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Chapter: 7. Integrals

Exercise 7.1

1. Find an antiderivative of the function $f(x)=\sin 2x$ by inspection method.

Solution:

Recall the functions whose derivative is $\sin 2x$

$$\frac{d}{dx}(\cos 2x)=-2\sin 2x \qquad \bullet\frac{d}{dx}(\cos ax)=-a\sin ax$$

It implies that

$$\sin 2x=-\frac{1}{2}\frac{d}{dx}(\cos 2x)$$
$$=\frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, antiderivative of $f(x)=\sin 2x$ is $-\frac{1}{2}\cos 2x$

2. Find the antiderivative of the function $f(x)=\cos 3x$ by inspection method.

Solution:

Recall the functions whose derivative is $\cos 3x$

$$\frac{d}{dx}(\sin 3x)=3\cos 3x \qquad \bullet\frac{d}{dx}(\sin ax)=a\cos ax$$

It implies that

$$\cos 3x=\frac{1}{3}\frac{d}{dx}(\sin 3x)$$
$$=\frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, antiderivative of $f(x)=\cos 3x$ is $\frac{1}{3}\sin 3x$

3. Find the antiderivative of the function $f(x)=e^{2x}$ by inspection method.

Solution:

Recall the functions whose derivative is e^{2x}

$$\frac{d}{dx}\left(e^{2x}\right)=2e^{2x} \qquad \bullet\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

It implies that

$$\begin{aligned} e^{2x} &= \frac{1}{2}\frac{d}{dx}\left(e^{2x}\right) \\ &= \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right) \end{aligned}$$

Therefore, antiderivative of $f(x)=e^{2x}$ is $\frac{1}{2}e^{2x}$

4. Find the antiderivative of the function $f(x)=(ax+b)^2$ by inspection method.

Solution:

Recall the functions whose derivative is $f(x)=(ax+b)^2$

$$\frac{d}{dx}\left[(ax+b)^3\right]=3(ax+b)^2 \qquad \bullet\frac{d}{dx}(ax+b)^n=n(ax+b)^{n-1}$$

It implies that

$$\begin{aligned} (ax+b)^2 &= \frac{1}{3}\frac{d}{dx}(ax+b)^3 \\ &= \frac{d}{dx}\left(\frac{1}{3}(ax+b)^3\right) \end{aligned}$$

Therefore, antiderivative of $f(x)=(ax+b)^2$ is $\frac{1}{3}(ax+b)^3$

5. Find the antiderivative of the function $\sin 2x - 4e^{3x}$ by inspection method.

Solution:

Recall the functions whose derivative is $f(x) = \sin 2x$

$$\frac{d}{dx}\cos 2x = -2\sin 2x \qquad \bullet \frac{d}{dx}\cos ax = -a\sin ax$$

It implies that

$$\sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$
$$= \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Recall the functions whose derivative is $f(x) = 4e^{3x}$

$$\frac{d}{dx}e^{3x} = 3e^{3x} \qquad \bullet \frac{d}{dx}e^{ax} = ae^{ax}$$

It implies that

$$e^{3x} = \frac{1}{3}\frac{d}{dx}\left(e^{3x}\right)$$
$$4e^{3x} = \frac{4}{3}\frac{d}{dx}\left(e^{3x}\right) \qquad \bullet \text{Multiply 4 on both sides}$$
$$= \frac{d}{dx}\left(\frac{4}{3}e^{3x}\right)$$

Therefore, antiderivative of $\sin 2x - 4e^{3x}$ is $-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}$

6. Find $\int\left(4e^{3x} + 1\right)dx$

Solution:

Consider the given integral $\int\left(4e^{3x} + 1\right)dx$

$$\int\left(4e^{3x}+1\right)dx=4\int e^{3x}dx+\int dx$$

$$=4\left(\frac{1}{3}e^{3x}\right)+x+C \qquad \bullet\int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$=\frac{4}{3}e^{3x}+x+C$$

Therefore, $\int\left(4e^{3x}+1\right)dx=\frac{4}{3}e^{3x}+x+C$

7. Find $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Solution:

Consider the given integral $\int x^2\left(1-\frac{1}{x^2}\right)dx$

$$\int x^2\left(1-\frac{1}{x^2}\right)dx=\int x^2dx-\int dx$$

$$=\frac{x^3}{3}-x+C \qquad \bullet\int x^n dx=\frac{x^{n+1}}{n+1}+c$$

Therefore, $\int x^2\left(1-\frac{1}{x^2}\right)dx=\frac{x^3}{3}-x+C$

8. Find $\int\left(ax^2+bx+c\right)dx$

Solution:

Consider the given integral $\int\left(ax^2+bx+c\right)dx$

$$\int\left(ax^2+bx+c\right)dx=a\int x^2dx+b\int xdx+c\int dx$$

$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+D$$

$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+D$$

Therefore, $\int\left(ax^2+bx+c\right)dx=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+D$

9. Find $\int\left(2x^{2}+e^{x}\right)dx$

Solution:

The given integral is $\int\left(2x^{2}+e^{x}\right)dx$

$$\int\left(2x^{2}+e^{x}\right)dx=2\int x^{2}dx+\int e^{x}dx \qquad \bullet\int x^{n}dx=\frac{x^{n+1}}{n+1}+c$$

$$=2\frac{x^{3}}{3}+e^{x}+C$$

Therefore, $\int\left(2x^{2}+e^{x}\right)dx=\frac{2x^{3}}{3}+e^{x}+C$

10. Find $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^{2}dx$

Solution:

The given integral is $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^{2}dx$

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^{2}dx=\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int xdx+\int\frac{1}{x}dx-\int 2dx$$

$$=\frac{x^{2}}{2}+\ln x-2x+C \qquad \bullet\int\frac{1}{x}dx=\ln x+c$$

Therefore, $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^{2}dx=\frac{x^{2}}{2}+\ln x-2x+C$

11. Find $\int\frac{x^{3}+5x^{2}-4}{x^{2}}dx$

Solution:

Consider the integral $\int \frac{x^3+5x^2-4}{x^2}dx$

$$\int \frac{x^3+5x^2-4}{x^2}dx = \int \frac{x^3+5x^2-4}{x^2}dx$$
$$= \int x+5-\frac{4}{x^2}dx$$
$$= \frac{x^2}{2}+5x+\frac{8}{x}+C$$

Therefore, $\int \frac{x^3+5x^2-4}{x^2}dx = \frac{x^2}{2}+5x+\frac{8}{x}+C$

12. Find $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

Solution:

The given integral is $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx = \int \frac{x^3}{\sqrt{x}}+\frac{3x}{\sqrt{x}}+\frac{4}{\sqrt{x}}dx$$
$$= \int x^{\frac{5}{2}}dx+3\int x^{\frac{1}{2}}dx+4\int x^{\frac{1}{2}}dx$$
$$= \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+3\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+4\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$$
$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}+3\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+4\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$$
$$= \frac{2}{7}x^{\frac{7}{2}}+3\left(\frac{2}{3}\right)x^{\frac{5}{2}}+4\left(\frac{2}{1}\right)x^{\frac{1}{2}}+C$$
$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{5}{2}}+8x^{\frac{1}{2}}+C$$

Therefore, $\int \frac{x^3+3x+4}{\sqrt{x}}dx = \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{5}{2}}+8\sqrt{x}+C$

13. Find , $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

Solution:

The given integral is $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

$$\begin{aligned}\int \frac{x^3 - x^2 + x - 1}{x - 1} dx &= \int \frac{x^2(x-1) + (x-1)}{x - 1} dx \\ &= \int \frac{(x-1)(x^2+1)}{x-1} dx \\ &= \int (x^2 + 1) dx \\ &= \frac{x^3}{3} + x + C\end{aligned}$$

Therefore, $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx = \frac{x^3}{3} + x + C$

14. Find $\int (1-x)\sqrt{x}dx$

Solution:

The given integral is $\int (1-x)\sqrt{x}dx$

$$\begin{aligned}\int (1-x)\sqrt{x}dx &= \int \sqrt{x} - x\sqrt{x}dx \\ &= \int \sqrt{x}dx - \int x^{\frac{3}{2}}dx \\ &= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C \\ &= \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C\end{aligned}$$

Therefore, $\int (1-x)\sqrt{x}dx = \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$

15. Find $\int \sqrt{x}\left(3x^2+2x+3\right)dx$

Solution:

The given integral is $\int \sqrt{x}\left(3x^2+2x+3\right)dx$

$$\int \sqrt{x}\left(3x^2+2x+3\right)dx = \int 3x^2\sqrt{x}+2x\sqrt{x}+3\sqrt{x}dx$$

$$=3\int x^{\frac{5}{2}}dx+2\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$$

$$=3\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+2\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$$=3\left(\frac{2}{7}\right)x^{\frac{7}{2}}+2\left(\frac{2}{5}\right)x^{\frac{5}{2}}+3\left(\frac{3}{2}\right)x^{\frac{3}{2}}+C$$

$$=\frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+\frac{9}{2}x^{\frac{3}{2}}+C$$

Therefore, $\int \sqrt{x}\left(3x^2+2x+3\right)dx = \frac{6}{7}x^{\frac{7}{2}}+\frac{4}{5}x^{\frac{5}{2}}+\frac{9}{2}x^{\frac{3}{2}}+C$ Adf

16. Find $\int\left(2x-3\cos x+e^x\right)dx$

Solution:

The given integral is $\int\left(2x-3\cos x+e^x\right)dx$

$$\int\left(2x-3\cos x+e^x\right)dx = 2\int xdx-\int 3\cos xdx+\int e^x dx$$

$$=2\frac{x^2}{2}-3\left(\sin x\right)+e^x+C$$

$$=x^2-3\sin x+e^x+C$$

Therefore, $\int\left(2x-3\cos x+e^x\right)dx = x^2-3\sin x+e^x+C$

17. Find $\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$

Solution:

The given integral is $\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$

$$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx=2\int x^2dx-3\int \sin x dx+5\int x^{\frac{1}{2}}dx$$

$$=2\frac{x^3}{3}-3\left(-\cos x\right)+5\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$$=\frac{2x^3}{3}+3\cos x+\frac{10}{3}x^{\frac{3}{2}}+C$$

Therefore, $\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx=\frac{2x^3}{3}+3\cos x+\frac{10}{3}x^{\frac{3}{2}}+C$

18. Find $\int \sec x\left(\sec x+\tan x\right)dx$

Solution:

The given integral is $\int \sec x\left(\sec x+\tan x\right)dx$

$$\int \sec x\left(\sec x+\tan x\right)dx=\int \sec^2 x dx+\int \sec x\tan x dx$$

$$=\tan x+\sec x+C$$

Therefore, $\int \sec x\left(\sec x+\tan x\right)dx=\tan x+\sec x+C$

19. Find $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

Solution:

The given integral is $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx = \int \tan^2 x dx$$
$$= \int \sec^2 x - 1 dx$$
$$= \int \sec^2 x dx - \int dx$$
$$= \tan x - x + C$$

Therefore, $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx = \tan x - x + C$

20. Find $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

Solution:

The given integral is $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx = 2\int \sec^2 x dx - 3\int \tan x \sec x dx$$
$$= 2\tan x - 3\sec x + C$$

Therefore, $\int \frac{2 - 3\sin x}{\cos^2 x} dx = 2\tan x - 3\sec x + C$

21. The antiderivative of $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$

1) $\frac{1}{3} x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$

2) $\frac{2}{3} x^{\frac{2}{3}} + \frac{1}{2} x^{\frac{1}{2}} + C$

3) $\frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

4) $\frac{3}{2} x^{\frac{3}{2}} + \frac{1}{2} x^{\frac{1}{2}} + C$

Solution:

The given integral is $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$

$$\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$$

$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

Hence, $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$

Therefore, the option 3 is correct.

22. If $\frac{d}{dx}\left(f\left(x\right)\right)=4x^{3}-\frac{3}{x^{4}}$ such that $f\left(2\right)=0$. Then $f\left(x\right)$ is

1) $x^{4}+\frac{1}{x^{3}}-\frac{129}{8}$

2) $x^{3}+\frac{1}{x^{4}}+\frac{129}{8}$

3) $x^{4}+\frac{1}{x^{3}}+\frac{129}{8}$

4) $x^{3}+\frac{1}{x^{4}}-\frac{129}{8}$

Solution:

Given $\frac{d}{dx}\left(f\left(x\right)\right)=4x^{3}-\frac{3}{x^{4}}$

It implies that

$$f\left(x\right)=\int 4x^{3}-\frac{3}{x^{4}}dx$$

$$=4\int x^{3}dx-3\int\frac{1}{x^{4}}dx$$

$$=4\frac{x^{3+1}}{3+1}-3\frac{x^{-4+1}}{-4+1}+C$$

$$=x^{4}+\frac{1}{x^{3}}+C$$

Given $f(2)=0$

$$f(2)=2^4+2^{-3}+C$$

$$0=16+\frac{1}{8}+C$$

$$C=\frac{128+1}{8}$$

$$=-\frac{129}{8}$$

Hence, $f(x)=x^4+\frac{1}{x^4}-\frac{129}{8}$

Therefore, option 1 is correct.