Chapter: 7. Integrals

Exercise: 7.6

1. Find the integral of the function $x\sin x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral

$$\begin{aligned} I &= \int x\sin xdx \\ &= x\int \sin xdx - \int 1\int \sin xdxdx \qquad \bullet f(x)=x, g(x)=\sin x \\ &= x(-\cos x) - \int -\cos xdx \\ &= -x\cos x + \sin x + C \end{aligned}$$

Therefore, $\int x\sin xdx = -x\cos x + \sin x + C$

2. Find the integral of the function $x\sin 3x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral

$$\begin{aligned} I &= \int x\sin 3xdx \\ &= x\int \sin 3xdx - \int 1\int \sin 3xdxdx \qquad \bullet f(x)=x, g(x)=\sin 3x \\ &= \frac{x}{3}(-\cos 3x) - \frac{1}{3}\int -\cos 3xdx \\ &= -\frac{1}{3}x\cos x + \frac{1}{9}\sin 3x + C \end{aligned}$$

Therefore, $\int x\sin 3xdx = -\frac{1}{3}x\cos x + \frac{1}{9}\sin 3x + C$

3. Find the integral of the function x^2e^x

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral

$$I = \int x^2e^x dx$$

$$= x^2\int e^x dx - \int 2x\int e^x dxdx \quad \bullet f(x) = x^2, g(x) = e^x$$

$$= x^2(e^x) - 2\int xe^x dx$$

$$= x^2e^x - 2(xe^x) + 2\int e^x dx$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

Therefore, $\int x^2e^x dx = x^2e^x - 2xe^x + 2e^x + C$

4. Find the integral of the function $x\log x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral

$$I = \int x\log x\, dx$$

$$= \log x\int xdx - \int \frac{1}{x}\int xdxdx \quad \bullet f(x) = \log x, g(x) = x$$

$$= \frac{x^2}{2}(\log x) - \frac{1}{2}\int xdx$$

$$= \frac{x^2\log x}{2} - \frac{x^2}{4} + C$$

Therefore, $\int x\log x\, dx = \frac{x^2\log x}{2} - \frac{x^2}{4} + C$

5. Find the integral of the function $x\log 2x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral

$$I = \int x\log 2x\,dx$$

$$= \log 2x\int xdx - \int \frac{2}{2x}\int xdxdx \qquad \bullet f(x) = \log 2x, g(x) = x$$

$$= \frac{x^2}{2}(\log 2x) - \frac{1}{2}\int xdx$$

$$= \frac{x^2\log 2x}{2} - \frac{x^2}{4} + C$$

Therefore, $\int x\log 2x\,dx = \frac{x^2\log 2x}{2} - \frac{x^2}{4} + C$

6. Find the integral of the function $x^2\log x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral

$$I = \int x^2\log x\,dx$$

$$= \log x\int x^2dx - \int \frac{1}{x}\int x^2dxdx \qquad \bullet f(x) = \log x, g(x) = x^2$$

$$= \frac{x^3}{3}(\log x) - \frac{1}{3}\int x^2dx$$

$$= \frac{x^3\log x}{3} - \frac{x^3}{9} + C$$

Therefore, $\int x^2\log x\,dx = \frac{x^3\log x}{3} - \frac{x^3}{9} + C$

7. Find the integral of the function $x\sin^{-1}x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x) = \sin^{-1}x, g(x) = x$

$$\int x\sin^{-1}xdx = \sin^{-1}x\int xdx - \int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int xdx\right\}dx$$

$$= \sin^{-1}x\left(\frac{x^2}{2}\right) - \int\frac{1}{\sqrt{1-x^2}}\frac{x^2}{2}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\int\left\{\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\int\left\{\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\left\{\int\sqrt{1-x^2}dx - \int\frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$= \frac{x^2\sin^{-1}x}{2} + \frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - \sin^{-1}x\right\} + C$$

$$= \frac{1}{4}\left(2x^2-1\right)\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + C$$

Therefore, $\int x\sin^{-1}xdx = \frac{1}{4}\left(2x^2-1\right)\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + C$

8. Find the integral of the function $x\tan^{-1}x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x) = \tan^{-1}x, g(x) = x$

$$\int x\tan^{-1}xdx = \tan^{-1}x\int xdx - \int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int xdx\right\}dx$$

$$= \tan^{-1}x\left(\frac{x^2}{2}\right) - \int\frac{1}{1+x^2}.\frac{x^2}{2}dx = \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\frac{x^2}{1+x^2}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\left(x - \tan^{-1}x\right) + C$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1} + C$$

Therefore, $\int x\tan^{-1}xdx = \frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1} + C$

9. Find the integral of the function $x\cos^{-1}x$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x) = \cos^{-1}x, g(x) = x$

$$\int x\cos^{-1}x\,dx=\cos^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x\,dx\right\}dx$$

$$=\cos^{-1}x\left(\frac{x^2}{2}\right)-\int\frac{-1}{\sqrt{1-x^2}}\frac{x^2}{2}dx$$

$$=\frac{x^2\cos^{-1}x}{2}+\frac{1}{2}\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\left\{\frac{1-x^2}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\left\{\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\left\{\int\sqrt{1-x^2}\,dx-\int\frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x-\sin^{-1}x\right\}+C$$

$$=\frac{1}{4}\left(2x^2-1\right)\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+C$$

Therefore, $\int x\cos^{-1}x\,dx=\frac{1}{4}\left(2x^2-1\right)\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+C$

10. Find the integral of the function $\left(\sin^{-1}x\right)^2$

Solution:

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx=f(x)\int g(x)dx-\int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x)=\left(\sin^{-1}x\right)^2, g(x)=1$

$$\int\left(\sin^{-1}x\right)^2dx=\left(\sin^{-1}x\right)^2\int 1dx-\int\left\{\left(\frac{d}{dx}\left(\sin^{-1}x\right)^2\right)\int 1dx\right\}dx$$

$$=\left(\sin^{-1}x\right)^2(x)-\int 2\sin^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right)x\,dx$$

$$=x\left(\sin^{-1}x\right)^2-2\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$$

Again use integration by parts. Use $f(x)=\sin^{-1}x, g(x)=\frac{2x}{\sqrt{1-x^2}}$

It implies that

$$x\left(\sin^{-1}x\right)^2-2\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx=x\left(\sin^{-1}x\right)^2-\int\sin^{-1}x\left(\frac{2x}{\sqrt{1-x^2}}\right)dx$$

$$=x\left(\sin^{-1}x\right)^2-\sin^{-1}x\left(-2\sqrt{1-x^2}\right)$$

$$+\int\frac{1}{\sqrt{1-x^2}}\left(-2\sqrt{1-x^2}\right)dx$$

$$=x\left(\sin^{-1}x\right)^2-2\sin^{-1}x\left(-\sqrt{1-x^2}\right)-2\int 1dx$$

$$=x\left(\sin^{-1}x\right)^2+2\sqrt{1-x^2}\sin^{-1}x-2x+C$$

Therefore, $\int\left(\sin^{-1}x\right)^2dx=x\left(\sin^{-1}x\right)^2+2\sqrt{1-x^2}\sin^{-1}x-2x+C$

11. Find the integral of the function $\frac{x\cos^{-1}x}{\sqrt{1-x^2}}$

Solution:

Consider the integral $\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

Suppose that $\cos^{-1}x=t$, so that $-\frac{1}{\sqrt{1-x^2}}dx=dt$ and $x=\cos t$

The integral becomes $\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx=-\int t\cdot\cos t dt$

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx=f(x)\int g(x)dx-\int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(t)=t, g(t)=\cos t$

$$-\int t\cdot \cos t dt = -t\int \cos t dt + \int \left\{ \left(\frac{d}{dt} t \right) \int \cos t dt \right\} dt$$

$$= -t\sin t + \int \sin t dt$$

$$= -t\sin t - \cos t + C$$

$$= -\cos x\cdot\sqrt{1-x^2} - x + C$$

Therefore, $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2}\cos x - x + C$

12. Find the integral of the function $x\sec^2 x$

Solution:

Given integral is $\int x\sec^2 xdx$

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x)=x, g(x)=\sec^2 x$

$$\int x\sec^2 xdx = x\int \sec^2 xdx - \int 1\cdot\int \sec^2 xdxdx$$

$$= x\tan x - \int \tan xdx$$

$$= x\tan x + \log|\cos x| + C$$

Therefore, $\int x\sec^2 xdx = x\tan x + \log|\cos x| + C$

13. Find the integral of the function $\tan^{-1} x$

Solution:

To find the integral of the function $\tan^{-1} x$, consider the integral $\int 1\cdot\tan^{-1} xdx$

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x)=\tan^{-1} x, g(x)=1$

Hence, the required integral becomes

$$\int 1\cdot \tan^{-1} x dx = \tan^{-1} x \cdot \int 1 dx - \int \frac{x}{1+x^2} dx$$
$$= x\tan^{-1} x - \frac{1}{2}\int \frac{2x}{1+x^2} dx$$
$$= x\tan^{-1} x - \frac{1}{2}\log\left|1+x^2\right| + C$$

Therefore, $\int \tan^{-1} x dx = x\tan^{-1} x - \frac{1}{2}\log\left|1+x^2\right| + C$

14. Find the integral of the function $x(\log x)^2 dx$

Solution:

Consider the integral of the function $\int x(\log x)^2 dx$

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x) = (\log x)^2, g(x) = x$

The given integral becomes

$$\int x(\log x)^2 dx = (\log x)^2 \cdot \frac{x^2}{2} - \int 2\log x\left(\frac{1}{x}\right)\left(\frac{x^2}{2}\right)dx$$
$$= \frac{x^2}{2}(\log x)^2 - \int x\log x dx$$
$$= \frac{x^2}{2}(\log x)^2 - \log x\left(\frac{x^2}{2}\right) + \int \frac{1}{x}\frac{x^2}{2}dx$$
$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}(\log x) + \frac{x^2}{4} + C$$

Therefore, $\int x(\log x)^2 dx = \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}(\log x) + \frac{x^2}{4} + C$

15. Find the integral of the function $(x^2+1)\log x$

Solution:

Consider the integral of the function $\int (x^2+1)\log x dx$

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x) = (\log x), g(x) = x^2+1$

The given integral becomes

$$\begin{aligned}\int (x^2+1)(\log x)dx &= (\log x)\cdot\left(\frac{x^3}{3}+x\right) - \int \frac{1}{x}\left(\frac{x^3}{3}+x\right)dx \\ &= \log x\left(\frac{x^3+3x}{3}\right) - \int \frac{x^2}{3}+1dx \\ &= \log x\left(\frac{x^3+3x}{3}\right) - \frac{x^3}{9} - x + C\end{aligned}$$

Therefore, $\int (x^2+1)(\log x)dx = \log x\left(\frac{x^3+3x}{3}\right) - \frac{x^3}{9} - x + C$

16. Find the integral of the function $e^x(\sin x + \cos x)$

Solution:

Consider the integral $\int e^x(\sin x + \cos x)dx$

Use the formula $\int e^x(f(x)+f'(x))dx = e^x f(x) + C$

Let $f(x) = \sin x \Rightarrow f'(x) = \cos x$

Hence, the given integral becomes

$$\begin{aligned}\int e^x(\sin x + \cos x)dx &= \int e^x(f(x)+f'(x))dx \\ &= e^x f(x) + C \\ &= e^x \sin x + C\end{aligned}$$

Therefore, $\int e^x(\sin x+\cos x)dx = e^x \sin x + C$

17. Find the integral of the function $\dfrac{xe^x}{(1+x)^2}$

Solution:

Consider the integral and then rewrite it as

$$\int \frac{xe^x}{(1+x)^2}dx = \int e^x\left(\frac{x}{(1+x)^2}\right)dx$$
$$= \int e^x\left(\frac{x+1-1}{(1+x)^2}\right)dx$$
$$= \int e^x\left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right)dx$$

Use the formula $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+C$

Let $f(x)=\dfrac{1}{1+x} \Rightarrow f'(x) = -\dfrac{1}{(1+x)^2}$

Hence, the given integral becomes

$$\int e^x\left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right)dx = \int e^x\left(f(x)+f'(x)\right)dx$$
$$= e^x f(x)+C$$
$$= e^x\left(\frac{1}{1+x}\right)+C$$

Therefore, $\int \dfrac{xe^x}{(1+x)^2}dx = \dfrac{e^x}{1+x}+C$

18. Find the integral of the function $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

Solution: Consider the integrand

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}e^x\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$=\frac{1}{2}e^x\left(1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right)$$

$$=\frac{1}{2}e^x\left(\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right)$$

Use the formula $\int e^x\left(f(x)+f'(x)\right)dx=e^xf(x)+C$

Let $f(x)=2\tan\frac{x}{2}\Rightarrow f'(x)=\sec^2\frac{x}{2}$

Hence, the given integral becomes

$$\int\frac{1}{2}e^x\left(\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right)dx=\frac{1}{2}\int e^x\left(f(x)+f'(x)\right)dx$$

$$=\frac{1}{2}e^xf(x)+C$$

$$=e^x\tan\frac{x}{2}+C$$

Therefore, $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx=e^x\tan\frac{x}{2}+C$

19. Find the integral of the function $e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Solution:

The given integral is $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

Use the formula $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+C$

Let $f(x)=\frac{1}{x} \Rightarrow f'(x)=-\frac{1}{x^2}$

Hence, the given integral becomes

$$\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = \int e^x\left(f(x)+f'(x)\right)dx$$
$$= e^x f(x)+C$$
$$= \frac{e^x}{x}+C$$

Therefore, $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = \frac{e^x}{x}+C$

20. Find the integral of the function $\frac{(x-3)e^x}{(x-1)^3}$

Solution:

Consider the integral and rewrite it as

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx = e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$
$$= \int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx$$

Use the formula $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+C$

Let $f(x)=\frac{1}{(x-1)^2}\Rightarrow f'(x)=\frac{-2}{(x-1)^3}$

Hence, the given integral becomes

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx=\int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx$$

$$=\int e^x\left(f(x)+f'(x)\right)dx$$

$$=e^x f(x)+C$$

$$=\frac{e^x}{(x-1)^2}+C$$

Therefore, $\int\frac{(x-3)e^x}{(x-1)^3}dx=\frac{e^x}{(x-1)^2}+C$

21. Find the integral of the function $e^{2x}\sin x$

Solution:

Consider the integral $\int e^{2x}\sin x dx$

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx=f(x)\int g(x)dx-\int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(x)=\sin x, g(x)=e^{2x}$

The given integral becomes

$$\int e^{2x}\sin x dx=\sin x\cdot\frac{e^{2x}}{2}-\int\cos x\left(\frac{e^{2x}}{2}\right)dx$$

$$=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\int e^{2x}\cos x dx$$

$$=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left(\cos x\left(\frac{e^{2x}}{2}\right)-\frac{1}{2}\int e^{2x}(-\sin x)dx\right)$$

$$=\frac{e^{2x}\sin x}{2}-\frac{1}{4}e^{2x}\cos x-\frac{1}{4}\int e^{2x}\sin x dx$$

$$\left(1+\frac{1}{4}\right)\int e^{2x}\sin x dx=\frac{e^{2x}\sin x}{2}-\frac{1}{4}e^{2x}\cos x$$

$$\frac{5}{4}\int e^{2x}\sin x dx=\frac{e^{2x}\sin x}{2}-\frac{1}{4}e^{2x}\cos x$$

$$\int e^{2x}\sin x dx=\frac{2}{5}e^{2x}\sin x-\frac{1}{5}e^{2x}\cos x$$

Therefore, $\int e^{2x}\sin x dx=\frac{2}{5}e^{2x}\sin x-\frac{1}{5}e^{2x}\cos x$

22. Find the integral of the function $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Solution:

Consider the integral $\int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Put $x=\tan\theta$, so that $dx=\sec^2\theta d\theta$

The given integral becomes

$$\int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx=\int\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta d\theta$$

$$=\int\sin^{-1}(\sin 2\theta)\sec^2\theta d\theta$$

$$=2\int\theta\sec^2\theta d\theta$$

To find the integral of product of two functions, use the integration by parts

$$\int f(x)g(x)dx=f(x)\int g(x)dx-\int f'(x)\int g(x)dxdx$$

Consider the integral and take $f(\theta)=\theta, g(\theta)=\sec^2\theta$

The integral becomes

$$
\begin{aligned}
2\int \theta \sec^2 \theta d\theta &= 2\theta \tan\theta - 2\int \tan\theta d\theta \\
&= 2\theta \tan\theta - 2\log\sec\theta + C \\
&= 2x\tan^{-1} x - \log\left(1+x^2\right) + C
\end{aligned}
$$

Therefore, $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = 2x\tan^{-1} x - \log\left(1+x^2\right) + C$

23. $\int x^2 e^{x^3} dx$

A) $\frac{1}{3}e^{x^3} + C$

B) $\frac{1}{3}e^{x^2} + C$

C) $\frac{1}{2}e^{x^3} + C$

D) $\frac{1}{3}e^{x^2} + C$

Solution:

Given $\int x^2 e^{x^3} dx$

Put $x^3 = t$, so that $3x^2 dx = dt$

The given integral becomes

$$
\begin{aligned}
\int x^2 e^{x^3} dx &= \frac{1}{3}\int \left(3x^2\right) e^{x^3} dx \\
&= \frac{1}{3}\int e^t dt \\
&= \frac{1}{3}\left(e^t\right) + C \\
&= \frac{1}{3}e^{x^3} + C
\end{aligned}
$$

This is matching with the option (A)

24. $\int e^x \sec x(1+\tan x)dx =$

A) $e^x \cos x + C$

B) $e^x \sec x + C$

C) $e^x \sin x + C$

D) $e^x \tan x + C$

Solution:

The given integral is $\int e^x \sec x(1+\tan x)dx$

Use the formula $\int e^x\{f(x)+f'(x)\}dx = e^x f(x)+C$

Consider the integral and rewrite it as

$$\begin{aligned}\int e^x \sec x(1+\tan x)dx &= \int e^x(\sec x+\sec x\tan x)dx\\ &= \int e^x(f(x)+f'(x))dx\\ &= e^x f(x)+C\\ &= e^x \sec x + C\end{aligned}$$

This is matching with the option (B)