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Chapter: 7. Integrals

Exercise: 7.7

1. Find the integral value of the function $\sqrt{4-x^2}$

Solution:

Consider the integral $\int \sqrt{4-x^2}\,dx = \int \sqrt{(2)^2-(x)^2}\,dx$

Use the formula $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

Hence,

$$\begin{aligned}\int \sqrt{4-x^2}\,dx &= \int \sqrt{(2)^2-(x)^2}\,dx\\ &= \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)+C\end{aligned}$$

Therefore, $\int \sqrt{4-x^2}\,dx = \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)+C$

2. Find the integral of the function $\sqrt{1-4x^2}$

Solution: Consider the integral

$$\begin{aligned}\int \sqrt{1-4x^2}\,dx &= \int \sqrt{1-(2x)^2}\,dx\\ &= \frac{1}{2}\int \sqrt{1-t^2}\,dt\end{aligned}$$

Use the formula $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

Hence,

$$\begin{aligned}\frac{1}{2}\int \sqrt{1-t^2}\,dt &= \frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}(t)+C\\ &= \frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}(2x)+C\\ &= \frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}(2x)+C\end{aligned}$$

Therefore, $\int \sqrt{1-4x^2}\,dx = \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}(2x) + C$

3. Find the integral of the function $\int \sqrt{x^2+4x+6}\,dx$

Solution:

The given integral is $\int \sqrt{x^2+4x+6}\,dx$

$$\begin{aligned}\int \sqrt{x^2+4x+6}\,dx &= \int \sqrt{x^2+4x+4+2}\,dx \\ &= \int \sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\,dx\end{aligned}$$

Use the formula $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)+C$

Hence, the given integral can be written as

$$\begin{aligned}\int \sqrt{x^2+4x+6}\,dx &= \int \sqrt{x^2+4x+4+2}\,dx \\ &= \int \sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\,dx \\ &= \frac{x+2}{2}\sqrt{x^2+4x+6} + \frac{2}{2}\log\left(x+2+\sqrt{x^2+4x+6}\right)+C \\ &= \frac{x+2}{2}\sqrt{x^2+4x+6} + \log\left|x+2+\sqrt{x^2+4x+6}\right|+C\end{aligned}$$

Therefore, $\int \sqrt{x^2+4x+6}\,dx = \frac{x+2}{2}\sqrt{x^2+4x+6} + \log\left|x+2+\sqrt{x^2+4x+6}\right|+C$

4. Find the integral of the function $\sqrt{x^2+4x+1}$

Solution:

The given integral is $\int \sqrt{x^2+4x+1}\,dx$

$$\begin{aligned}\int \sqrt{x^2+4x+1}\,dx &= \int \sqrt{x^2+4x+4-3}\,dx \\ &= \int \sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx\end{aligned}$$

Use the formula $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)+C$

Hence, the given integral can be written as

$$\int \sqrt{x^2+4x+1}\,dx = \int \sqrt{x^2+4x+4-3}\,dx$$
$$= \int \sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$$
$$= \frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left(x+2+\sqrt{x^2+4x+1}\right)+C$$

Therefore, $\int \sqrt{x^2+4x+1}\,dx = \frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right|+C$

5. $\sqrt{1-4x-x^2}$

Solution:

Consider, $= \int \sqrt{1-4x-x^2}\,dx$

$$= \int \sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$

$$= \int \sqrt{1+4-(x+2)^2}\,dx$$

$$= \int \sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

Since, $\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}\,\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

6. $\sqrt{x^2+4x-5}$

Solution:

Let $I = \int \sqrt{x^2+4x-5}\,dx$

$$= \int \sqrt{\left(x^2+4x+4\right)-9}\,dx = \int \sqrt{(x+2)^2-(3)^2}\,dx$$

Since, $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x-5} - \frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right| + C$$

7. $\sqrt{1+3x-x^2}$

Solution:

Put, $I = \int \sqrt{1+3x-x^2}\,dx$

$$= \int \sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}dx$$

$$= \int \sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx = \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

Since, $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\therefore I = \frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2} + \frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$$

$$= \frac{2x-3}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + C$$

8. $\sqrt{x^2+3x}$

Solution:

Let $I = \int \sqrt{x^2+3x}dx$

$$= \int \sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}dx$$

$$=\int\sqrt{\left(x+\frac{3}{4}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

Since, $\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

$$=\frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

9. $\sqrt{1+\frac{x^2}{9}}$

Solution:

Let $I=\int\sqrt{1+\frac{x^2}{9}}\,dx=\frac{1}{3}\int\sqrt{9-x^2}\,dx=\frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$

Since, $\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$

$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

10. $\int\sqrt{1+x^2}$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}}+C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{2}{3}}+C$

D. $\frac{x^3}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

Solution:

Since, $\sqrt{a^2+x^2}dx=\frac{x}{2}\sqrt{a^2+x^2}\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$\therefore\int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

Thus, the correct answer is A.

11. $\int\sqrt{x^2-8x+7}dx$ is equal

A. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Solution:

Let $I=\int\sqrt{x^2-8x+7dx}$

$=\int\sqrt{(x^2-8x+16)-9dx}$

$=\int\sqrt{(x-4)^2-(3)^2}dx$

Since, $\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\int\sqrt{x^2-8x+7}\right|+C$

Thus, the correct answer is D.