Chapter 6: Applications of Derivatives.

Exercise 6.4

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal

$$(i)\sqrt{25.3},(ii)\sqrt{49.5},(iii)\sqrt{0.6},(iv)(0.009)^{\frac{1}{3}},(v)(0.999)^{\frac{1}{10}},(vi)(15)^{\frac{1}{4}},(vii)(26)^{\frac{1}{3}}$$

$$(viii)(255)^{\frac{1}{4}},(ix)(82)^{\frac{1}{4}},(x)(401)^{\frac{1}{2}},(xi)(0.0037)^{\frac{1}{2}},(xii)(26.57)^{\frac{1}{3}},(xiii)(81.5)^{\frac{1}{4}},(xiv)(3.968)^{\frac{3}{2}}$$

$$(xv)(32.15)^{\frac{1}{5}}$$

Solution:

$$(i)\sqrt{25.3}$$

$$y=\sqrt{x}.\ Let\ x=25\ and\ \Delta x=0.3$$

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{25.3}-\sqrt{25}=\sqrt{25.3}-5$$

$$\Rightarrow\sqrt{25.3}=\Delta y+5$$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(0.3)$$

$$=\frac{2}{2\sqrt{25}}(0.3)=0.03$$

$$\sqrt{25.3}\approx 0.03+5=5.03$$

$$(ii)\sqrt{49.5}$$

$$y=\sqrt{x}.\ Let\ x=49\ and\ \Delta x=0.5$$

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{49.5}-\sqrt{49}=\sqrt{49.5}-7$$

$$\Rightarrow\sqrt{49.5}=7+\Delta y$$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(0.5)$$

$$=\frac{1}{2\sqrt{49}}(0.5)=\frac{1}{14}(0.5)=0.035$$

$$\sqrt{49.5}\approx 7+0.035=7.035$$

$(iii)\sqrt{0.6}$

$y=\sqrt{x}.Let\ x=1\ and\ \Delta x=-0.4$

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{0.6}-1$$

$$\Rightarrow\sqrt{0.6}=1+\Delta y$$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(\Delta x)$$

$$=\frac{1}{2}(-0.4)=-0.2$$

$$\sqrt{0.6}\approx 1+(-0.2)=1-0.2=0.8$$

$(iv)(0.009)^{\frac{1}{3}}$

$y=x^{\frac{1}{3}}.Let x=0.008\ and\ \Delta x=0.001$

$$\Delta y=(x+\Delta x)^{\frac{1}{3}}-(x)^{\frac{1}{3}}=(0.009)^{\frac{1}{3}}-(0.008)^{\frac{1}{3}}=(0.009)^{\frac{1}{3}}-0.2$$

$$\Rightarrow(0.009)^{\frac{1}{3}}=0.2+\Delta y$$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{3(x)^{\frac{2}{3}}}(\Delta x)$$

$$=\frac{1}{3\times 0.04}(0.001)=\frac{0.001}{0.12}=0.008$$

$(0.009)^{\frac{1}{3}}$ *is* $0.2+0.008=0.208$

$(v)(0.999)^{\frac{1}{10}}$

$y=(x)^{\frac{1}{10}}.Let\ x=1\ and\ \Delta x=-0.001$

$\Delta y=(x+\Delta x)^{\frac{1}{10}}-(x)^{\frac{1}{10}}=(0.999)^{\frac{1}{10}}-1$

$\Rightarrow(0.999)^{\frac{1}{10}}=1+\Delta y$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{10(x)^{\frac{9}{10}}}(\Delta x)$$

$$=\frac{1}{10}(-0.001)=-0.0001$$

$(0.999)^{\frac{1}{10}}$ *is* $1+(-0.0001)=0.9999$

$(vi)(15)^{\frac{1}{4}}$

$y=x^{\frac{1}{4}}.Let\ x=16\ and\ \Delta x=-1$

$\Delta y=(x+\Delta x)^{\frac{1}{4}}-x^{\frac{1}{4}}=(15)^{\frac{1}{4}}-(16)^{\frac{1}{4}}=(15)^{\frac{1}{4}}-2$

$\Rightarrow(15)^{\frac{1}{4}}=2+\Delta y$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$=\frac{1}{4(16)^{\frac{3}{4}}}(-1)=\frac{-1}{4\times 8}=\frac{-1}{32}=-0.03125$$

$(15)^{\frac{1}{4}}$ *is* $2+(-0.03125)=1.96875$

$(vii)(26)^{\frac{1}{3}}$

$y=(x)^{\frac{1}{3}}$ .Let $x=27$ and $\Delta x=-1$

$\Delta y=(x+\Delta x)^{\frac{1}{3}}-(x)^{\frac{1}{3}}=(26)^{\frac{1}{3}}-(27)^{\frac{1}{3}}=(26)^{\frac{1}{3}}-3$

$\Rightarrow (26)^{\frac{1}{3}}=3+\Delta y$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{3(x)^{\frac{2}{3}}}(\Delta x)$$

$$=\frac{1}{3(27)^{\frac{2}{3}}}(-1)=\frac{-1}{27}=-0.0\overline{370}$$

$(26)^{\frac{1}{3}}$ is $3+(-0.0370)=2.9629$

$(viii)(255)^{\frac{1}{4}}$

$y=(x)^{\frac{1}{4}}$ .Let $x=256$ and $\Delta x=-1$

$\Delta y=(x+\Delta x)^{\frac{1}{4}}-(x)^{\frac{1}{4}}=(255)^{\frac{1}{4}}-(256)^{\frac{1}{4}}=(255)^{\frac{1}{4}}-4$

$\Rightarrow (255)^{\frac{1}{4}}=4+\Delta y$

$$dy\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$=\frac{1}{4(256)^{\frac{3}{4}}}(-1)=\frac{-1}{4\times 4^{3}}=-0.0039$$

$(255)^{\frac{1}{4}}$ is $4+(-0.0039)=3.9961$

$(ix)\,(82)^{\frac{1}{4}}$

$y = x^{\frac{1}{4}}$. Let $x = 81$ and $\Delta x = 1$

$$\Delta y = (x+\Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}} = (82)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (82)^{\frac{1}{4}} - 3$$

$$\Rightarrow (82)^{\frac{1}{4}} = \Delta y + 3$$

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$= \frac{1}{4(81)^{\frac{3}{4}}}(1) = \frac{1}{4(3)^3} = \frac{1}{108} = 0.009$$

$(82)^{\frac{1}{4}}$ is $3 + 0.009 = 3.009$

$(x)(401)^{\frac{1}{2}}$

$y = x^{\frac{1}{2}}$. Let $x = 400$ and $\Delta x = 1$

$$\Delta y = \sqrt{x+\Delta x} - \sqrt{x} = \sqrt{401} - \sqrt{400} = \sqrt{401} - 20$$

$$\Rightarrow \sqrt{401} = 20 + \Delta y$$

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x)$$

$$= \frac{1}{2\times 20}(1) = \frac{1}{40} = 0.025$$

$\sqrt{401}$ is $20 + 0.025 = 20.025$

$(xi)(0.037)^{\frac{1}{2}}$

$y = x^{\frac{1}{2}}$. Let $x = 0.0036$ and $\Delta x = 0.0001$

$$\Delta y = (x+\Delta x)^{\frac{1}{2}} - (x)^{\frac{1}{2}} = (0.0037)^{\frac{1}{2}} - (0.0036)^{\frac{1}{2}} = (0.0037)^{\frac{1}{2}} - 0.06$$

$$\Rightarrow (0.0037)^{\frac{1}{2}} = 0.06 + \Delta y$$

$$dy\left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(\Delta x)$$

$$= \frac{1}{2\times 0.06}(0.0001)$$

$$= \frac{0.0001}{0.12} = 0.00083$$

$$(0.0037)^{\frac{1}{2}}\ is\ 0.06 + 0.00083 = 0.6083$$

$$(xii)\ (26.57)^{\frac{1}{3}}$$

$$y = x^{\frac{1}{3}}.Let\ x = 27\ and\ \Delta x = -0.43$$

$$\Delta y = (x+\Delta x)^{\frac{1}{3}} - x^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - 3$$

$$\Rightarrow (26.57)^{\frac{1}{3}} = 3 + \Delta y$$

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{3(x)^{\frac{2}{3}}}(\Delta x)$$

$$= \frac{1}{3(9)}(-0.43)$$

$$= \frac{-0.43}{27} = -0.015$$

$$(26.57)^{\frac{1}{3}}\ is\ 3 + (-0.015) = 2.984$$

$$(xiii)\ (81.5)^{\frac{1}{4}}$$

$$y = x^{\frac{1}{4}}.\ Let x = 81\ and\ \Delta x = 0.5$$

$$\Delta y=(x+\Delta x)^{\frac{1}{4}}-(x)^{\frac{1}{4}}=(81.5)^{\frac{1}{4}}-(81)^{\frac{1}{4}}=(81.5)^{\frac{1}{4}}-3$$

$$\Rightarrow (81.5)^{\frac{1}{4}}=3+\Delta y$$

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{4(x)^{\frac{3}{4}}}(\Delta x)$$

$$=\frac{1}{4(3)^{3}}(0.5)=\frac{0.5}{108}=0.0046$$

$$(81.5)^{\frac{1}{4}}\ is\, 3+0.0046=3.0046$$

$$(xiv)\,(3.968)^{\frac{3}{2}}$$

$$y=x^{\frac{3}{2}}.\ Let\ x=4\ and\ \Delta x=-0.032$$

$$\Delta y=(x+\Delta x)^{\frac{3}{2}}-x^{\frac{2}{3}}=(3.968)^{\frac{3}{2}}-(4)^{\frac{2}{3}}=(3.968)^{\frac{3}{2}}-8$$

$$\Rightarrow (3.968)^{\frac{3}{2}}=8+\Delta y$$

$$dy\left(\frac{dy}{dx}\right)\Delta x=\frac{3}{2}(x)^{\frac{1}{2}}(\Delta x)$$

$$=\frac{3}{2}(2)(-0.032)$$

$$=-0.096$$

$$(3.968)^{\frac{3}{2}}\ is\ 8+(-0.096)=7.904$$

$$(xv)(32.15)^{\frac{1}{15}}$$

$$y=x^{\frac{1}{15}}.Let\ x=32\ and\ \Delta x=-0.15$$

$$\Delta y=(x+\Delta x)^{\frac{1}{5}}-x^{\frac{1}{5}}=(32.15)^{\frac{1}{5}}-(32)^{\frac{1}{5}}=(32.15)^{\frac{1}{5}}-2$$

$$\Rightarrow (32.15)^{\frac{1}{15}} = 2 + \Delta y$$

$$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{5(x)^{\frac{4}{5}}}.(\Delta x)$$

$$= \frac{1}{5 \times (2)^4}(0.15)$$

$$= \frac{0.15}{80} = 0.00187$$

$$(32.15)^{\frac{1}{15}}\ is 2 + 0.00187 = 2.00187$$

2. Find the approximate value of (2.01), where $f(x) = 4x^2 + 5x + 2$

Solution:

$$x = 2\ and \Delta x = 0.01$$

$$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$$

$$\Delta y = f(x + \Delta x) - f(x)$$

$$f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x)\Delta x$$

$$\Rightarrow f(2.01) \approx (4x^2 + 5x + 2) + (8x + 5)\Delta x$$

$$= \left[4(2)^2 + 5(2) + 2\right] + \left[8(2) + 5\right](0.01)$$

$$= (16 + 10 + 2) + (16 + 5)(0.01)$$

$$= 28 + (21)(0.01)$$

$$= 28 + 0.21$$

$$= 28.21$$

$f(2.01)$ *is* 28.21

3. Find the approximate value of $f(5.001)$, where $f(x)=x^3-4x^2+15$

Solution:

$x=5$ *and* $\Delta x=0.001$

$f(5.001)=f(x+\Delta x)=(x+\Delta x)^3-7(x+\Delta x)^2+15$

$\Delta y=f(x+\Delta x)-f(x)$

$\therefore f(x+\Delta x)=f(x)+\Delta y$

$\approx f(x)+f'(x)\Delta x$

$\Rightarrow f(5.001)\approx(x^3-7x^2+15)+(3x^2-14x)\Delta x$

$=\left[(5)^3-7(5)^2+15\right]+\left[3(5)^2-14(5)\right](0.001)$

$=(125-175+15)+(75-70)(0.001)$

$=-35+(5)(0.001)$

$=-35+0.005$

$=-34.995$

$f(5.001)$ *is* -34.995

4. Find the approximate change in the volume V of a cube side x meters caused by increasing side by 1%

Solution:

$V=x^3$

$\therefore dV=\left(\frac{dV}{dx}\right)\Delta x$

$$=\left(3x^2\right)\Delta x$$

$$=\left(3x^2\right)\left(0.01x\right)$$

$$=0.03x^3$$

So, the approx change in the volume of the cube is $0.03x^3m^3$

5. Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%

Solution:

$$S=6x^2$$

$$\therefore \frac{dS}{dx}=\left(\frac{dS}{dx}\right)\Delta x$$

$$=\left(12x\right)\Delta x$$

$$=\left(12x\right)\left(0.01x\right)$$

$$=0.12x^2$$

So, the approx change in volume of cube is $0.12x^2m^2$

6. If the radius of a sphere is measured as 7m with an error of 0.02m, then find the approximate error in calculating its volume

Solution:

$$r=7m \text{ and } \Delta r=0.02m$$

$$V=\frac{4}{3}\pi r^3$$

$$\therefore \frac{dV}{dr}=4\pi r^2$$

$$\therefore dV=\left(\frac{dV}{dr}\right)\Delta r$$

$$=\left(4\pi r^{2}\right)\Delta r$$

$$=4\pi(7)^{2}(0.02)m^{3}=3.92\pi m^{3}$$

So, the approx error in calculating the volume is $3.92\pi m^{3}$

7. If the radius of a sphere is measured as 9m with an error of 0.03m, then find the approximate error in calculating in surface area

Solution:

$$r=9m \text{ and } \Delta r=0.03m$$

$$\therefore \frac{dS}{dr}=\frac{d}{dr}\left(4\pi r^{2}\right)=8\pi r$$

$$\therefore dS=\left(\frac{dS}{dr}\right)\Delta r$$

$$=(8\pi r)\Delta r$$

$$=8\pi(9)(0.03)m^{2}$$

$$=2.16\pi m^{2}$$

So, the approx error in calculating the surface area is $2.16\pi m^{2}$

8. If $f(x)=3x^{2}+15x+5$, then the approximate value of (3.02) is

$(A)47.66,\ (B)57.66,\ (C)67.66\ \ (D)77.66$

Solution:

$$f(3.02)=f(x+\Delta x)=3(x+\Delta x)^{2}+15(x+\Delta x)+5$$

$$\Delta y=f(x+\Delta x)-f(x)$$

$$\Rightarrow f(x+\Delta x)=f(x)+\Delta y$$

$$\approx f(x)+f'(x)\Delta x$$

$$\Rightarrow f(3.02) \approx (3x^2 + 15x + 5) + (6x + 15)\Delta x$$

$$= [3(3^2) + 15(3) + 5] + [6(3) + 15](0.02)$$

$$= (27 + 45 + 5) + (18 + 15)(0.02)$$

$$= 77 + (33)(0.02)$$

$$= 77 + 0.66$$

$$= 77.66$$

So, approx value of (3.02) is 77.66

The correct answer is D

9. The approximate change in the volume of a cube of side x meters caused by increasing the side by 3% is

$(A) 0.06x^3m^3$ $(B) 0.6x^3m^3$ $(C) 0.09x^3m^3$ $(D) 0.9x^3m^3$

Solution:

$$V = x^3$$

$$\therefore dV = \left(\frac{dV}{dx}\right)\Delta x$$

$$= (3x^2)\Delta x$$

$$= (3x^2)(0.03x)$$

$$= 0.09x^3m^3$$

So, the approx change in the volume of the cube is $0.09x^3m^3$

The correct answer is C