Chapter: 7. Integrals

Exercise: 7.9

1. Find the value of $\int_{-1}^{1}(x+1)\,dx$

Solution:

Consider the integral $\int_{-1}^{1}(x+1)\,dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)\,dx = F(x)+C$ then $\int_a^b f(x)\,dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_{-1}^{1}(x+1)\,dx = \left[\frac{x^2}{2}+x\right]_{-1}^{1}$$
$$= \frac{1}{2}+1-\frac{1}{2}+1$$
$$= 2$$

Therefore, $\int_{-1}^{1}(x+1)\,dx = 2$

2. Find the value of $\int_{2}^{3}\frac{1}{x}\,dx$

Solution: Consider the integral $\int_{2}^{3}\frac{1}{x}\,dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)\,dx = F(x)+C$ then $\int_a^b f(x)\,dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_{2}^{3}\frac{1}{x}\,dx = [\log x]_2^3$$
$$= \log 3 - \log 2$$
$$= \log\frac{3}{2}$$

Therefore, $\int_2^3 \frac{1}{x} dx = \log \frac{3}{2}$

3. Find the value of $\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx$

Solution: Consider the integral $\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x) dx = F(x) + C$ then $\int_a^b f(x) dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx = \left[x^4 - \frac{5x^3}{3} + 3x^2 + 9x\right]_1^2$$

$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$

$$= \frac{64}{3}$$

Therefore, $\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right) dx = \frac{64}{3}$

4. Find the value of $\int_0^{\frac{x}{4}} \sin 2x dx$

Solution: Consider the integral $\int_0^{\frac{x}{4}} \sin 2x dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x) dx = F(x) + C$ then $\int_a^b f(x) dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_0^{\frac{\pi}{4}} \sin 2x dx = \left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{2}\left(\cos \frac{\pi}{2} - \cos 0\right)$$

$$= \frac{1}{2}$$

Therefore, $\int_0^{\frac{\pi}{4}} \sin 2x dx = \frac{1}{2}$

5. $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$

Solution: Consider the integral $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x) dx = F(x) + C$ then $\int_a^b f(x) dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_0^{\frac{\pi}{2}} \cos 2x \, dx = \left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= \frac{1}{2}$$

Therefore, $\int_0^{\frac{\pi}{4}} \cos 2x dx = \frac{1}{2}$

6. Find the value of $\int_4^5 e^x dx$

Solution: Consider the integral $\int_4^5 e^x dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x) dx = F(x) + C$ then $\int_a^b f(x) dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_4^5 e^x dx = \left[e^x\right]_4^5$$

$$= e^5 - e^4$$

$$= e^4(e - 1)$$

Therefore, $\int_4^5 e^x dx = e^4(e-1)$

7. Find the value of $\int_0^{\frac{\pi}{4}} \tan x \, dx$

Solution: Consider the integral $\int_0^{\frac{\pi}{4}} \tan x \, dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x) dx = F(x) + C$ then $\int_a^b f(x) dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_0^{\frac{\pi}{4}} \tan x \, dx = [\log(\sec x)]_0^{\frac{\pi}{4}}$$

$$= \log\left(\sec \frac{\pi}{4}\right) - \log(\sec 0)$$

$$= \log\left(\sqrt{2}\right) - \log(1)$$

$$= \frac{1}{2} \log 2$$

Therefore, $\int_0^{\frac{\pi}{4}} \tan x \, dx = \frac{1}{2} \log 2$

8. Find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

Solution: Consider the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x) dx = F(x) + C$ then $\int_a^b f(x) dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos ec x\, dx = \left[\log\left(\csc x - cot x\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= \log\left(\csc\frac{\pi}{4} - \cot\frac{\pi}{4}\right) - \log\left(\csc\frac{\pi}{6} - \cot\frac{\pi}{6}\right)$$

$$= \log\left(\sqrt{2} - 1\right) - \log\left(2 - \sqrt{3}\right)$$

$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

Therefore, $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos ec x\, dx = \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$

9. Find the value of $\int_0^1 \frac{dx}{\sqrt{1 - x^2}}$

Solution: Consider the integral $\int_0^1 \frac{dx}{\sqrt{1 - x^2}}$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x) + C$ then $\int_a^b f(x)dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_0^1 \frac{dx}{\sqrt{1 - x^2}} = \left[sin^{-1}x\right]_0^1$$

$$= \frac{\pi}{2} - 0$$

$$= \frac{\pi}{2}$$

Therefore, $\int_0^1 \frac{dx}{\sqrt{1 - x^2}} = \frac{\pi}{2}$

10. Find the value of $\int_0^1 \frac{dx}{1 + x^2}$

Solution: Consider the integral $\int_0^1 \frac{dx}{1 + x^2}$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1$$

$$= \frac{\pi}{4}$$

Therefore, $\int_0^1 \frac{dx}{1+x^2} = \frac{\pi}{4}$

11. Find the value of $\int_2^3 \frac{dx}{x^2-1}$

Solution: Consider the integral $\int_2^3 \frac{dx}{x^2-1}$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_2^3 \frac{dx}{x^2-1} = \frac{1}{2}\left[\log\frac{x-1}{x+1}\right]_2^3$$

$$= \frac{1}{2}\left(\log\left(\frac{2}{4}\right)-\log\left(\frac{1}{3}\right)\right)$$

$$= \frac{1}{2}\log\left(\frac{3}{2}\right)$$

Therefore, $\int_2^3 \frac{dx}{x^2-1} = \frac{1}{2}\log\left(\frac{3}{2}\right)$

12. Find the value of $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Solution: Consider the integral $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_0^{\frac{\pi}{2}} \cos^2 xdx = \frac{1}{2}\int_0^{\frac{\pi}{2}} 1+\cos 2xdx$$

$$= \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(\frac{\pi}{2}+0\right)$$

$$= \frac{\pi}{4}$$

Therefore, $\int_0^{\frac{\pi}{2}} \cos^2 xdx = \frac{\pi}{4}$

13. Find the value of $\int_2^3 \frac{xdx}{x^2+1}$

Solution: Consider the integral $\int_2^3 \frac{xdx}{x^2+1}$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_2^3 \frac{xdx}{x^2+1} = \frac{1}{2}\int_2^3 \frac{2xdx}{x^2+1}$$

$$= \frac{1}{2}\left(\log\left(x^2+1\right)\right)_2^3$$

$$= \frac{1}{2}\log(10) - \frac{1}{2}\log(5)$$

$$= \frac{1}{2}\log 2$$

Therefore, $\int_2^3 \frac{xdx}{x^2+1} = \frac{1}{2}\log 2$

14. Find the value of $\int_0^1 \frac{2x+3}{5x^2+1}dx$

Solution: Consider the integral $\int_0^1 \frac{2x+3}{5x^2+1}dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_0^1 \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1}dx + \frac{3}{5}\int_0^1 \frac{1}{x^2+\left(\frac{1}{\sqrt{5}}\right)^2}dx$$

$$= \left[\frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\left(\frac{\sqrt{5}}{1}\right)\tan^{-1}\left(\frac{x}{\frac{1}{\sqrt{5}}}\right)\right]_0^1$$

$$= \frac{1}{5}\log(6) - \log(1) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)$$

Therefore, $\int_0^1 \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)$

15. Find the value of $\int_0^1 xe^{x^2}dx$

Solution: Consider the integral $\int_0^1 xe^{x^2}dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_0^1 xe^{x^2}dx = \frac{1}{2}\int_0^1 2xe^{x^2}dx$$
$$= \frac{1}{2}\left[e^{x^2}\right]_0^1$$
$$= \frac{1}{2}(e-1)$$

Therefore, $\int_0^1 xe^{x^2}dx = \frac{1}{2}(e-1)$

16. Find the value of $\int_1^2 \frac{5x^2}{x^2+4x+3}$

Solution: Consider the integral $\int_1^2 \frac{5x^2}{x^2+4x+3}$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_1^2 \frac{5x^2}{x^2+4x+3}dx = \int_1^2 \frac{5x^2+20x+15-20x-15}{x^2+4x+3}dx$$
$$= 5\int_1^2 1dx - 10\int_1^2 \frac{2x+4}{x^2+4x+3}dx + 25\int_1^2 \frac{1}{x^2+4x+3}dx$$
$$= 5(x)_1^2 - 10\left(\log\left(x^2+4x+3\right)\right)_1^2 + 25\int_1^2 \frac{1}{(x+2)^2-1}dx$$
$$= 5-10\log\left(\frac{15}{8}\right)+25\left(\frac{1}{2}\right)\left(\log\left(\frac{x+2-1}{x+2+1}\right)\right)_1^2$$
$$= 5-10\log\left(\frac{15}{8}\right)+\frac{25}{2}\log\left(\frac{3}{5}\right)-\frac{25}{2}\log\left(\frac{1}{2}\right)$$

Therefore, $\int_1^2 \frac{5x^2}{x^2+4x+3}dx = 5-10\log\left(\frac{15}{8}\right)+\frac{25}{2}\log\left(\frac{3}{5}\right)-\frac{25}{2}\log\left(\frac{1}{2}\right)$

17. Find the value of $\int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$

Solution: Consider the integral $\int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x) + C$ then $\int_a^b f(x)dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right)dx = \left[2\tan x + \frac{x^4}{4} + 2x\right]_0^{\frac{\pi}{4}}$$
$$= 2 + \frac{1}{4}\left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)$$
$$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

Therefore, $\int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right)dx = 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$

18. Find the value of $\int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right)dx$

Solution: Consider the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos ecx\, dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x) + C$ then $\int_a^b f(x)dx = F(b) - F(a)$"

Hence the integral becomes

$$\int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right)dx = -\int_0^{\pi} \cos x dx$$
$$= -(\sin x)_0^{\pi}$$
$$= 0$$

Therefore, $\int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right)dx = 0$

19. Find the value of $\int_0^2 \frac{6x+3}{x^2+4}dx$

Solution: Consider the integral $\int_0^2 \frac{6x+3}{x^2+4}dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_0^2 \frac{6x+3}{x^2+4}dx = 3\int_0^2 \frac{2x+1}{x^2+4}dx$$
$$= 3\int_0^2 \frac{2x}{x^2+4}dx + 3\int_0^2 \frac{1}{x^2+4}dx$$
$$= \left[3\log\left(x^2+4\right)+\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$$
$$= 3\log(8)+\frac{3}{2}\left(\frac{\pi}{4}\right)-3\log 4$$
$$= 3\log 2+\frac{3\pi}{8}$$

Therefore, $\int_0^2 \frac{6x+3}{x^2+4}dx = 3\log 2+\frac{3\pi}{8}$

20. Find the value of $\int_0^1 \left(xe^x+\sin\frac{\pi x}{4}\right)dx$

Solution: Consider the integral $\int_0^1 \left(xe^x+\sin\frac{\pi x}{4}\right)dx$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_a^b f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_0^1 \left(xe^x+\sin\frac{\pi x}{4}\right)dx = \left[e^x(x-1)-\frac{4}{\pi}cos\frac{\pi x}{4}\right]_0^1$$
$$= -1(0-1)-\frac{4}{\pi}cos\left(\frac{\pi}{4}\right)+\frac{4}{\pi}$$
$$= 1-\frac{2\sqrt{2}}{\pi}+\frac{4}{\pi}$$

Therefore, $\int_0^1 \left(xe^x+\sin\frac{\pi x}{4}\right)dx = 1-\frac{2\sqrt{2}}{\pi}+\frac{4}{\pi}$

21. The value of $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2}$

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{12}$

Solution: Consider the integral $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2}$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_{a}^{b} f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \left[\tan^{-1} x\right]_{1}^{\sqrt{3}}$$
$$= \tan^{-1}\left(\sqrt{3}\right) - \tan^{-1}(1)$$
$$= \frac{\pi}{3} - \frac{\pi}{4}$$
$$= \frac{\pi}{12}$$

Therefore, this is matching with the option (D)

22. The value of $\int_{0}^{\frac{2}{3}} \frac{dx}{4+9x^2}$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{12}$ C) $\frac{\pi}{24}$ D) $\frac{\pi}{4}$

Solution: Consider the integral $\int_{0}^{\frac{2}{3}} \frac{dx}{4+9x^2}$

Use the fundamental theorem of integral calculus which states that "if $\int f(x)dx = F(x)+C$ then $\int_{a}^{b} f(x)dx = F(b)-F(a)$"

Hence the integral becomes

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{\frac{4}{9}+x^2}$$

$$== \frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

$$= \frac{1}{6}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{\frac{2}{3}} \qquad \bullet\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

$$= \frac{1}{6}\left(\tan^{-1}1 - \tan^{-1}(0)\right)$$

$$= \frac{1}{6}\left[\frac{\pi}{4}\right]$$

$$= \frac{\pi}{24}$$

Therefore, this is matching with the option (C)