Chapter: 7. Integrals

Exercise: 7.2

1. Integrate the function $\frac{2x}{1+x^2}$

Solution:

The given integral is $\int \frac{2x}{1+x^2} dx$

Put $t = 1 + x^2$ so that $dt = 2xdx$

Hence,

$$\begin{aligned}\int \frac{2x}{1+x^2} dx &= \int \frac{1}{t} dt \\ &= \log|t| + C \\ &= \log\left|1+x^2\right| + C\end{aligned}$$

Therefore, $\int \frac{2x}{1+x^2} dx = \log\left|1+x^2\right| + C$

2. Integrate the function $\frac{(\log x)^2}{x}$

Solution: The given integral is $\int \frac{(\log x)^2}{x} dx$

Put $t = \log x$ so that $dt = \frac{1}{x} dx$

Hence,

$$\begin{aligned}\int \frac{(\log x)^2}{x} dx &= \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{(\log x)^3}{3} + C\end{aligned}$$

Therefore, $\int \frac{(\log x)^2}{x} dx = \frac{|(\log x)|^3}{3} + C$

3. Integrate the function $\frac{1}{x + x\log x}$

Solution:

The given integral is $\int \frac{1}{x + x\log x} dx = \int \frac{1}{x(1+\log x)} dx$

Put $t = 1 + \log x$ so that $dt = \frac{1}{x} dx$

Hence,

$$\int \frac{1}{x + x\log x} dx = \int \frac{1}{x(1+\log x)} dx$$
$$= \int \frac{1}{t} dx$$
$$= \log t + C$$
$$= \log(1 + \log x) + C$$

Therefore, $\int \frac{1}{x + x\log x} dx = \log|(1+\log x)| + C$

4. Integrate the function $\sin x \sin(\cos x)$

Solution:

The given integral is $\int \sin x \sin(\cos x) dx$

Put $t = \cos x$ so that $dt = -\sin x dx$

Hence,

$$\int \sin x \sin(\cos x) dx = -\int \sin t dt$$
$$= \cos t + C$$
$$= \cos(\cos x) + C$$

Therefore, $\int \sin x \sin(\cos x)\,dx = \cos(\cos x) + C$

5. Integrate the function $\sin(ax+b)\cos(ax+b)$

Solution:

The given integral is $\int \sin(ax+b)\cos(ax+b)\,dx$

Put $t = \sin(ax+b)$ so that $dt = a\cos(ax+b)\,dx$

Hence,

$$\begin{aligned}\int \sin(ax+b)\cos(ax+b)\,dx &= \frac{1}{a}\int t\,dt \\ &= \frac{1}{a}\frac{t^2}{2} + C \\ &= \frac{1}{2a}\sin^2(ax+b) + C\end{aligned}$$

Therefore, $\int \sin(ax+b)\cos(ax+b)\,dx = \frac{1}{2a}\sin^2(ax+b) + C$

6. Integrate the function $\sqrt{ax+b}$

Solution:

The given integral is $\int \sqrt{ax+b}\,dx$

Put $t = (ax+b)$ so that $dt = a\,dx$

Hence,

$$\begin{aligned}\int \sqrt{ax+b}\,dx &= \frac{1}{a}\int \sqrt{t}\,dt \\ &= \frac{1}{a}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \\ &= \frac{2}{3a}(ax+b)^{\frac{3}{2}}\end{aligned}$$

Therefore, $\int \sqrt{ax+b}dx = \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$

7. Integrate the function $x\sqrt{x+2}$

Solution:

The given integral is $\int x\sqrt{x+2}dx$

Put $t = x+2$ so that $dt = dx$

Hence,

$$\begin{aligned}\int x\left(\sqrt{x+2}\right)dx &= \int (t-2)\sqrt{t}dt \\ &= \int t^{\frac{3}{2}}dt - 2\int t^{\frac{1}{2}}dt \\ &= \frac{2}{5}t^{\frac{5}{2}} - 2\left(\frac{2}{3}\right)t^{\frac{3}{2}} + C \\ &= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C\end{aligned}$$

Therefore, $\int x\left(\sqrt{x+2}\right)dx = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

8. Integrate the function $x\sqrt{1+2x^2}$

Solution: The given integral is $\int x\sqrt{1+2x^2}dx$

Put $t = 1+2x^2$ so that $dt = 4xdx$

Hence,

$$\begin{aligned}\int x\sqrt{1+2x^2}dx &= \frac{1}{4}\int 4x\sqrt{1+2x^2}dx \\ &= \frac{1}{4}\int \sqrt{t}dt\end{aligned}$$

$$=\frac{1}{4}\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C$$

$$=\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}+C$$

Therefore, $\int x\sqrt{1+2x^2}dx=\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}+C$

9. Integrate the function $(4x+2)\sqrt{x^2+x+1}$

Solution:

The given integral is $\int(4x+2)\sqrt{x^2+x+1}dx=2\int(2x+1)\sqrt{x^2+x+1}dx$

Put $t=x^2+x+1$ so that $dt=(2x+1)dx$

Hence,

$$\begin{aligned}\int(4x+2)\sqrt{x^2+x+1}dx&=2\int(2x+1)\sqrt{x^2+x+1}dx\\&=2\int\sqrt{t}dt\\&=2\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C\\&=\frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C\end{aligned}$$

Therefore, $\int(4x+2)\sqrt{x^2+x+1}dx=\frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$

10. Integrate the function $\frac{1}{x-\sqrt{x}}$

Solution:

The given integral is $\int\frac{1}{x-\sqrt{x}}\cdot dx=\int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}\cdot dx$

Put $t=\sqrt{x}-1$ so that $dt=\frac{1}{2\sqrt{x}}dx$

Hence,

$$\int\frac{1}{x-\sqrt{x}}\cdot dx=\int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}\cdot dx$$
$$=2\int\frac{1}{2\sqrt{x}}\left(\frac{1}{\sqrt{x}-1}\right)dx$$
$$=2\int\frac{1}{t}dt$$
$$=2\log\left(t\right)+C$$
$$=2\log\left|\left(\sqrt{x}-1\right)\right|+C$$

Therefore, $\int\frac{1}{x-\sqrt{x}}\cdot dx=2\log\left|\left(\sqrt{x}-1\right)\right|+C$

11. Integrate the function $\frac{x}{\sqrt{x+4}}$

Solution:

The given integral is $\int\frac{x}{\sqrt{x+4}}dx$

$$\int\frac{x}{\sqrt{x+4}}\cdot dx=\int\frac{x+4-4}{\sqrt{x+4}}\cdot dx$$
$$=\int\frac{x+4}{\sqrt{x+4}}-\frac{4}{\sqrt{x+4}}\cdot dx$$
$$=\int\sqrt{x+4}dx-4\int\frac{1}{\sqrt{x+4}}dx$$
$$=\frac{\left(x+4\right)^{\frac{1}{2}+1}}{\frac{3}{2}}-4\frac{\left(x+4\right)^{-\frac{1}{2}+1}}{\frac{1}{2}}+C$$
$$=\frac{2}{3}\left(x+4\right)^{\frac{3}{2}}-8\sqrt{x+4}+C$$

Therefore, $\int\frac{x}{\sqrt{x+4}}dx=\frac{2}{3}\sqrt{x+4}\left(x-8\right)+C$

12. Integrate the function $\left(x^3-1\right)^{\frac{1}{3}} x^5$

Solution:

The given integral is $\int\left(x^3-1\right)^{\frac{1}{3}} x^5 dx$

Put $t=x^3-1$ so that $dt=3x^2dx$

Hence,

$$\begin{aligned}
\int\left(x^3-1\right)^{\frac{1}{3}} x^5 dx &= \frac{1}{3}\int\left(x^3-1\right)^{\frac{1}{3}}\left(3x^2\right)x^3 dx \\
&= \frac{1}{3}\int t^{\frac{1}{3}}(t+1)\,dt \\
&= \frac{1}{3}\int t^{\frac{4}{3}}+t^{\frac{1}{3}}dt \\
&= \frac{1}{3}\left(\frac{t^{\frac{7}{3}}}{\frac{7}{3}}\right)+\frac{1}{3}\left(\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right) \\
&= \frac{1}{7}t^{\frac{7}{3}}+\frac{1}{4}t^{\frac{4}{3}}+C \\
&= \left(x^3-1\right)\left(\frac{1}{7}\left(x^3-1\right)^{\frac{4}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{1}{3}}\right)+C
\end{aligned}$$

Therefore, $\int\left(x^3-1\right)^{\frac{1}{3}} x^5 dx=\left(x^3-1\right)\left(\frac{1}{7}\left(x^3-1\right)^{\frac{4}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{1}{3}}\right)+C$

13. Integrate the function $\frac{x^2}{\left(2+3x^3\right)^3}$

Solution:

The given integral is $\int\frac{x^2}{\left(2+3x^3\right)^3}dx$

Put $t=2+3x^3$ so that $dt=9x^2dx$

Hence,

$$\int \frac{x^2}{\left(2+3x^3\right)^3}dx = \frac{1}{9}\int \frac{9x^2}{\left(2+3x^3\right)^3}dx$$

$$= \frac{1}{9}\int t^{-3}dt$$

$$= \frac{1}{9}\left(\frac{t^{-3+1}}{-3+1}\right)$$

$$= -\frac{1}{18t^2}$$

$$= -\frac{1}{18\left(2+3x^3\right)^2} + C$$

Therefore, $\int \frac{x^2}{\left(2+3x^3\right)^3}dx = -\frac{1}{18\left(2+3x^3\right)^2} + C$

14. Find the integral of the function $\frac{1}{x(\log x)^m}$

Solution:

The given integral is $\int \frac{1}{x(\log x)^m} dx$

Put $t = \log x$ so that $dt = \frac{1}{x}dx$

Hence,

$$\int \frac{1}{x(\log x)^m} dx = \int \frac{1}{t^m} dt$$

$$= \frac{t^{-m+1}}{-m+1} + C$$

$$= \frac{(\log x)^{-m+1}}{-m+1} + C$$

Therefore, $\int \frac{1}{x(\log x)^m} dx = \frac{(\log x)^{-m+1}}{-m+1} + C$

15. Integrate the function $\frac{x}{9-4x^2}$

Solution:

The given integral is $\int \frac{x}{9-4x^2}dx$

Put $t=9-4x^2$, so that $dt=-8x$

Hence, the integral can be rewrite as

$$\begin{aligned}\int \frac{x}{9-4x^2}dx &= -\frac{1}{8}\int \frac{-8x}{9-4x^2}dx \\ &= -\frac{1}{8}\int \frac{dt}{t} \\ &= -\frac{1}{8}\log(t)+C \\ &= -\frac{1}{8}\log(9-4x^2)+C\end{aligned}$$

Therefore, $\int \frac{x}{9-4x^2}dx = -\frac{1}{8}\log(9-4x^2)+C$

16. Find the integral of the function e^{2x+3}

Solution:

The given integral is $\int e^{2x+3}dx$

Put $t=2x+3$, so that $dt=2dx$

Hence, the given integral can be rewrite as

$$\begin{aligned}\int e^{2x+3}dx &= \frac{1}{2}\int e^{2x+3}(2dx) \\ &= \frac{1}{2}\int e^{t}dt \\ &= \frac{1}{2}e^{t}+C \\ &= \frac{1}{2}e^{2x+3}+C\end{aligned}$$

Therefore, $\int e^{2x+3}dx = \frac{1}{2}e^{2x+3}+C$

17. Find the integral of the function $\frac{x}{e^{x^2}}$

Solution:

The given integral is $\int \frac{x}{e^{x^2}} dx$

Put $t = x^2$, so that $dt = 2xdx$

Hence the integral can be rewrite as

$$\begin{aligned}\int \frac{x}{e^{x^2}} dx &= \frac{1}{2}\int \frac{2x}{e^{x^2}} dx \\ &= \frac{1}{2}\int \frac{dt}{e^t} \\ &= \frac{1}{2}\int e^{-t} dt \\ &= -\frac{1}{2}e^{-t} + C \\ &= -\frac{1}{2}e^{-x^2} + C\end{aligned}$$

Therefore, $\int \frac{x}{e^{x^2}} dx = -\frac{1}{2}e^{-x^2} + C$

18. Find the integral of the function $\frac{e^{\tan^{-1}x}}{1+x^2}$

Solution:

The given integral is $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

Put $t = \tan^{-1}x$, so that $dt = \frac{1}{1+x^2} dx$

Hence the integral can be rewrite as

$$\begin{aligned}\int \frac{e^{\tan^{-1}x}}{1+x^2} dx &= \int e^t dt \\ &= e^t + C \\ &= e^{\tan^{-1}x} + C\end{aligned}$$

Therefore, $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx = e^{\tan^{-1} x} + C$

19. Find the integral of the function $\frac{e^{2x}-1}{e^{2x}+1}$

Solution:

The given integral is

$$\int \frac{e^{2x}-1}{e^{2x}+1} dx = \int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} dx$$, by dividing both numerator and denominator with e^x

Put $t = e^x + e^{-x}$ so that $dt = \left(e^x - e^{-x}\right) dx$

Hence the integral can be rewrite as

$$\begin{aligned} \int \frac{e^{2x}-1}{e^{2x}+1} dx &= \int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} dx \\ &= \int \frac{dt}{t} \\ &= \ln(t) + C \\ &= \ln\left(e^x + e^{-x}\right) + C \end{aligned}$$

Therefore, $\int \frac{e^{2x}-1}{e^{2x}+1} dx = \ln\left(e^x + e^{-x}\right) + C$

20. Find the integral of the function $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

Solution:

The given integral is

$$\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$$, by dividing both numerator and denominator with e^x

Put $t = e^{2x} + e^{-2x}$ so that $dt = 2\left(e^{2x} - e^{-2x}\right) dx$

Hence the integral can be rewrite as

$$\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx = \frac{1}{2}\int \frac{2\left(e^{2x}-e^{-2x}\right)}{e^{2x}+e^{-2x}}dx$$
$$= \frac{1}{2}\int \frac{dt}{t}$$
$$= \frac{1}{2}\log(t)+C$$
$$= \frac{1}{2}\log\left(e^{2x}+e^{-2x}\right)+C$$

Therefore, $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx = \frac{1}{2}\log\left(e^{2x}+e^{-2x}\right)+C$

21. Find the integral of the function $\tan^2(2x-3)$

Solution:

The given integral is $\int \tan^2(2x-3)\,dx = \int\left(\sec^2(2x-3)-1\right)dx$

Put $t = 2x-3$ so that $dt = 2dx$

Hence the integral can be rewrite as

$$\int \tan^2(2x-3)\,dx = \int\left(\sec^2(2x-3)-1\right)dx$$
$$= \frac{1}{2}\int\left(\sec^2(2x-3)-1\right)2dx$$
$$= \frac{1}{2}\int\left(\sec^2(t)-1\right)dt$$
$$= \frac{1}{2}\left(\tan(t)-t\right)+c$$
$$= \frac{1}{2}\tan(2x-3)-(2x-3)+c$$
$$= \frac{1}{2}\tan(2x-3)-x+C$$

Therefore, $\int \tan^2(2x-3)\,dx = \frac{1}{2}\tan(2x-3)-x+C$

22. Find the integral of the function $\sec^2(7-4x)$

Solution:

The given integral is $\int \sec^2(7-4x)\,dx$

Put $t = 7-4x$ so that $dt = -4dx$

Hence the integral can be rewrite as

$$\begin{aligned}\int \sec^2(7-4x)\,dx &= -\frac{1}{4}\int \sec^2(7-4x)(-4dx)\\ &= -\frac{1}{4}\int \sec^2 t\,dt\\ &= -\frac{1}{4}\tan(t)+C\\ &= -\frac{1}{4}\tan(7-4x)+C\end{aligned}$$

Therefore, $\int \sec^2(7-4x)\,dx = -\frac{1}{4}\tan(7-4x)+C$

23. Find the integral of the function $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

Solution:

The given integral is $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Put $t = \sin^{-1}x$, so that $dt = \frac{1}{\sqrt{1-x^2}}\,dx$

Hence the integral can be rewrite as

$$\begin{aligned}\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx &= \int t\,dt\\ &= \frac{t^2}{2}+C\\ &= \frac{(\sin^{-1}x)^2}{2}+C\end{aligned}$$

Therefore, $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \frac{\left(\sin^{-1} x\right)^2}{2} + C$

24. Find the integral of the function $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

Solution:

The given integral is $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx$

Put $t = (3\cos x + 2\sin x)$ so that $dt = (2\cos x - 3\sin x) dx$

Hence the integral can be rewrite as

$$\begin{aligned}\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx &= \frac{1}{2} \int \frac{2\cos x - 3\sin x}{3\cos x + 2\sin x} dx \\ &= \frac{1}{2} \int \frac{dt}{t} \\ &= \frac{1}{2} \ln(t) + C \\ &= \frac{1}{2} \ln\left|3\cos x + 2\sin x\right| + C\end{aligned}$$

Therefore, $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx = \frac{1}{2} \ln\left|3\cos x + 2\sin x\right| + C$

25. Find the integral of the function $\frac{1}{\cos^2 x (1 - \tan x)^2}$

Solution:

The given integral is $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx = \int \frac{\sec^2 x}{(1 - \tan x)^2} dx$

Put $t = 1 - \tan x$ so that $dt = -\sec^2 x dx$

Hence the integral can be rewrite as

$$\int \frac{1}{\cos^2 x(1-\tan x)^2}dx = -\int \frac{-\sec^2 x}{(1-\tan x)^2}dx$$
$$= -\int \frac{dt}{t^2}$$
$$= -\frac{t^{-1}}{-1}$$
$$= \frac{1}{t}$$
$$= \frac{1}{1-\tan x}$$

Therefore, $\int \frac{1}{\cos^2 x(1-\tan x)^2}dx = \frac{1}{1-\tan x}$

26. Find the integral of the function $\frac{\cos\sqrt{x}}{\sqrt{x}}$

Solution:

The given integral is $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

Put $t=\sqrt{x}$ so that $dt = \frac{1}{2\sqrt{x}}dx$

Hence the integral can be rewrite as

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\int \frac{\cos\sqrt{x}}{2\sqrt{x}}dx$$
$$= 2\int \cos t dt$$
$$= 2(\sin t)+C$$
$$= 2\sin\left(\sqrt{x}\right)+C$$

Therefore, $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\sin\left(\sqrt{x}\right)+C$

27. Find the integral of the function $\sqrt{\sin 2x}\cos 2x$

Solution:

The given integral is $\int \sqrt{\sin 2x}\cos 2x dx$

Put $t=\sin 2x$, so that $dt = 2\cos 2x$

The integral can be rewrite as

$$\begin{aligned}\int \sqrt{\sin 2x}\cos 2x dx &= \frac{1}{2}\int \sqrt{\sin 2x}\, 2\cos 2x dx \\ &= \frac{1}{2}\int \sqrt{t} dt \\ &= \frac{1}{2}\cdot\frac{2}{3}t^{\frac{3}{2}}+C \\ &= \frac{1}{3}t^{\frac{3}{2}}+C \\ &= \frac{1}{3}(\sin 2x)^{\frac{3}{2}}+C\end{aligned}$$

Therefore, $\int \sqrt{\sin 2x}\cos 2x dx = \frac{1}{3}(\sin 2x)^{\frac{3}{2}}+C$

28. Find the integral of the function $\frac{\cos x}{\sqrt{1+\sin x}}$

Solution:

The given integral is $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$

Put $t = 1+\sin x$, so that $dt = \cos x dx$

The integral can be rewrite as

$$\begin{aligned}\int \frac{\cos x}{\sqrt{1+\sin x}} dx &= \int \frac{dt}{\sqrt{t}} \\ &= 2\sqrt{t}+C \\ &= 2\sqrt{1+\sin x}+C\end{aligned}$$

Therefore, $\int \frac{\cos x}{\sqrt{1+\sin x}} dx = 2\sqrt{1+\sin x}+C$

29. Find the integral of the function $\cot x \log \sin x$

Solution:

The given integral is $\int \cot x \log \sin x dx$

Put $t = \log \sin x$, so that $dt = \frac{1}{\sin x} \cos x dx$

The integral can be rewrite as

$$\begin{aligned}\int \cot x \log \sin x dx &= \int \log \sin x \frac{1}{\sin x} \cos x dx \\ &= \int t dt \\ &= \frac{t^2}{2} + C \\ &= \frac{1}{2}(\log \sin x)^2 + C\end{aligned}$$

Therefore, $\int \cot x \log \sin x dx = \frac{1}{2}(\log \sin x)^2 + C$

30. Find the integral of the function $\frac{\sin x}{1+\cos x}$

Solution:

The given integral is $\int \frac{\sin x}{1+\cos x} dx$

Put $t = 1+\cos x$, so that $dt = -\sin x$

Hence, the given integral becomes

$$\begin{aligned}\int \frac{\sin x}{1+\cos x} dx &= -\int \frac{-\sin x}{1+\cos x} dx \\ &= -\int \frac{dt}{t} \\ &= -\ln(t) + C \\ &= -\ln(1+\cos x) + C\end{aligned}$$

Therefore, $\int \frac{\sin x}{1+\cos x} dx = -\ln(1+\cos x) + C$

31. Find the integral of the function $\frac{\sin x}{(1+\cos x)^2}$

Solution:

The given integral is $\int \frac{\sin x}{(1+\cos x)^2} dx$

Put $t = 1+\cos x$, so that $dt = -\sin x$

Hence, the given integral becomes

$$\int \frac{\sin x}{(1+\cos x)^2} dx = -\int \frac{-\sin x}{(1+\cos x)^2} dx$$
$$= -\int \frac{dt}{t^2}$$
$$= \frac{1}{t} + C$$
$$= \frac{1}{1+\cos x} + C$$

Therefore, $\int \frac{\sin x}{(1+\cos x)^2} dx = \frac{1}{1+\cos x} + C$

32. Find the integral of the function $\frac{1}{1+\cot x}$

Solution:

The given integral is $\int \frac{1}{1+\cot x} dx$

$$\int \frac{1}{1+\cot x} dx = \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$$
$$= \int \frac{\sin x}{\sin x + \cos x} dx$$
$$= \frac{1}{2}\int \frac{2\sin x}{\sin x + \cos x} dx$$
$$= \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{\sin x + \cos x} dx$$

Put $sinx + cosx = t$, so that $(\cos x - \sin x)dx = dt$

The above integral becomes

$$\frac{1}{2}\int 1 - \left(\frac{\cos x - \sin x}{\sin x + \cos x}\right)dx = \frac{x}{2} - \frac{1}{2}\int \frac{dt}{t} + C$$
$$= \frac{x}{2} - \frac{1}{2}\ln(t) + C$$
$$= \frac{x}{2} - \frac{1}{2}\ln(\sin x + \cos x) + C$$

Therefore, $\int \frac{1}{1+\cot x}dx = \frac{x}{2} - \frac{1}{2}\ln(\sin x + \cos x) + C$

33. Find the integral of the function $\frac{1}{1-\tan x}$

Solution:

The given integral is $\int \frac{1}{1-\tan x}dx$

$$\int \frac{1}{1-\tan x}dx = \int \frac{1}{1 - \frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)}{\cos x - \sin x}dx$$

Put $cosx - \sin x = t$, so that $(-\sin x - \cos x)dx = dt$

The above integral becomes

$$\frac{1}{2}\int 1 - \left(\frac{-\cos x - \sin x}{\cos x - \sin x}\right)dx = \frac{x}{2} - \frac{1}{2}\int \frac{dt}{t} + C$$

$$= \frac{x}{2} - \frac{1}{2}\ln(t) + C$$
$$= \frac{x}{2} - \frac{1}{2}\ln(\cos x - \sin x) + C$$

Therefore, $\int \frac{1}{1-\tan x}dx = \frac{x}{2} - \frac{1}{2}\ln(\cos x - \sin x) + C$

34. Find the integral of the function $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Solution:

The given integral is $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$

The above integral rewrite and simplify as below

$$\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}\cos^2 x}dx$$
$$= \int \frac{\sqrt{\tan x \sec^2 x}}{\tan x}dx$$
$$= \int \frac{\sec^2 x}{\sqrt{\tan x}}dx$$

Put $\tan x = t$, so that $\sec^2 x dx = dt$

$$\int \frac{\sec^2 x}{\sqrt{\tan x}}dx = \int \frac{dt}{\sqrt{t}}$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

Therefore, $\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx = 2\sqrt{\tan x} + C$

35. Find the integral of the function $\frac{(1+\log x)^2}{x}$

Solution:

The given integral is $\int \frac{(1+\log x)^2}{x} dx$

Put $1+\log x = t$, so that $\frac{1}{x} dx = dt$

Hence,

$$\int \frac{(1+\log x)^2}{x} dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{(1+\log x)^3}{3} + C$$

Therefore, $\int \frac{(1+\log x)^2}{x} dx = \frac{(1+\log x)^3}{3} + C$

36. Find the integral of the function $\frac{(x+1)(x+\log x)^2}{x}$

Solution:

The given integral is $\int \frac{(x+1)(x+\log x)}{x} dx$

Put $t = x + \log x$, so that $dt = \left(1+\frac{1}{x}\right)dx$

The integral can be rewrite it as

$$\int \frac{(x+1)(x+\log x)}{x}dx = \int (x+\log x)\left(1+\frac{1}{x}\right)dx$$
$$= \int t dt$$
$$= \frac{t^2}{2}+C$$
$$= \frac{(x+\log x)^2}{2}+C$$

Therefore, $\int \frac{(x+1)(x+\log x)}{x}dx = \frac{(x+\log x)^2}{2}+C$

37. Find the integral of the function $\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

Solution:

The given integral is $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}dx$

Put $\tan^{-1} x^4 = t$, so that $\frac{1}{1+x^8}(4x^3)dx = dt$

The integral becomes

$$\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}dx = \frac{1}{4}\int \sin t dt$$
$$= \frac{1}{4}(-\cos t)+C$$
$$= \frac{1}{4}\left(-\cos\left(\tan^{-1} x^4\right)\right)+C$$

Therefore, $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}dx = \frac{1}{4}\left(-\cos\left(\tan^{-1} x^4\right)\right)+C$

38. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx =$

1) $10^x - x^{10} + C$

2) $10^x + x^{10} + C$

3) $\left(10^x + x^{10}\right)^{-1} + C$

4) $\log\left(10^x + x^{10}\right) + C$

Solution:

The given integral is $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$

Put $t = x^{10} + 10^x$, so that $dt = \left(10x^9 + 10^x \log_e 10\right) dx$

The integral becomes

$$\begin{aligned}\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx &= \int \frac{1}{t} dt \\ &= \log(t) + C \\ &= \log\left(x^{10} + 10^x\right) + C\end{aligned}$$

Therefore, the option 4 is correct.

39. $\int \frac{dx}{\sin^2 x \cos^2 x} =$

1) $\tan x + \cot x + C$

2) $\tan x - \cot x + C$

3) $\tan x \cot x + C$

4) $\tan x - \cot 2x + C$

Solution:

The given integral becomes

$$\begin{aligned}\int \frac{dx}{\sin^2 x \cos^2 x} &= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx \\ &= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx \\ &= \int \left(\sec^2 x + \csc^2 x\right) dx \\ &= \tan x - \cot x + C\end{aligned}$$

Therefore, option 2 is correct.