-1)^3 - 3(-1)^2 + 7 = -1 - 3 + 9 + 7 = 12$$

Hence, the required points are $(3,-20) \; and \; (-1,12)$

8. Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $(2,0) \; and \; (4,4)$

Solution:

Slope of chord = $\frac{4-0}{4-2} = \frac{4}{2} = 2$

Slope of tangent = $\frac{dy}{dx} = 2(x-2)$

$$\therefore 2(x-2) = 2 \Rightarrow x - 2 = 1 \Rightarrow x = 3$$

When $x=3, y=(3-2)^2=1$

Hence, the point is (3,1)

9. Find the point on the curve $y=x^3-11x+5$ at which the tangents is $y=x-11$

Solution:

Equation of tangent is y = x – 11

$\therefore$ Slope of the tangent = 1

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^3-11x+5\right)=3x^2-11$$

$$\therefore 3x^2-11=1 \Rightarrow 3x^2=12 \Rightarrow x^2=4 \Rightarrow x=\pm 2$$

$$x=2, y=(2)^3-11(2)+5=8-22+5=-9$$

$$x=-2, y=(-2)^3-11(-2)+5=-8+22+5=19$$

So, the points are $(2,-9)$ *and* $(-2,19)$

10. Find the equation of all lines having slope – 1 that are tangents to the curve

$$y=\frac{1}{x-1}, x\neq 1$$

Solution:

$$\frac{dy}{dx}=\frac{-1}{(x-1)^2}$$

$$\frac{-1}{(x-1)^2}=-1 \Rightarrow (x-1)^2 \Rightarrow x-1=\pm 1 \Rightarrow x=2,0$$

$$x=0, y=-1 \text{ and } x=2, y=1$$

$$y-(-1)=-1(x-0)$$

$$\Rightarrow y+1=-x$$

$$\Rightarrow y+x+1=0$$

$$y-1=-1(x-2)$$

$$\Rightarrow y-1=x+2$$

$$\Rightarrow y+x-3=0$$

So, the equations of the required lines are $y+x+1=0$ *and* $y+x-3=0$

11. Find the equation of all lines having slope 2 which are tangents to the curve

$y=\frac{1}{x-3}, x\neq 3$

Solution:

$$\frac{dy}{dx}=\frac{-1}{(x-3)^2}$$

$$\frac{-1}{(x-3)^2}=2\Rightarrow 2(x-3)^2=-1\Rightarrow (x-3)^2=\frac{-1}{2}$$

Which is not possible

So, there is no tangent to the curve of slope 2.

12. Find the equations of all lines having slope 0 which are tangent to the curve

$y=\frac{1}{x^2-2x+3}$

Solution:

$$\frac{dy}{dx}=\frac{-(2x-2)}{(x^2-2x+3)^2}=\frac{-2(x-1)}{(x^2-2x+3)^2}$$

$$\frac{-2(x-1)}{(x^2-2x+3)^2}=0\Rightarrow -2(x-1)=0\Rightarrow x=1$$

When $x=1, y=\frac{1}{1-2+3}=\frac{1}{2}$

$$y-\frac{1}{2}=0(x-1)$$

$$\Rightarrow y-\frac{1}{2}=0$$

$$\Rightarrow y=\frac{1}{2}$$

So, the equation of the line is $y=\frac{1}{2}$

13. Find points on the curve $\frac{x^2}{9}+\frac{y^2}{16}=1$ at which the tangents are

i. Parallel to x – axis ii. Parallel to y – axis

Solution:

$$\frac{2x}{9}+\frac{2y}{16}\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{-16x}{9y}$$

$$(i)\ \frac{dy}{dx}=\frac{-16x}{9y}=0\Rightarrow x=0$$

$$\frac{x^2}{9}+\frac{y^2}{16}=1\ for\ x=0\Rightarrow y^2=16\Rightarrow y=\pm4$$

So, the points are (0, 4) and (0, -4)

$$(ii)\ \frac{dx}{dy}=0\Rightarrow\frac{-1}{\left(\frac{-16x}{9y}\right)}=\frac{9y}{16x}=0\Rightarrow y=0$$

$$\frac{x^2}{9}+\frac{y^2}{16}=1\ for\ y=0\Rightarrow x=\pm3$$

So, the points are $(3,0)$ *and* $(-3,0)$

14. Find the equations of the tangents and normal to the given curves at the indicated points.

$I.\ \ y=x^4-6x^3+13x^2-10x+5\ at\ (0,5)$

$II.\ \ y=x^4-6x^3+13x^2-10x+5\ at\ (1,3)$

$III.\ \ y=x^3\ at\ (1,1)$

$IV.\ \ y=x^2\ at\ (0,0)$

$V.\ \ x=\cos t, y=\sin t\ at\ t=\frac{\pi}{4}$

Solution:

$$I.\ \frac{dy}{dx}=4x^3-18x^2+26x-10$$

$$\left.\frac{dy}{dx}\right]_{(0,5)}=-10$$

Slope of tangent at (0, 5) is – 10

$$y-5=-10(x-0)$$

$$\Rightarrow y-5=-10x$$

$$\Rightarrow 10x+y=5$$

Slope of normal at (0, 5) is $\frac{-1}{-10}=\frac{1}{10}$

$$y-5=\frac{1}{10}(x-0)$$

$$\Rightarrow 10y-50=x$$

$$\Rightarrow x-10y+50=0$$

$$II.\frac{dy}{dx}=4x^3-18x^2+26x-10$$

$$\left.\frac{dy}{dx}\right]_{(1,3)}=4-18+26-10=2$$

Slope of tangent at (1, 3) is 2

$$y-3=2(x-1)$$

$$\Rightarrow y-3=2x-2$$

$$\Rightarrow y=2x+1$$

Slope of normal at (1, 3) is $-\frac{1}{2}$

$$y-3=\frac{1}{2}(x-1)$$

$\Rightarrow 2y-6=x+1$

$\Rightarrow x+2y-7=0$

$III.\ \frac{dy}{dx}=3x^2$

$\left.\frac{dy}{dx}\right]_{(1,1)}=3(1)^2=3$

Slope of tangent at (1, 1) is 3

$y-1=3(x-1)$

$\Rightarrow y=3x-2$

Slope of normal at (1, 1) is $-\frac{1}{3}$

$y-1=\frac{-1}{3}(x-1)$

$\Rightarrow 3y-3=-x+1$

$\Rightarrow x+3y-4=0$

$IV.\ \frac{dy}{dx}=2x$

$\left.\frac{dy}{dx}\right]_{(0,0)}=0$

Slope of tangent at (0, 0) is 0

$y-0=0(x-0)$

$\Rightarrow y=0$

Slope of normal at (0, 0) is $-\frac{1}{0}$, which is undefined

$\therefore x=0$

$V.\ x=\cos t \quad y=\sin t$

$$\therefore \frac{dx}{dt}=-\sin t \quad \frac{dy}{dt}=\cos t$$

$$\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{\cos t}{-\sin t}=-\cot t$$

$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}}=-\cot t=-1$$

Slope of tangent at $t=\frac{\pi}{4}$ is -1

$$t=\frac{\pi}{4}, x=\frac{1}{\sqrt{2}}, y=\frac{1}{\sqrt{2}}$$

$$y-\frac{1}{\sqrt{2}}=-1\left(x-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow x+y-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=0$$

$$\Rightarrow x+y-\sqrt{2}=0$$

Slope of normal at $t=\frac{\pi}{4}$ is $\frac{-1}{-1}=1$

$$y-\frac{1}{\sqrt{2}}=1\left(x-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow x=y$$

15. Find the equation of the tangent line to the curve $y=x^2-2x+7$ which is

a. Parallel to the line $2x-y+9=0$

b. Perpendicular to the line $5y-15x=13$

Solution:

$$\frac{dy}{dx}=2x-2$$

$$(a)\ 2x-y+9=0\Rightarrow y=2x+9$$

Slope of line = 2

$$\therefore 2=2x-2\Rightarrow 2x=4\Rightarrow x=2$$

$$x=2\Rightarrow y=4-4+7=7$$

Equation of tangent through (2, 7) is

$$y-7=2(x-2)\Rightarrow y-2x-3=0$$

$$(b)\ 5y-15x=13\Rightarrow y=3x+\frac{13}{5}$$

Slope of line = 3

$$\therefore 2x-2=\frac{-1}{3}$$

$$\Rightarrow 2x=\frac{-1}{3}+2$$

$$\Rightarrow 2x=\frac{5}{3}$$

$$\Rightarrow x=\frac{5}{6}$$

$$\Rightarrow y=\frac{25}{36}+\frac{10}{6}+7=\frac{25-60+252}{36}=\frac{217}{36}$$

Equation of tangent through $\left(\frac{5}{6},\frac{217}{36}\right)$ is

$$y-\frac{217}{36}=\frac{1}{3}\left(x-\frac{5}{6}\right)$$

$$\Rightarrow \frac{36y-217}{36}=\frac{-1}{18}(6x-5)$$

$\Rightarrow 36y - 217 = -2(6x - 5)$

$\Rightarrow 36y - 217 = -12x + 10$

$\Rightarrow 36y + 12x - 227 = 0$

16. Show that the tangents to the curve $y = 7x^3 + 11$ at the points where $x = 2$ *and* $x = -2$ are parallel

Solution:

$\therefore \frac{dy}{dx} = 21x^2$

$\left.\frac{dy}{dx}\right]_{x=2} = 21(2)^2 = 84$

$\left.\frac{dy}{dx}\right]_{x=-2} = 21(-2)^2 = 84$

Clearly, the tangents are parallel

17. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point

Solution:

$\therefore \frac{dy}{dx} = 3x^2$

According to the equation, $y = \frac{dy}{dx} = 3x^2$

Also, $y = x^3$

$\therefore 3x^2 = x^3$

$x^2(x-3) = 0$

$x = 0, x = 3$

$x = 0, y = 0$ *and* $x = 3, y = 3(3)^2 = 27$

So the points are (0, 0) and (3, 27)

18. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangents passes through the origin

Solution:

$$\frac{dy}{dx} = 12x^2 - 10x^4$$

Equation of tangent through (X, Y) is

$$Y - y = \left(12x^2 - 10x^4\right)\left(X - x\right)$$

For passing through origin, X = 0, Y= 0

$$-y = \left(12x^2 - 10x^4\right)\left(-x\right)$$

$$y = 12x^3 - 10x^5$$

Also, $y = 4x^3 - 2x^5$

$$\therefore 12x^3 - 10x^5 = 4x^3 - 2x^5$$

$$\Rightarrow 8x^5 - 8x^3 = 0$$

$$\Rightarrow x^5 - x^3 = 0$$

$$\Rightarrow x^3\left(x^2 - 1\right) = 0$$

$$\Rightarrow x =, \pm 1$$

$$x = 0, y = 4(0)^3 - 2(0)^5 = 0$$

$$x = 1, y = 4(1)^3 - 2(1)^5 = 2$$

$$x = -1, y = 4(-1)^3 - 2(-1)^5 = -2$$

So, the points are (0, 0), (1, 2) and $(-1, -2)$

19. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x- axis

Solution:

$$2x + 2y\frac{dy}{dx} - 2 = 0$$

$$\Rightarrow y\frac{dy}{dx} = 1 - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$$

For parallel to X axis,

$$\therefore \frac{1-x}{y} = 0 \Rightarrow 1 - x = 0 \Rightarrow x = 1$$

$$x^2 + y^2 - 2x - 3 = 0$$

$$\Rightarrow y^2 = 4, y = \pm 2$$

So, the points are (1, 2) and (1, -2)

20. Find the equation of the normal at the point $\left(am^2, am^3\right)$ for the curve $ay^2 = x^3$

Solution:

$$2ay\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

Slope of tangent at $\left(am^2, am^3\right)$ is

$$\left.\frac{dy}{dx}\right]_{\left(am^2, am^3\right)} = \frac{3\left(am^2\right)^2}{2a\left(am^3\right)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}$$

Slope of normal $= \frac{-2}{3m}$

$$y - am^3 = \frac{-2}{3m}\left(x - am^2\right)$$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$

$$\Rightarrow 2x + 3my - am^2\left(2 + 3m^2\right) = 0$$

21. Find the equation of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

Solution:

$$\frac{dy}{dx} = 3x^2 + 2$$

Slope of the normal $= \frac{-1}{3x^2 + 2}$

$$x + 14y + 4 = 0 \Rightarrow y = -\frac{1}{14}x - \frac{4}{14}$$

$$\therefore \frac{-1}{3x^2 + 2} = \frac{-1}{14}$$

$$\Rightarrow 3x^2 + 2 = 14$$

$$\Rightarrow 3x^2 = 12$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$$x = 2, y = 8 + 4 + 6 = 18$$

$$x = -2, y = -8 - 4 + 6 = -6$$

Equation of normal through (2, 18) is

$$y - 18 = \frac{-1}{14}(x - 2)$$

$\Rightarrow 14y-252=x+2$

$\Rightarrow x+14y-254=0$

Equation of normal through $(-2,-6)$ is

$y-(-6)=\frac{-1}{14}[x-(-2)]$

$\Rightarrow y+6=\frac{-1}{14}(x+2)$

$\Rightarrow 14y+84=-x-2$

$\Rightarrow x+14y+86=0$

22. Find the equations of the tangent and normal to the parabola $y^2=4ax$ at the point $(at^2,2at)$

Solution:

$2y\frac{dy}{dx}=4a$

$\Rightarrow \frac{dy}{dx}=\frac{2a}{y}$

$\left.\frac{dy}{dx}\right]_{(at^2,2at)}=\frac{2a}{2at}=\frac{1}{t}$

Slope of tangent $=\frac{1}{t}$

Equation of tangent is

$y-2at=\frac{1}{t}(x-at^2)$

$\Rightarrow ty-2at^2=x-at^2$

$\Rightarrow ty=x+at^2$

Slope of normal $= -\frac{1}{\left(\frac{1}{t}\right)} = -t$

Equation of normal is

$$y - 2at = -t\left(x - at^2\right)$$

$$\Rightarrow y - 2at = -tx + at^3$$

$$\Rightarrow y = -tx + 2at + at^3$$

23. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$. [Hint: Tow curves intersect at right angle if the tangents to the curves at the point of intersection are perpendicular to each other]

Solution:

The curves are $x = y^2 \text{ and } xy = k$

Putting $x = y^2 \text{ and } xy = k,$

$$y^3 = k \Rightarrow y = k^{\frac{1}{3}}$$

$$\therefore x = k^{\frac{2}{3}}$$

So, the point of intersection is $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)$

Differentiating $x = y^2$

$$1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

So, slope of tangent to $x = y^2 at \left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)$ is

$$\left.\frac{dx}{dy}\right]_{\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right)} = \frac{1}{2k^{\frac{1}{3}}}$$

Differentiating xy = k,

$$x\frac{dy}{dx}+y=0 \Rightarrow \frac{dy}{dx}=\frac{-y}{x}$$

Slope of tangent to $xy=k\ at\left(k^{\frac{2}{3}},k^{\frac{1}{3}}\right)$ is

$$\left.\frac{dx}{dy}\right]_{\left(k^{\frac{2}{3}},k^{\frac{1}{3}}\right)}=\left.\frac{-y}{x}\right]_{\left(k^{\frac{2}{3}},k^{\frac{1}{3}}\right)}=-\frac{k^{\frac{1}{3}}}{k^{\frac{2}{3}}}=\frac{-1}{k^{\frac{1}{3}}}$$

$\left(\frac{1}{2k^{\frac{1}{3}}}\right)\left(\frac{-1}{k^{\frac{1}{3}}}\right)=-1$ for perpendicularity condition

$$\Rightarrow 2k^{\frac{2}{3}}=1$$

$$\Rightarrow \left(2k^{\frac{2}{3}}\right)^3=(1)^3$$

$$\Rightarrow 8k^2=1$$

So, the curves intersect at right angles if $8k^2=1$

24. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (x_0,y_0)

Solution:

$$\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx}=\frac{2x}{a^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{b^2x}{a^2y}$$

Slope of tangent at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0,y_0)} = \frac{b^2x_0}{a^2y_0}$

Equation of tangent at (x_0, y_0) is

$$y - y_0 = \frac{b^2x_0}{a^2y_0}(x - x_0)$$

$$\Rightarrow a^2yy_0 - a^2y_0^2 = b^2xx_0 - b^2x_0^2$$

$$\Rightarrow b^2xx_0 - a^2yy_0 - b^2x_0^2 + a^2y_0^2 = 0$$

$$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} - \left(\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2}\right) = 0$$

$$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} - 1 = 0$$

$$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$$

Slope of normal at $(x_0, y_0) = \frac{-a^2y_0}{b^2x_0}$

Equation of normal at (x_0, y_0) is

$$y - y_0 = \frac{-a^2y_0}{b^2x_0}(x - x_0)$$

$$\Rightarrow \frac{y - y_0}{a^2y_0} = \frac{-(x - x_0)}{b^2x_0}$$

$$\Rightarrow \frac{y - y_0}{a^2y_0} - \frac{(x - x_0)}{b^2x_0} = 0$$

25. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$

Solution:

Slope of tangent at (x, y) is

$$\frac{dy}{dx}=\frac{3}{2\sqrt{3x-2}}$$

The given line is $4x-2y+5=0$

$$4x-2y+5=0 \Rightarrow y=2x+\frac{5}{2}$$

Slope of line = 2

$$\frac{3}{2\sqrt{3x-2}}=2$$

$$\Rightarrow \sqrt{3x-2}=\frac{3}{4}$$

$$\Rightarrow 3x-2=\frac{9}{16}$$

$$\Rightarrow 3x=\frac{9}{16}+2=\frac{41}{16}$$

$$\Rightarrow x=\frac{41}{48}$$

$$x=\frac{41}{48}, y=\sqrt{3\left(\frac{41}{48}\right)-2}=\sqrt{\frac{41}{16}-2}=\sqrt{\frac{41-32}{16}}=\sqrt{\frac{9}{16}}=\frac{3}{4}$$

Equation of tangent through $\left(\frac{41}{48}, \frac{3}{4}\right)$ is

$$y-\frac{3}{4}=2\left(x-\frac{41}{48}\right)$$

$$\Rightarrow \frac{4y-3}{4}=2\left(\frac{48x-41}{48}\right)$$

$$\Rightarrow 4y-3=\left(\frac{48x-41}{6}\right)$$

$\Rightarrow 24y-18=48x-41$

$\Rightarrow 48x-24y=23$

26. The slope of the normal to the curve $y=2x^2+3\sin x$ at x= 0 is

$(A)=3,(B)=\frac{1}{3},(C)=-3,(D)=-\frac{1}{3}$

Solution:

$$\left.\frac{dy}{dx}\right]_{x=0}=4x+3\cos x\Big]_{x=0}=0+3\cos 0=3$$

Slope of normal $=\frac{-1}{3}$

The correct answer is D.

27. The line $y=x+1$ is a tangent to the curve $y^2=4x$ at the point

$(A)(1,2),(B)(2,1),(C)(1,-2),(D)(-1,2)$

Solution:

$$2y\frac{dy}{dx}=4\Rightarrow\frac{dy}{dx}=\frac{2}{y}$$

Given line is $y=x+1$

Slope of line = 1

$$\frac{2}{y}=1$$

$\Rightarrow y=2$

$y=x+1\Rightarrow x=y-1\Rightarrow x=2-1=1$

So, line $y=x+1$ is tangent to curve at point (1, 2)

The correct answer if A