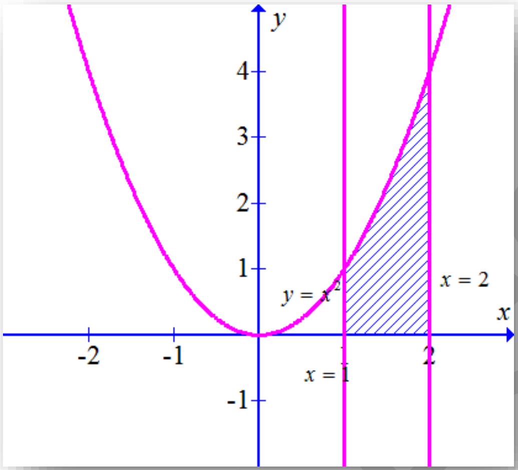
Chapter: 8. Applications of Integrals

Exercise 8. Miscellaneous

1. (i) Find the area under the curve $y=x^2$, the lines $x=1, x=2$ and the $x-$ axis.

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $y=f(x)$, the lines $x=a, x=b$ and $x-$ axis is $\left|\int_a^b f(x)\,dx\right|$.

The area of the shaded region is the region bounded by the curve $y=x^2$, the lines $x=1, x=2$ and the $x-$ axis in the first quadrant.

Hence, the required area is $A=\int_1^2 x^2 dx$

$$A=\int_1^2 x^2 dx$$

$$=\left[\frac{x^3}{3}\right]_1^2 \qquad \bullet\int x^n dx=\frac{x^{n+1}}{n+1}; n\neq -1$$

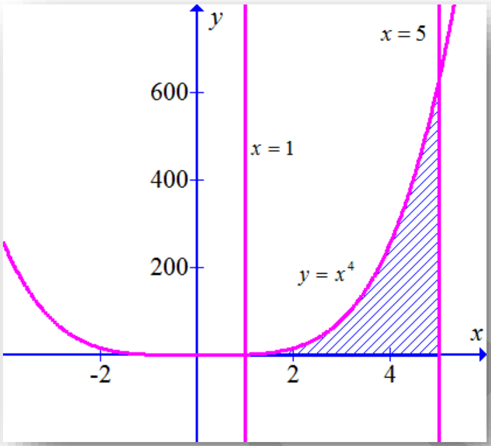
$$=\frac{1}{3}(8-1)$$

$$=\frac{7}{3}\text{square units}$$

Therefore, the area of the region bounded by the curve $y = x^2$ and the lines $x = 1, x = 2$ and the $x-$ axis in the first quadrant is $\frac{7}{3}$ square units

(ii) Find the area under the curve $y = x^4$, the lines $x = 1, x = 5$ and the $x-$ axis.

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $y = f(x)$, the lines $x = a, x = b$ and $x-$ axis is $\left|\int_a^b f(x)\,dx\right|$.

The area of the shaded region is the region bounded by the curve $y = x^4$, the lines $x = 1, x = 5$ and the $x-$ axis in the first quadrant.

Hence, the required area is $A = \int_1^5 x^4 dx$

$$A = \int_1^5 x^4 dx$$

$$= \left[\frac{x^5}{5}\right]_1^5 \qquad \bullet \int x^n dx = \frac{x^{n+1}}{n+1}; n \neq -1$$

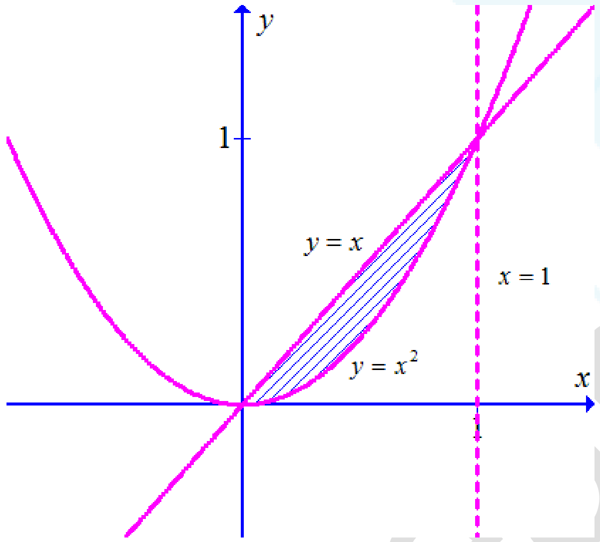
$$= \frac{1}{5}(3125 - 1)$$

$$= 624.8 \text{ square units}$$

Therefore, the area of the region bounded by the curve $y = x^4$ and the lines $x = 1, x = 5$ and the $x-$ axis in the first quadrant is 624.8 square units

2. Find the area between the curves $y = x$ and $y = x^2$.

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded by two curves $y = f(x)$ and $y = g(x)$ the lines $x = a, x = b$ is $\int_a^b |f(x) - g(x)| dx$.

The point of intersection of curve $y = x^2$ and the line $y = x$

$$x = x^2$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0 \text{ or } x = 1$$

The area of the shaded region is the region bounded by the curve $y = x^2$, the lines $y = x, x = 0, x = 1$.

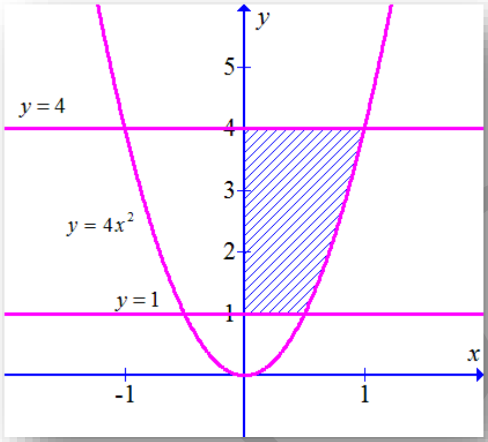
Hence, the required area is $A = \int_0^1 (x - x^2) dx$

$$A = \int_0^1 (x - x^2) dx$$
$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$
$$= \left(\frac{1}{2} - \frac{1}{3}\right)$$
$$= \frac{1}{6} \text{square units}$$

Therefore, the area of the region bounded by the curve $y = x^2$ and the line $y = x$ is $\frac{1}{6}$ square units

3. Find the area of the region lying in the first quadrant bounded by $y = 4x^2$, lines $y = 1, y = 4$ and the $y -$ axis .

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $x = g(y)$, lines $y = a, y = b$ and $y -$ axis in the first quadrant is defined as $\int_a^b g(y)\,dy$

The area of the shaded region is the region bounded by the curve $y = 4x^2$, the lines $y = 1, y = 4$ and the $y -$ axis in the first quadrant.

Hence, the required area is $A = \frac{1}{2}\int_1^4 \sqrt{y}dy$

$$A = \frac{1}{2}\int_1^4 \sqrt{y}dy$$

$$= \frac{1}{2}\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^4$$

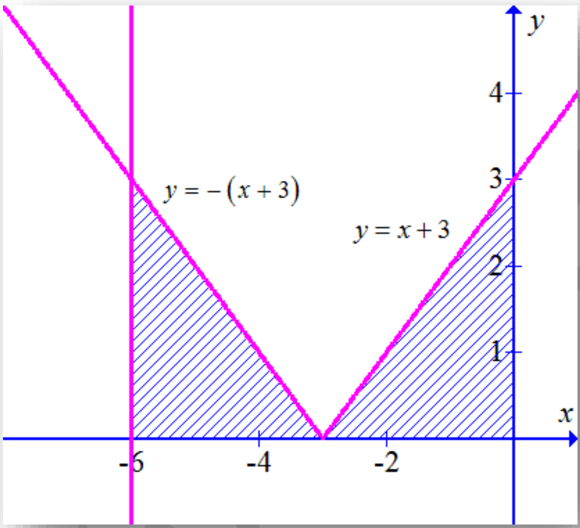
$$= \frac{1}{2}\left(\frac{2}{3}\right)(8-1)$$

$$= \frac{7}{3}$$

The area of the shaded region is the region bounded by the curve $y = 4x^2$, the lines $y = 1, y = 4$ and the $y-$ axis in the first quadrant. $\frac{7}{3}$ square units

4. Sketch the graph of $y = |x+3|$, and evaluate $\int_{-6}^{0} |x+3|\,dx$

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $y = f(x)$, the lines $x = a, x = b$ and $x-$ axis is $\left|\int_a^b f(x)\,dx\right|$.

The given equation $y = |x+3|$ can be rewrite as below

$$y = \begin{cases} x+3 & x > -3 \\ -(x+3) & x < -3 \end{cases}$$

Hence,

$$\int_{-6}^{0} |x+3|\,dx = \left|\int_{-6}^{-3} (x+3)\,dx - \int_{-3}^{0} (x+3)\,dx\right|$$

$$= \left|\left(\frac{x^2}{2} + 3x\right)_{-6}^{-3} - \left(\frac{x^2}{2} + 3x\right)_{-3}^{0}\right|$$

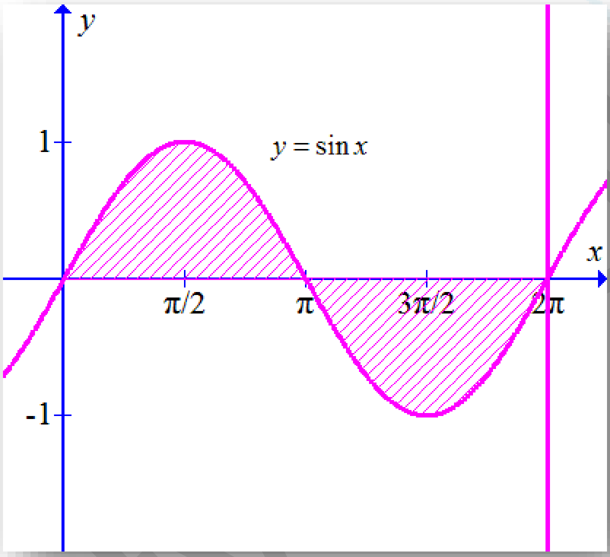
$$= \left|\frac{9}{2} - 9 - \frac{36}{2} + 18 + \frac{9}{2} - 9\right|$$

$$= 9$$

Therefore, the area of the region bounded by the curve $y=|x+3|$ between the lines $x=-6, x=0$ above the $x-$ axis is 9 square units

5. Find the area bounded by the curve $y=\sin x$ between the lines $x=0$ and $x=2\pi$

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $y=f(x)$, the lines $x=a, x=b$ and $x-$ axis is $\left|\int_a^b f(x)\,dx\right|$.

The area of the shaded region is 2 times the region bounded by the curve $y=\sin x$, the lines $x=0, x=\pi$ and the $x-$ axis in the first quadrant.

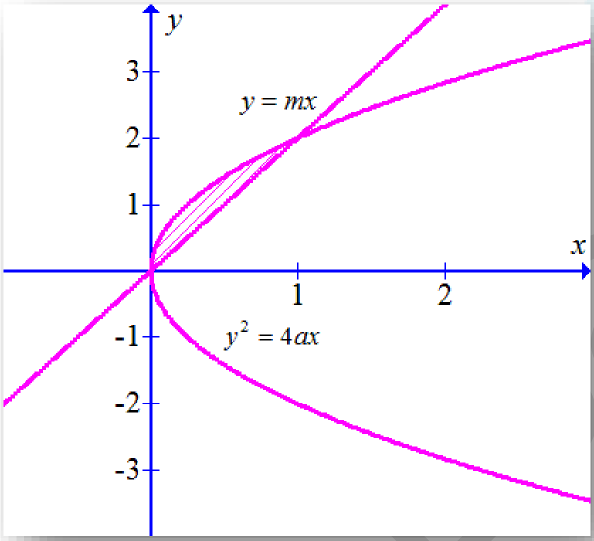
Hence, the required area is $A=2\left(\int_0^{\pi}\sin x dx\right)$

$$A=2\left(\int_0^{\pi}\sin x dx\right)$$
$$=2(-\cos x)_0^{\pi}$$
$$=2(1+1)$$
$$=4$$

Therefore, the area of the region bounded by the curve $y = \sin x$ between the lines $x = 0, x = 2\pi$ is 4 square units

6. Find the area of the region enclosed between the curves $y^2 = 4ax$ and the line $y = mx$

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y = f(x)$ and $y = g(x)$ between the

Lines $x = a, x = b$ is $\left|\int_a^b (f(x) - g(x))dx\right|$.

The curve $y^2 = 4ax$ and the line $y = mx$ intersect:

$$(mx)^2 = 4ax$$

$$m^2x^2 = 4ax$$

$$m^2x = 4a$$

$$x = \frac{4a}{m^2}$$

The area of the shaded region is the area of the region bounded between the curve $y^2 = 4ax$ and the line $y = mx$ is sum of the following regions

Hence, the required area is

$$A=\int_0^{\frac{4a}{m^2}} 2\sqrt{ax}-mx$$

$$=2\sqrt{a}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^{\frac{4a}{m^2}}-m\left(\frac{x^2}{2}\right)_0^{\frac{4a}{m^2}}$$

$$=2\sqrt{a}\left(\frac{2}{3}\right)\left(\frac{8a^{\frac{3}{2}}}{m^3}\right)-m\left(\frac{8a^2}{m^4}\right)$$

$$=\frac{32a^2}{3m^3}-\frac{8a^2}{m^3}$$

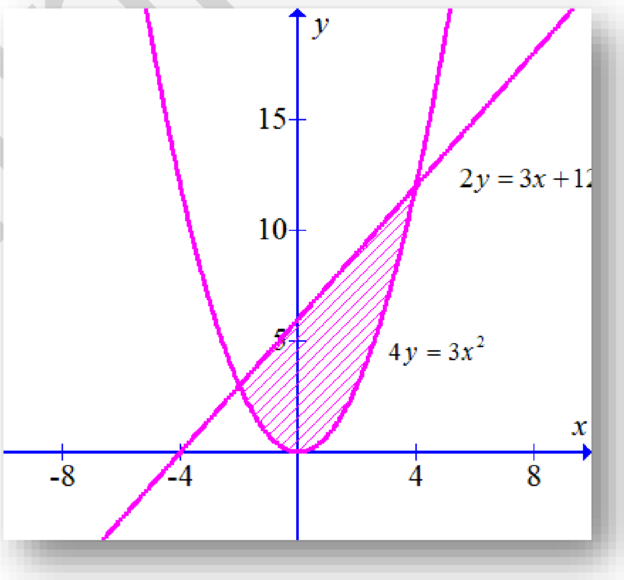
$$=\frac{8a^2}{3m^3}$$

Therefore, the enclosed between the curves $y^2=4ax$ and the line $y=mx$ is $\frac{8a^2}{3m^3}$ square units

7. Find the area enclosed by the parabola $4y=3x^2$ and the line $2y=3x+12$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y = f(x)$ and $y = g(x)$ between the

Lines $x = a, x = b$ is $\left|\int_a^b (f(x) - g(x))dx\right|$.

Points of intersection of the curve $4y = 3x^2$ and the line $2y = 3x + 12$

$$3x^2 = 2(3x + 12)$$

$$3x^2 - 6x - 24 = 0$$

$$x^2 - 2x - 8 = 0$$

$$x^2 - 4x + 2x - 8 = 0$$

$$x(x-4) + 2(x-4) = 0$$

$$(x+2)(x-4) = 0$$

It implies that $x = -2, x = 4$

The area of the shaded region is two times the area of the region bounded by the curves $y = \frac{3x^2}{4}$, the lines $y = \frac{3x+12}{2}$, $x = -2, x = 4$.

Hence, the required area is

$$A = \int_{-2}^{4} \left(\frac{3x^2}{4} - \frac{3x+12}{2} \right) dx$$

$$= \frac{3}{4} \int_{-2}^{4} (x^2 - 2x - 8) dx$$

$$= \frac{3}{4} \left(\frac{x^3}{3} - x^2 - 8x \right)_{-2}^{4}$$

$$= \frac{3}{4} \left(\frac{64}{3} - 16 - 32 + \frac{8}{3} + 4 - 16 \right)$$

$$= \left| \frac{3}{4} \left(\frac{72}{3} - 60 \right) \right|$$

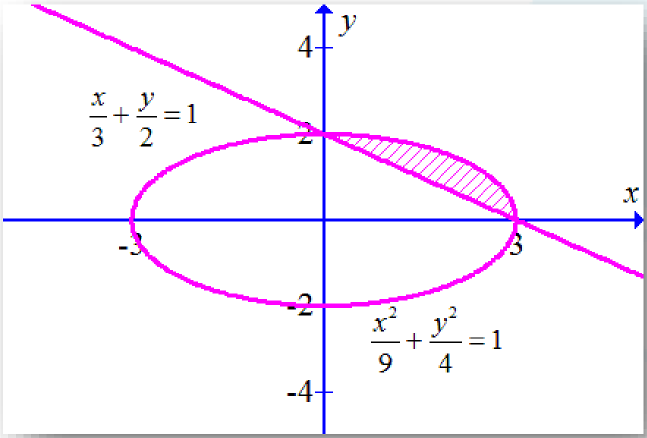
$$= 27$$

Therefore, the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$ is 27 square units

8. Find the area of the smallest region in the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y=f(x)$ and $y=g(x)$ between the

Lines $x=a, x=b$ is $\left|\int_a^b (f(x)-g(x))dx\right|$.

The area of the shaded region is the region between the curves $y=\frac{2}{3}\sqrt{9-x^2}$,

$y=2\left(\frac{3-x}{3}\right)$ and the lines $x=0, x=3$

Hence, the area of the shaded region is $A=\frac{2}{3}\int_0^3\left(\sqrt{9-x^2}-(3-x)\right)dx$

$$A=\frac{2}{3}\int_0^3\left(\sqrt{9-x^2}-(3-x)\right)dx$$

$$=\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}-\frac{9}{2}\sin^{-1}\frac{x}{3}-3x+\frac{x^2}{2}\right]_0^3$$

$$=\frac{2}{3}\left[-\frac{9}{2}\left(\frac{\pi}{2}\right)-9+\frac{9}{2}\right]$$

$$=\frac{2}{3}\left(\frac{9}{2}\right)\left(\frac{\pi}{2}-1\right)$$

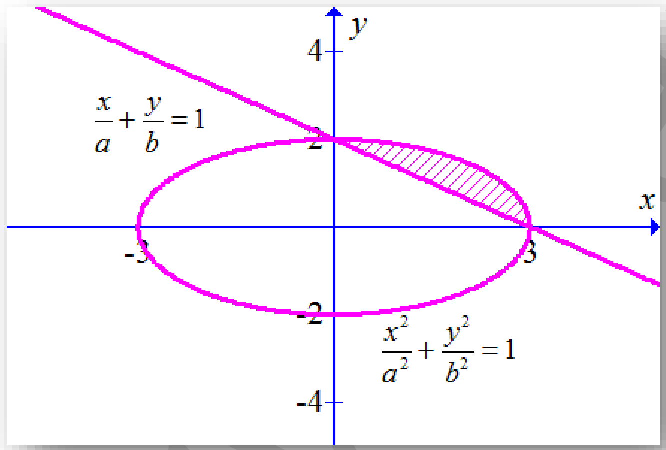
$$=\frac{3}{2}(\pi-2)$$

Therefore, the area of the smallest region in the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ is $\frac{3}{2}(\pi-2)$square units

9. Find the area of the smallest region in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y=f(x)$ and $y=g(x)$ between the

Lines $x=a, x=b$ is $\left|\int_a^b (f(x)-g(x))dx\right|$.

The area of the shaded region is the region between the curves $y=\frac{b}{a}\sqrt{a^2-x^2}$ and the line $y=\frac{b}{a}(a-x)$

Hence, the area of the shaded region is $A=\frac{b}{a}\int_0^3\left(\sqrt{a^2-x^2}-(a-x)\right)dx$

$$A=\frac{b}{a}\int_0^3\left(\sqrt{a^2-x^2}-(a-x)\right)dx$$

$$=\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}-\frac{a^2}{2}\sin^{-1}\frac{x}{a}-ax+\frac{x^2}{2}\right]_0^a$$

$$=\frac{b}{a}\left[-\frac{a^2}{2}\left(\frac{\pi}{2}\right)-a^2+\frac{a^2}{2}\right]$$

$$=\frac{b}{a}\left(\frac{a^2}{2}\right)\left(\frac{\pi}{2}-1\right)$$

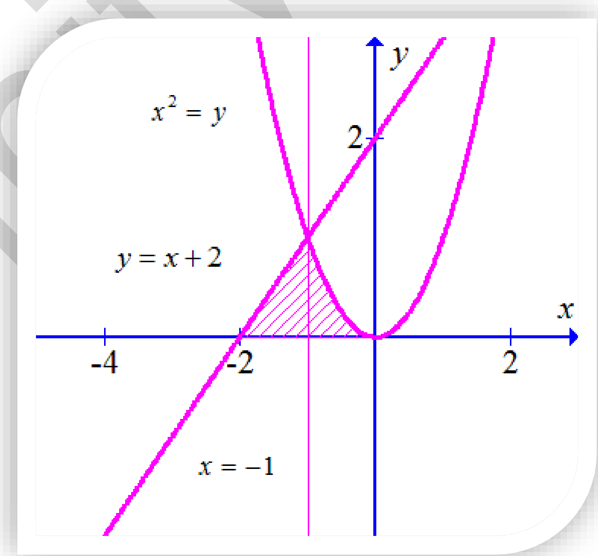
$$=\frac{ab}{4}(\pi-2)$$

Therefore, the area of the smallest region in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$ is $\frac{ab}{4}(\pi-2)$ square units

10. Find the area bounded by the curve $x^2=y$ and the line $y=x+2$ and $x-$ axis.

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y=f(x)$ and $y=g(x)$ between the

Lines $x=a, x=b$ is $\left|\int_a^b (f(x)-g(x))dx\right|$.

Points of intersection of the curve $x^2=y$ and the line $y=x+2$

$$x^2=x+2$$
$$x^2-x-2=0$$
$$x^2-2x+x-2=0$$
$$x(x-2)+1(x-2)=0$$
$$(x+1)(x-2)=0$$

Hence, the points of intersection of both curve and the line are $x=-1, x=2$

The area of the shaded region is the sum of the areas of the region bounded by the curve $y=x+2$, lines $x=-2, x=-1$ and $x-$ axis and the area of the region bounded by the curves $x^2=y$, the lines $x=-1, x=0$ and $x-$ axis in the second quadrant.

Hence, the required area is $A=\int_{-2}^{1}(x+2)dx+\int_{-1}^{0}x^2dx$

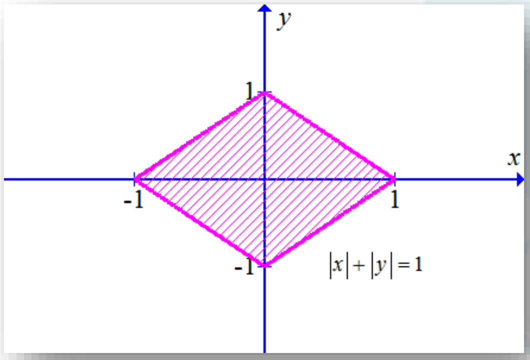
$$A=\int_{-2}^{1}(x+2)dx+\int_{-1}^{0}x^2dx$$
$$=\left(\frac{x^2}{2}+2x\right)_{-2}^{-1}+\left[\frac{x^3}{3}\right]_{-1}^{0}$$
$$=\left(\frac{1}{2}-2-2+4+\frac{1}{3}\right)$$
$$=\left(\frac{5}{6}\right)\text{square units}$$

Therefore, the area of the region bounded by the curve $x^2=y$ the line $y=x+2$ and $x-$ axis is $\frac{5}{6}$ square units

11. Using the method of integration, find the area bounded by the curve $|x|+|y|=1$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $y = f(x)$, the lines $x = a, x = b$ and $x-$ axis is $\left|\int_a^b f(x)\,dx\right|$.

The area of the shaded region is four times the area of the triangle in the first quadrant

Hence, the required area is $A = 4\int_0^1 (x+1)\,dx$

$$A = 4\int_0^1 (-x+1)\,dx$$

$$= 4\left(-\frac{x^2}{2} + x\right)_0^1$$

$$= 4\left(-\frac{1}{2} + 1\right)$$

$$= 4\left(\frac{1}{2}\right)$$

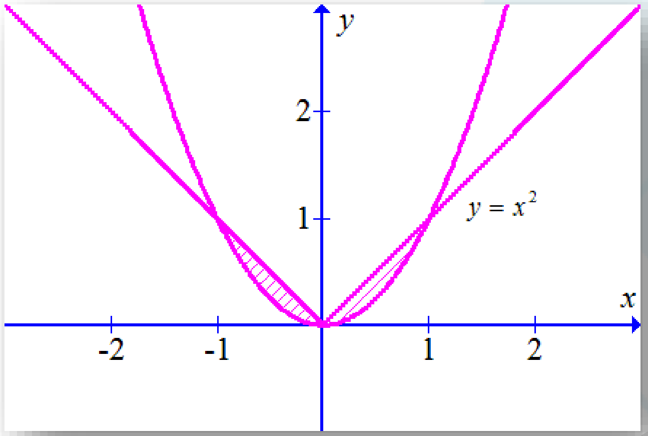
$= 2$ square units

Therefore, the area bounded by the curve $|x| + |y| = 1$ is 2 square units

12. Find the area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y = f(x)$ and $y = g(x)$ between the

Lines $x = a, x = b$ is $\left|\int_a^b (f(x) - g(x))dx\right|$.

The area of the shaded region is two times the area of the region bounded by the line $y = x^2$ and the line $y = x$ in the first quadrant

Hence, the required area is $A = 2\int_0^1 (x^2 - x)dx$

$$A = 2\int_0^1 (x^2 - x)dx$$

$$= 2\left(\frac{x^3}{3} - \frac{x^2}{2}\right)_0^1$$

$$= 2\left(\frac{1}{3} - \frac{1}{2}\right)$$

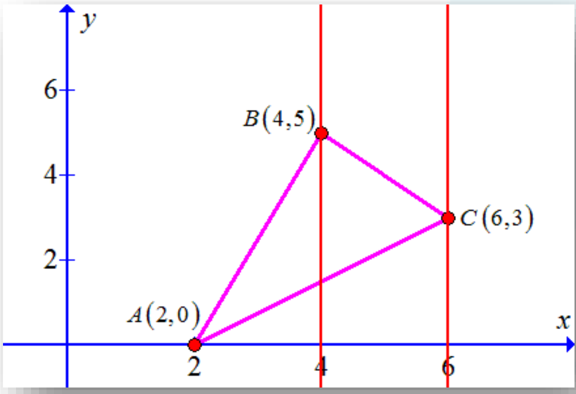
$$= \frac{1}{3}\text{square units}$$

Therefore the area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$ is $\frac{1}{3}$square units

13. Using the method of integration, find the area of the triangle formed by the points $A(2,0), B(4,5), C(6,3)$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y=f(x)$ and $y=g(x)$ between the Lines $x=a, x=b$ is $\left|\int_a^b (f(x)-g(x))dx\right|$.

The equations of the sides of the triangle are as below

Equation of AB is $y=\frac{5}{2}(x-2)$

Equation of BC is $y=-x+9$

Equation of CA is $y=\frac{3}{4}(x-2)$

The area of the triangle is sum of the following areas

(i) The area of the region between the lines $AB, AC, x=2, x=4$

(ii) The area of the region between the lines $BC, AC, x=4, x=6$

Hence, the area of the triangle is

$$A=\int_2^4 (AB-AC)dx+\int_4^6 (BC-AC)dx$$

$$=\int_2^4\left(\frac{5}{2}(x-2)-\frac{3}{4}(x-2)\right)dx+\int_4^6\left((-x+9)-\frac{3}{4}(x-2)\right)dx$$

$$=\frac{5}{2}\int_2^4 (x-2)dx-\frac{3}{4}\int_2^6 (x-2)dx+\int_4^6 (-x+9)dx$$

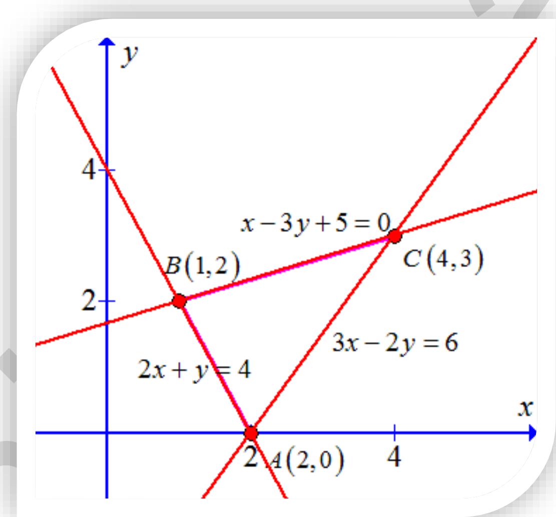
$$=\frac{5}{2}\left(\frac{x^2}{2}-2x\right)_2^4-\frac{3}{4}\left(\frac{x^2}{2}-2x\right)_2^6+\left(-\frac{x^2}{2}+9x\right)_4^6$$

$$=\frac{5}{2}(2)-\frac{3}{4}(8)+(-18+54+8-36)$$
$$=5-6+8$$
$$=7$$

Therefore, the area of the triangle formed by the points $A(2,0),B(4,5),C(6,3)$ is 7 square units.

14. Using the method of integration find the area of the region bounded by the lines $2x+y=4, 3x-2y=6, x-3y+5=0$

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y=f(x)$ and $y=g(x)$ between the Lines $x=a, x=b$ is $\left|\int_a^b (f(x)-g(x))dx\right|$.

The vertices of triangle are

Equation of AB is $y=-2x+4$

Equation of BC is $y=\frac{x+5}{3}$

Equation of CA is $y=\frac{3x-6}{2}$

The area of the triangle is sum of the following areas

(i) The area of the region between the lines $AB, BC, x=1, x=2$

(ii) The area of the region between the lines $BC, AC, x=2, x=4$

Hence, the area of the triangle is

$$A=\int_1^2 (BC-AB)\,dx+\int_2^4 (BC-AC)\,dx$$

$$=\int_1^2\left(\frac{1}{3}(x+5)-(-2x+4)\right)dx+\int_2^4\left(\frac{1}{3}(x+5)-\left(\frac{3x-6}{2}\right)\right)dx$$

$$=\frac{1}{3}\int_1^4 (x+5)\,dx+\int_1^2 (2x-4)\,dx-\frac{3}{2}\int_2^4 (x-2)\,dx$$

$$=\frac{1}{3}\left(\frac{x^2}{2}+5x\right)_1^4+\left(x^2-4x\right)_1^2-\frac{3}{2}\left(\frac{x^2}{2}-2x\right)_2^4$$

$$=\frac{1}{3}\left(28-\frac{11}{2}\right)+(-4+3)-\frac{3}{2}(2)$$

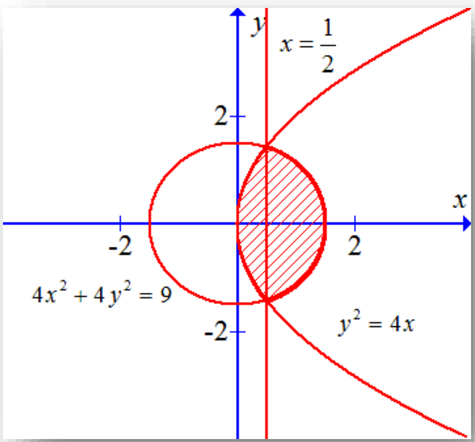
$$=\frac{15}{2}-1-3$$

$$=\frac{7}{2}$$

Therefore, the area of the region bounded by the lines $2x+y=4, 3x-2y=6$ and $x-3y+5=0$ is $\frac{7}{2}$ square units

15. Find the area of the region $\{(x,y): y^2\le 4x \text{ and } 4x^2+4y^2\le 9\}$

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y = f(x)$ and $y = g(x)$ between the Lines $x = a, x = b$ is $\left|\int_a^b \left(f(x) - g(x)\right)dx\right|$.

Point of intersection of $y^2 = 4x, 4x^2 + 4y^2 = 9$

$$4x^2 + 4(4x) = 9$$
$$4x^2 + 16x - 9 = 0$$
$$4x^2 + 18x - 2x - 9 = 0$$
$$2x(2x+9) - 1(2x+9) = 0$$
$$(2x-1)(2x+9) = 0$$

Observing the figure both curves intersect at $x = \frac{1}{2}$

The area of the shaded region is sum of the following areas

(i) Two times the area of the region bounded by the curve $y^2 = 4x$, lines $x = 0, x = \frac{1}{2}$ and $x-$ axis in the first quadrant.

(ii) Two times the area of the region bounded by the curve $4x^2 + 4y^2 = 9$, lines $x = \frac{1}{2}, x = \frac{3}{2}$ and $x-$ axis in the first quadrant.

Hence, the area of the shaded region is $A = 2\left[\int_0^{\frac{1}{2}} 2\sqrt{x}dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{9-4x^2}dx\right]$

$$A=2\left[\int_0^{\frac{1}{2}} 2\sqrt{x}dx+\int_{\frac{1}{2}}^{\frac{3}{2}}\frac{1}{2}\sqrt{9-4x^2}dx\right]$$

$$=2\left[\int_0^{\frac{1}{2}} 2x^{\frac{1}{2}}dx+\int_{\frac{1}{2}}^{\frac{3}{2}}\frac{1}{2}\sqrt{3^2-(2x)^2}dx\right]$$

$$=2\left[2\left(\frac{2}{3}x^{\frac{3}{2}}\right)_0^{\frac{1}{2}}+\frac{1}{4}\left(\frac{2x}{2}\sqrt{9-4x^2}+\frac{9}{2}\sin^{-1}\left(\frac{2x}{3}\right)\right)_{\frac{1}{2}}^{\frac{3}{2}}\right]$$

$$=2\left[2\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{4}\left(\frac{9}{2}\left(\frac{\pi}{2}\right)-\frac{1}{2}2\sqrt{2}-\frac{9}{2}\sin^{-1}\frac{1}{3}\right)\right]$$

$$=2\left(\frac{2\sqrt{2}}{3}+\frac{9\pi}{16}-\sqrt{2}-\frac{9}{8}\sin^{-}\frac{1}{3}\right)$$

$$=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)+\frac{1}{3\sqrt{2}}$$

Therefore, the area of the region $\{(x,y): y^2\le 4x \text{ and } 4x^2+4y^2\le 9\}$ is

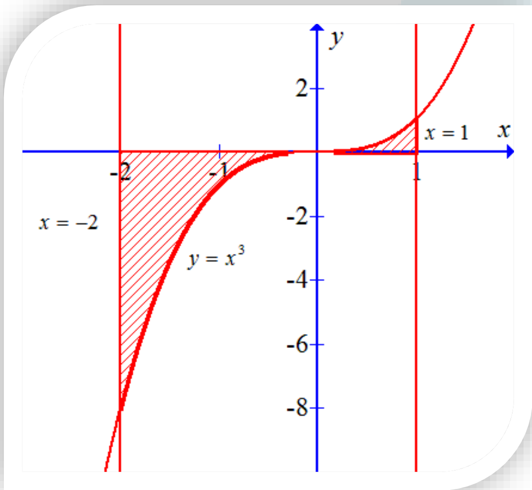
$$\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)+\frac{1}{3\sqrt{2}}$$

16. Area bounded by the curve $y=x^3$, the $x-$ axis and ordinates $x=-2$ and $x=1$ is in square units

1) -9 2) $-\frac{15}{4}$ 3) $\frac{15}{4}$ 4) $\frac{17}{4}$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $y = f(x)$, the lines $x = a, x = b$ and $x-$ axis is $\left|\int_a^b f(x)dx\right|$.

The area of the shaded region is sum of areas of the regions bounded by the curve $y = x^3$, the line $x = 0, x = 1$ and $x-$ axis in the first quadrant and bounded by the curve $y = -x^3$, the line $x = 0, x = -2$ and $x-$ axis in the third quadrant

Hence, the required area is $A = \int_{-2}^{0} -x^3dx + \int_0^1 x^3dx$

$$A = \int_{-2}^{0} -x^3dx + \int_0^1 x^3dx$$
$$= \left(-\frac{x^4}{4}\right)_{-2}^{0} + \left(\frac{x^4}{4}\right)_0^1$$
$$= \frac{16}{4} + \frac{1}{4}$$
$$= \frac{17}{4}\text{square units}$$

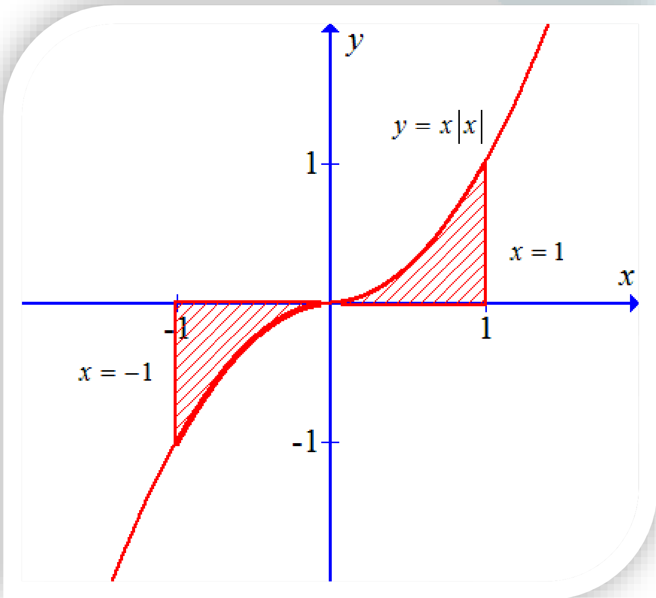
Therefore, the option 4 is correct

17. Area bounded by the curve $y = x|x|$, $x-$ axis and the ordinates $x = -1$ and $x = 1$ is given by in square units.

1) 0 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) $\frac{4}{3}$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded by the curve $y = f(x)$, the lines $x = a, x = b$ and $x-$ axis is $\left|\int_a^b f(x)\,dx\right|$.

The given equation of the curve $y = x|x|$ can be rewrite as

$$y = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$$

The area of the shaded region is two times the area of the region bounded by the curve $y = x^2$, the line $x = 0, x = 1$ and $x-$ axis.

Hence, the required area is $A = 2\int_0^1 x^2 dx$

$$A = 2\int_0^1 x^2 dx$$

$$= 2\left(\frac{x^3}{3}\right)_0^1$$

$$= \frac{2}{3}\text{square units}$$

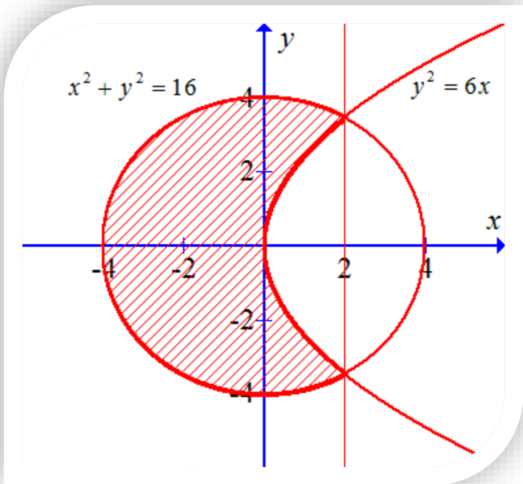
Therefore, the option 3 is correct.

18. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$

1) $\frac{4}{3}\left(4\pi - \sqrt{3}\right)$ 2) $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$ 3) $\frac{4}{3}\left(8\pi - \sqrt{3}\right)$ 4) $\frac{4}{3}\left(8\pi + \sqrt{3}\right)$

Solution:

The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y = f(x)$ and $y = g(x)$ between the Lines $x = a, x = b$ is $\left|\int_a^b \left(f(x) - g(x)\right)dx\right|$

The points of intersection of the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$

$$x^2 + 6x - 16 = 0$$

$$x^2 + 8x - 2x - 16 = 0$$

$$x(x+8) - 2(x+8) = 0$$

$$(x-2)(x+8) = 0$$

It implies that $x = 2$ or $x = -8$

Observing the graph, the point of intersection of circle and parabola is $x = 2$

The area of the shaded region is sum of the area of the semicircle $x^2 + y^2 = 16$ and two times the area of the region bounded by the circle $y = \sqrt{16 - x^2}$, parabola $y = \sqrt{6x}$, the lines $x = 0, x = 2$, $x -$ axis in the first quadrant.

Hence, the required area is $A = 8\pi + 2\int_0^2 \sqrt{16-x^2} - \sqrt{6x}dx$

$$A = 8\pi + 2\int_0^2 \sqrt{16-x^2} - \sqrt{6x}dx$$

$$= 8\pi + 2\left(\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right) - \sqrt{6}\left(\frac{2}{3}\right)x^{\frac{3}{2}}\right)_0^2$$

$$= 8\pi + 2\left(\sqrt{12} + 8\cdot\frac{\pi}{6} - \sqrt{6}\left(\frac{2}{3}\right)2\sqrt{2}\right)$$

$$= 8\pi + 2\left(2\sqrt{3} + \frac{4\pi}{3} - \frac{8\sqrt{3}}{3}\right)$$

$$= 2\left(\frac{16\pi}{3} - \frac{2\sqrt{3}}{3}\right)$$

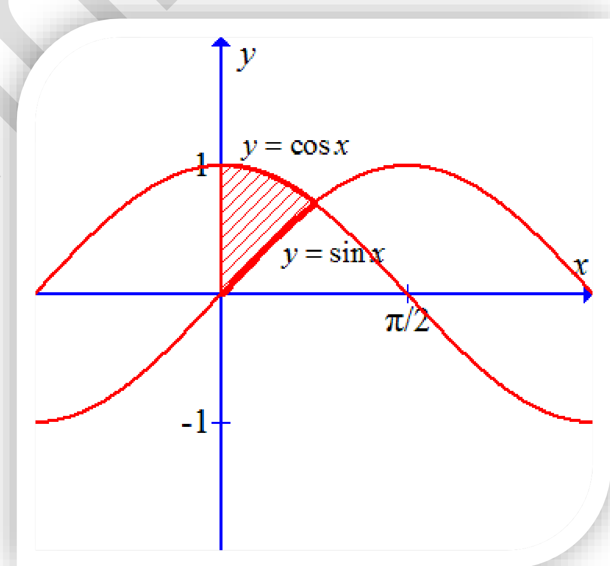
$$= \frac{4}{3}\left(8\pi - \sqrt{3}\right)$$

Therefore, the option 3 is correct.

19. The area bounded by the $y-$ axis , $y = \cos x, y = \sin x$ when $0 \le x \le \frac{\pi}{2}$

1) $2\left(\sqrt{2}-1\right)$ 2) $\left(\sqrt{2}-1\right)$ 3) $\left(\sqrt{2}+1\right)$ 4) $\sqrt{2}$

Solution: The required area of the region is the shaded region in the following figure



The area of the region bounded between two curves $y = f(x)$ and $y = g(x)$ between the Lines $x = a, x = b$ is $\left|\int_a^b \left(f(x) - g(x)\right)dx\right|$.

The area of the shaded region is the area of the region between the two curves $y = \sin x$ and $y = \cos x$, lines $x = 0, x = \frac{\pi}{4}$ in the first quadrant.

Hence, the required area is $A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$

$$A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$$

$$= (\sin x + \cos x)_0^{\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= \left(\sqrt{2} - 1\right)\text{square units}$$

Therefore, the option 2 is correct.