Chapter: 9. Differential Equations

Exercise: 9.1

1. Determine order and degree (if defined) of differential equation $\frac{d^4y}{dx^4}+\sin\left(y'''\right)=0$

Solution:

Rewrite the equation $\frac{d^4y}{dx^4}+\sin\left(y'''\right)=0$. As

$\Rightarrow y''''+\sin\left(y'''\right)=0$

The highest order between the two terms is of y'''' which is four.

The differential equation contains a trigonometric derivative term and is not completely polynomial in its derivative, thus degree is not defined.

2. Determine order and degree (if defined) of differential equation $y'+5y=0$.

Solution:

The given differential equation is $y'+5y=0$

The highest order term is y', thus the order is one.

As the derivative is of completely polynomial nature is and highest power of derivative is

of y' which is one. Thus degree is one

3. Determine order and degree (if defined) of differential equation $\left(\frac{ds}{dt}\right)^4+3s\frac{d^2s}{dt^2}=0$

Solution:

The given differential equation is $\left(\frac{ds}{dt}\right)^4+3s\frac{d^2s}{dt^2}=0$

The highest order term is $\frac{d^2s}{dt^2}$, thus the order is two.

As the derivative is of completely polynomial nature is and highest power of derivative term $\left(\frac{ds}{dt}\right)^4$ which is four. Thus the degree is four.

4. Determine order and degree (if defined) of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$$

Solution:

The given differential equation is $\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$

The highest order term is $\frac{d^2y}{dx^2}$, thus the order is two

The differential equation contains a trigonometric derivative term and is not completely polynomial in its derivative, thus degree is not defined.

5. Determine order and degree (if defined) of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2-\cos 3x+\sin 3x=0$$

Solution:

The given differential equation is $\left(\frac{d^2y}{dx^2}\right)^2-\cos 3x+\sin 3x=0$

$\left(\frac{d^2y}{dx^2}\right)^2-\cos 3x+\sin 3x=0$ the highest order term is $\frac{d^2y}{dx^2}$, thus the order is two

As the derivative is of completely polynomial nature is and highest power of derivative

term which is two. Thus degree is two $\left(\frac{d^2y}{dx^2}\right)^2$

6. Determine order and degree (if defined) of differential equation

$(y''')^2+(y'')^3+(y')^4+y^5=0$

Solution:

The given differential equation is $(y''')^2+(y'')^3+(y')^4+y^5=0$.

The highest order term is $(y''')^2$, thus the order is three.

The differential equation is of the polynomial form and the power of highest order term y''' is two, thus the degree is two

7. Determine order and degree (if defined) of differential equation $y'''+2y''+y'=0$

Solution:

The given differential equation is . y'''+2y''+y'=0

The highest order derivative in the differential equation is y'''. Thus its order is three.

The differential equation is polynomial with the highest order term y''' having a degree one. Thus the degree is one.

8. Determine order and degree (if defined) of differential equation $y'+y=e'$

Solution:

The given differential equation is $y'+y=e'$. Therefore

$\Rightarrow y'+y-e'=0$

The highest order derivative in the differential equation is y'. Thus its order is one.

The given equation is of polynomial form with the highest order term y' with degree one. Thus the degree is one.

9. Determine order and degree (if defined) of differential equation $y'+(y')^2+2y=0$

Solution:

The given differential equation is $y'+(y')^2+2y=0$

The highest order derivative in the differential equation is y'. Thus its order is one.

The given equation is of polynomial form with the highest order term y' with highest degree two. Thus the degree is two.

10. Determine order and degree (if defined) of differential equation $y''+2y'+\sin y=0$

Solution:

The given differential equation is $y''+2y'+\sin y=0$.

The highest order derivative in the differential equation is y''. Thus its order is two.

The given equation is of polynomial form with the highest order term y'' with the highest degree one. Thus the degree is one.

11. Find the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$

Solution:

The given differential equation is $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$

The differential equation is not polynomial in its derivative because of the term $\sin\left(\frac{dy}{dx}\right)$

thus its order is not defined.

The correct answer is (D).

12. Find the order of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$

Solution:

The given differential equation is $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$

The highest order term of the equation is $\frac{d^2y}{dx^2}$, thus the order is two

The correct answer is (A)