Chapter: 7. Integrals

Exercise: 7.3

1. Find the integral of the function $\sin^2(2x+5)$

Solution:

From trigonometry we have $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2}$$
$$= \frac{1-\cos(4x+10)}{2}$$

Use the integrals $\int \cos kx\,dx = \dfrac{1}{k}\sin kx + C$

Consider the integral

$$\int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2}\,dx$$
$$= \frac{1}{2}\int 1dx - \frac{1}{2}\int \cos(4x+10)\,dx$$
$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$
$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

Therefore, $\int \sin^2(2x+5)\,dx = \dfrac{1}{2}x - \dfrac{1}{8}\sin(4x+10) + C$

2. Find the integral of the function $\sin 3x.\cos 4x$

Solution:

From trigonometry, we have $\sin A\cos B = \dfrac{1}{2}\left\{\sin(A+B) + \sin(A-B)\right\}$

Use the integrals $\int \sin kx dx = -\frac{1}{k}\cos kx + C$

Consider the integral

$$\begin{aligned}\int \sin 3x \cos 4x \, dx &= \frac{1}{2}\int \{\sin(3x+4x)+\sin(3x-4x)\} dx \\ &= \frac{1}{2}\int \{\sin 7x + \sin(-x)\} dx \\ &= \frac{1}{2}\int \{\sin 7x - \sin x\} dx \\ &= \frac{1}{2}\left(-\frac{\cos 7x}{7} + \cos x\right) + C \\ &= -\frac{\cos 7x}{14} + \frac{\cos x}{2} + C\end{aligned}$$

Therefore, $\int \sin 3x \cos 4x \, dx = -\frac{\cos 7x}{14} + \frac{\cos x}{2} + C$

3. Find the integral of the function $\cos 2x \cos 4x \cos 6x$

Solution:

From trigonometry we have $\cos A \cos B = \frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

Consider the product $\cos 4x \cos 6x$

$$\begin{aligned}\cos 4x \cos 6x &= \frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\} \\ &= \frac{1}{2}\cos(10x)+\cos(-2x) \\ &= \frac{1}{2}(\cos 10x + \cos 2x)\end{aligned}$$

Hence, the given product can be written as

$$\cos 2x \cos 4x \cos 6x = \frac{1}{2}(\cos 2x \cos 10x + \cos 2x \cos 2x)$$
$$= \frac{1}{4}(2\cos 2x \cos 10x + 2\cos^2 2x)$$
$$= \frac{1}{4}(\cos 12x + cos(-8x) + (1 + \cos 4x))$$
$$= \frac{1}{4}(1 + \cos 12x + cos 8x + cos 4x)$$

Use the integrals $\int \cos kx dx = \frac{1}{k}\sin kx + C$

Consider the integral

$$\int \cos 2x \cos 4x \cos 6x dx = \frac{1}{4}\int 1 + \cos 12x + cos 8x + cos 4x dx$$
$$= \frac{1}{4}\left[x + \frac{\sin 12x}{12} + \frac{\sin 8x}{8} + \frac{\sin 4x}{4}\right] + C$$
$$= \frac{x}{4} + \frac{\sin 12x}{48} + \frac{\sin 8x}{32} + \frac{\sin 4x}{16} + C$$

Therefore, $\int \cos 2x \cos 4x \cos 6x dx = \frac{x}{4} + \frac{\sin 12x}{48} + \frac{\sin 8x}{32} + \frac{\sin 4x}{16} + C$

4. Find the integral of the function $\sin^3(2x+1)$

Solution:

Method 1:

From trigonometry, we have $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$\sin^3(2x+1) = \frac{1}{4}(3\sin(2x+1) - \sin(6x+3))$$

Use the integrals $\int \sin kx dx = -\frac{1}{k}\cos kx + C$

Consider

$$\int \sin^3(2x+1)dx = \frac{1}{4}\int(3\sin(2x+1) - \sin(6x+3))dx$$
$$= \frac{3}{4}\int \sin(2x+1)dx - \frac{1}{4}\int \sin(6x+3)dx$$

$$= \frac{3}{4}\left(-\frac{\cos(2x+1)}{2}\right) - \frac{1}{4}\left(-\frac{\cos(6x+3)}{6}\right) + C$$

$$= -\frac{3}{8}\cos(2x+1) - \frac{1}{24}\cos(6x+3) + C$$

Therefore, $\int \sin^3(2x+1)\,dx = -\frac{3}{8}\cos(2x+1) - \frac{1}{24}\cos(6x+3) + C$

Method: 2

Consider the integral

$$\begin{aligned} I &= \int \sin^3(2x+1) \\ &= \int \sin^2(2x+1).\sin(2x+1)\,dx \\ &= \int \left(1-\cos^2(2x+1)\sin(2x+1)\right)dx \\ &= \int \sin(2x+1)\,dx - \int \cos^2(2x+1)\cdot\sin(2x+1)\,dx \end{aligned}$$

Suppose that

$$\begin{aligned} \cos(2x+1) &= t \\ -2\sin(2x+1)\,dx &= dt \end{aligned}$$

Use the integrals $\int \sin kx dx = -\frac{1}{k}\cos kx + C$

$$\begin{aligned} \int \sin^3(2x+1)\,dx &= \int \sin(2x+1)\,dx + \frac{1}{2}\int \cos^2(2x+1)\cdot\left(-2\sin(2x+1)\right)dx \\ &= -\frac{\cos(2x+1)}{2} + c + \frac{1}{2}\int t^2 dt \\ &= -\frac{\cos(2x+1)}{2} + \frac{t^3}{6} + C \\ &= -\frac{\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C \end{aligned}$$

Therefore, $\int \sin^3(2x+1)\,dx = -\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

5. Find the integral of the function $\sin^3 x\cos^3 x$

Solution:

Consider the integral

$$\int \sin^3 x\cos^3 x dx = \int \sin^2 x\cos^3 x\sin x\, dx$$
$$= \int \cos^3 x\left(1-\cos^2 x\right)\sin x\, dx$$

Put $\cos x = t$, so that $-\sin x dx = dt$

The integral becomes

$$\int \sin^3 x\cos^3 x dx = \int t^3\left(1-t^2\right)\left(-dt\right)$$
$$= -\int \left(t^3 - t^5\right) dt$$
$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$
$$= -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$$

Therefore, $\int \sin^3 x\cos^3 x dx = -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

6. Find the integral of the function $\sin x\sin 2x\sin 3x$

Solution:

From the trigonometry, use the formula $\sin A\sin B = \frac{1}{2}\left\{\cos\left(A-B\right) - \cos\left(A+B\right)\right\}$

The given function is $\sin x\sin 2x\sin 3x$

$$\sin x\sin 2x\sin 3x = \frac{1}{2}\left(\sin x\left(2\sin 2x\sin 3x\right)\right)$$

$$= \frac{1}{2}\left(\sin x\left(\cos x - \cos 5x\right)\right)$$

$$= \frac{1}{2}\left(\sin x\cos x - \sin x\cos 5x\right)$$

$$=\frac{1}{4}\left(2\sin x\cos x-2\sin x\cos 5x\right)$$
$$=\frac{1}{4}\left(\sin 2x-\sin 6x+\sin 4x\right)$$

Use the integral $\int \sin kx dx=-\frac{1}{k}\cos kx+C$

Consider the integral

$$\int \sin x\sin 2x\sin 3x dx=\frac{1}{4}\int\left(\sin 2x-\sin 6x+\sin 4x\right)dx$$
$$=\frac{1}{4}\left(-\frac{\cos 2x}{2}\right)-\frac{1}{4}\left(-\frac{\cos 6x}{6}\right)+\frac{1}{4}\left(-\frac{\cos 4x}{4}\right)+C$$
$$=-\frac{1}{8}\cos 2x+\frac{1}{24}\cos 6x-\frac{1}{16}\cos 4x+C$$

Therefore, $\int \sin x\sin 2x\sin 3x dx=-\frac{1}{8}\cos 2x+\frac{1}{24}\cos 6x-\frac{1}{16}\cos 4x+C$

7. Find the integral of the function $\sin 4x\sin 8x$

Solution:

From trigonometry $2\sin A\sin B=\cos\left(A-B\right)-\cos\left(A+B\right)$

Hence,

$$\sin 4x\sin 8x=\frac{1}{2}\left(\cos\left(4x-8x\right)-\cos\left(4x+8x\right)\right)$$
$$=\frac{1}{2}\left(\cos\left(-4x\right)-\cos 12x\right)$$
$$=\frac{1}{2}\left(\cos 4x-\cos 8x\right)$$

Use the integral $\int \cos kx dx=\frac{1}{k}\sin kx+C$

Consider the integral

$$\int \sin 4x \sin 8x dx = \frac{1}{2}\int (\cos 4x - \cos 8x) dx$$
$$= \frac{1}{2}\left(\frac{\sin 4x}{4} - \frac{\sin 8x}{8}\right) + C$$
$$= \frac{1}{8}\sin 4x - \frac{1}{16}\sin 8x + C$$

Therefore, $\int \sin 4x \sin 8x dx = \frac{1}{8}\sin 4x - \frac{1}{16}\sin 8x + C$

8. Find the integral of the function $\frac{1-\cos x}{1+\cos x}$

Solution:

From the trigonometric formula $1-\cos x = 2\sin^2\frac{x}{2}, 1+\cos x = 2\cos^2\frac{x}{2}$

Hence,

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}$$
$$= \tan^2\left(\frac{x}{2}\right)$$
$$= \sec^2\left(\frac{x}{2}\right) - 1$$

Consider the integral

$$\int \frac{1-\cos x}{1+\cos x} dx = \int \sec^2\left(\frac{x}{2}\right) - 1 dx \qquad \bullet \int \sec^2(kx) dx = \frac{1}{k}\tan(kx) + c$$
$$= 2\tan\left(\frac{x}{2}\right) - x + C$$

9. Find the integral of the function $\frac{\cos x}{1+\cos x}$

Solution:

From trigonometry,

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$$

$$1 + \cos x = 2\cos^2\left(\frac{x}{2}\right)$$

Hence,

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= \frac{1}{2} - \frac{1}{2}\tan^2\frac{x}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\left(\sec^2\frac{x}{2} - 1\right)$$

$$= \frac{1}{2} - \frac{1}{2}\sec^2\frac{x}{2} + \frac{1}{2}$$

$$= 1 - \frac{1}{2}\sec^2\frac{x}{2}$$

Consider the integral

$$\int \frac{\cos x}{1+\cos x}dx = \int 1 - \frac{1}{2}\sec^2\frac{x}{2}dx \qquad \bullet \int \sec^2(kx)\,dx = \frac{1}{k}\tan kx + c$$

$$= x - \frac{1}{2}\left(2\tan\frac{x}{2}\right) + C$$

$$= x - \tan\frac{x}{2} + C$$

Therefore, $\int \frac{\cos x}{1+\cos x}dx = x - \tan\frac{x}{2} + C$

10. Find the integral of the function $\sin^4 x$

Solution:

From trigonometry

$$\sin^4 x = \left(\sin^2 x\right)^2$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2$$

$$=\frac{1}{4}\left(1+\cos^2 2x-2\cos 2x\right)$$

$$=\frac{1}{4}\left(1+\frac{1+\cos 4x}{2}-2\cos 2x\right)$$

$$=\frac{1}{8}\left(3+\cos 4x-4\cos 2x\right)$$

Use the integrals $\int \cos kx dx=\frac{1}{k}\sin kx+C$

Consider the integral

$$\int \sin^4 x dx=\frac{1}{8}\int 3+\cos 4x-4\cos 2x dx$$

$$=\frac{3x}{8}+\frac{\sin 4x}{32}-\frac{1}{2}\frac{\sin 2x}{2}+C$$

$$=\frac{3x}{8}+\frac{\sin 4x}{32}-\frac{\sin 2x}{4}+C$$

Therefore, $\int \sin^4 x dx=\frac{3x}{8}+\frac{\sin 4x}{32}-\frac{\sin 2x}{4}+C$

11. Find the integral of the function $\cos^4 2x$

Solution:

From trigonometry

$$\cos^4 2x=\left(\cos^2 2x\right)^2$$

$$=\left(\frac{1+\cos 4x}{2}\right)^2$$

$$=\frac{1}{4}\left(1+\cos^2 4x+2\cos 4x\right)$$

$$=\frac{1}{4}\left(1+\frac{1+\cos 8x}{2}+2\cos 4x\right)$$

$$=\frac{1}{8}\left(3+\cos 8x+2\cos 4x\right)$$

Use the integrals $\int \cos kx dx=\frac{1}{k}\sin kx+C$

Consider the integral

$$\int \cos^4 2x dx = \frac{1}{8}\int 3+\cos 8x+2\cos 4x dx$$
$$= \frac{3x}{8}+\frac{\cos 8x}{64}+\frac{1}{4}\frac{\cos 4x}{4}+C$$
$$= \frac{3x}{8}+\frac{\cos 8x}{64}+\frac{\cos 4x}{16}+C$$

Therefore, $\int \cos^4 2x dx = \frac{3x}{8}+\frac{\cos 8x}{64}+\frac{\cos 4x}{16}+C$

12. Find the integral of the function $\frac{\sin^2 x}{1+\cos x}$

Solution:

Consider the integral $\int \frac{\sin^2 x}{1+\cos x} dx$

$$\int \frac{\sin^2 x}{1+\cos x} dx = \int \frac{1-\cos^2 x}{1+\cos x} dx$$
$$= \int \frac{(1-\cos x)(1+\cos x)}{1+\cos x} dx$$
$$= \int (1-\cos x) dx$$
$$= x-\sin x+C$$

Therefore, $\int \frac{\sin^2 x}{1+\cos x} dx = x-\sin x+C$

13. Find the integral of the function $\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$

Solution:

Use the trigonometry

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{2\cos^2 x-1-\left(2\cos^2\alpha-1\right)}{\cos x-\cos\alpha}$$

$$=\frac{2\left(\cos^2 x-\cos^2\alpha\right)}{\cos x-\cos\alpha}$$

$$=2\left(\cos x+\cos\alpha\right)$$

Consider the integral

$$\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=2\int\left(\cos x+\cos\alpha\right)dx$$

$$=2\sin x+2x\cos\alpha+C$$

Therefore, $\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=2\sin x+2x\cos\alpha+C$

14. Find the integral of the function $\frac{\cos x-\sin x}{1+\sin 2x}$

Solution:

From trigonometry, use the formula $\cos^2 x+\sin^2 x=1, \sin 2x=2\sin x\cos x$

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\cos^2 x+\sin^2 x+2\sin x\cos x}$$

$$=\frac{\cos x-\sin x}{\left(\cos x+\sin x\right)^2}$$

Put $t=\cos x+\sin x$, so that $dt=\left(\cos x-\sin x\right)dx$

Hence,

$$\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{\left(\cos x+\sin x\right)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=-\frac{1}{t}+C$$

$$=-\frac{1}{\cos x+\sin x}+C$$

Therefore, $\int \frac{\cos x-\sin x}{1+\sin 2x}dx=-\frac{1}{\cos x+\sin x}+C$

15. Find the integral of the function $\tan^3 2x\sec 2x$

Solution:

From trigonometry

$$\begin{aligned}\tan^3 2x\sec 2x &= \tan^2 2x\tan 2x\sec 2x\\ &=\left(\sec^2 2x-1\right)\tan 2x\sec 2x\\ &=\sec^2 2x.\tan 2x\sec 2x-\tan 2x\sec 2x\end{aligned}$$

Consider the integral

$$\int \tan^3 2x\sec 2xdx=\int \sec^2 2x.\tan 2x\sec 2x-\tan 2x\sec 2xdx$$

Put $t=\sec 2x,$ so that $dt=2\sec 2x\tan 2x$

The above integral becomes

$$\begin{aligned}\int \tan^3 2x\sec 2xdx &= \int \sec^2 2x.\tan 2x\sec 2x-\tan 2x\sec 2xdx\\ &=\frac{1}{2}\int t^2\,dt-\frac{1}{2}\int dt\\ &=\frac{1}{2}\left(\frac{t^3}{3}\right)-\frac{t}{2}+C\\ &=\frac{\sec^3 2x}{6}-\frac{\sec 2x}{2}+C\end{aligned}$$

Therefore, $\int \tan^3 2x\sec 2xdx=\frac{\sec^3 2x}{6}-\frac{\sec 2x}{2}+C$

16. Find the integral of the function $\tan^4 x$

Solution:

$$\begin{aligned}\int \tan^4 xdx &= \int\left(\sec^2 x-1\right)^2 dx\\ &=\int \sec^4 x+1-2\sec^2 xdx\\ &=\int\left(\sec^2 x\right)\left(\sec^2 x\right)+1-2\sec^2 xdx\end{aligned}$$

$$= \int (1+\tan^2 x)(\sec^2 x) + 1 - 2\sec^2 x dx$$

$$= \int \sec^2 x + \tan^2 x \sec^2 x + 1 - 2\sec^2 x dx$$

$$= \int (\tan x)^2 \sec^2 x dx - \int \sec^2 x dx + \int 1 dx$$

Put $\tan x = t$, so that $\sec^2 x dx = dt$

Hence, the above integral becomes

$$\int \tan^4 x dx = \int t^2 dt - \tan x + x + C$$

$$= \frac{t^3}{3} - \tan x + x + C$$

$$= \frac{1}{3}\tan^3 x - \tan x + C$$

Therefore, $\int \tan^4 x dx = \frac{1}{3}\tan^3 x - \tan x + C$

17. Find the integral of the function $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Solution:

Consider the integral

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \frac{\sin x}{\cos^2 x} dx + \int \frac{\cos x}{\sin^2 x} dx$$

$$= \int \tan x \sec x dx + \int \cot x \csc x dx$$

$$= \sec x - \csc x + C$$

Therefore, $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \sec x - \csc x + C$

18. Find the integral of the function $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Solution:

Consider the integral

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \frac{\cos^2 x - \sin^2 x + 2\sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx$$

$$= \int \sec^2 x dx$$

$$= \tan x + C$$

Therefore, $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \tan x + C$

19. Find the integral of the function $\frac{1}{\sin x \cos^3 x}$

Solution:

Consider the integral is $\int \frac{1}{\sin x \cos^3 x} dx$

$$\int \frac{1}{\sin x \cos^3 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} dx$$

$$= \int \frac{\sin x}{\cos^3 x} dx + \int \frac{1}{\sin x \cos x} dx$$

$$= \int \tan x \sec^2 x dx + \int \frac{1}{\frac{\sin x}{\cos x} \cos^2 x} dx$$

$$= \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

Put $\tan x = t$, $\sec^2 x dx = dt$

$$\int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

$$= \int t dt + \int \frac{1}{t} dt$$

$$= \frac{t^2}{2} + \log|t| + C$$

$$= \frac{\tan^2 x}{2} + \log(\tan x) + C$$

Therefore, $\int \frac{1}{\sin x \cos^3 x} dx = \frac{\tan^2 x}{2} + \log(\tan x) + C$

20. Find the integral of the function $\frac{\cos 2x}{(\cos x + \sin x)^2}$

Solution:

Consider the integral

$$\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} dx$$

$$= \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)} dx$$

Put $\cos x + \sin x = t$, so that $(-\sin x + \cos x) dx = dt$

The above integral becomes

$$\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)} dx$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log|\cos x + \sin x| + C$$

Therefore, $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \log|\cos x + \sin x| + C$

21. Find the integral of the function $\sin^{-1}(\cos x)$

Solution:

Consider the integral

$$\int \sin^{-1}(\cos x) dx = \int \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx$$ • with restricted domain

$$=\int\left(\frac{\pi}{2}-x\right)dx$$

$$=\frac{\pi x}{2}-\frac{x^2}{2}+C$$

Therefore, $\int \sin^{-1}(\cos x)\,dx=\frac{\pi x}{2}-\frac{x^2}{2}+C$

22. The functional expression for the integral $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

Solution:

Rewrite the integrand as below.

$$\frac{1}{\cos(x-a)\cos(x-b)}=\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x-b)-(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-a)\right]$$

Use the integral $\int \tan x dx=-\log(\cos x)+c$

The given integral becomes

$$\int\frac{1}{\cos(x-a)\cos(x-b)}dx=\frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$

$$=\frac{1}{\sin(a-b)}\left[-\log\left|\cos(x-b)\right|+\log\left|\cos(x-a)\right|\right]$$

$$=\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

Therefore, $\int\frac{1}{\cos(x-a)\cos(x-b)}dx=\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$

23. The functional expression $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$ is equal to

A) $\tan x+\cot x+C$

B) $\tan x+\operatorname{cosec} x+C$

C) $-\tan x+\cot x+C$

D) $\tan x+\sec x+C$

Solution:

The given integral is

$$\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx=\int\left(\frac{\sin^2 x}{\sin^2 x\cos^2 x}-\frac{\cos^2 x}{\sin^2 x\cos^2 x}\right)dx$$

$$=\int\left(\sec^2 x-\cos ec^2 x\right)dx$$

$$=\tan x+\cot x+C$$

Thus, the correct answer is (A)

24. The functional expression for the $\int \frac{e^x(1+x)}{\cos^2\left(e^x x\right)}dx$ equals

A) $-\cot\left(xe^x\right)+C$

B) $\tan\left(xe^x\right)+C$

C) $\tan\left(e^x\right)+C$

D) $\cot\left(e^x\right)+C$

Solution:

The given integral is $\int \frac{e^x(1+x)}{\cos^2\left(e^x x\right)}dx$

Put $xe^x=t$, so that $\left(xe^x+e^x\right)dx=dt \Rightarrow e^x(x+1)dx=dt$

Hence, the integral becomes

$$\int \frac{e^x(1+x)}{\cos^2\left(e^x x\right)}dx=\int \frac{dt}{\cos^2 t}dt$$

$$=\int \sec^2 t dt$$

$$=\tan t+C$$

$$=\tan\left(xe^x\right)+C$$

Therefore, the option (B) is correct.