Chapter: 9. Differential Equations

Exercise: 9.2

1. Verify the function $y = e^x + 1$ is solution of differential equation $y'' - y = 0$

Solution:

The given function is $y = e^x + 1$

Take its derivative

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + 1\right)$$

$$\Rightarrow y' = e^x \text{..........}(1)$$

Take the derivative of the above equation

$$\frac{d}{dx}(y') = \frac{d}{dx}\left(e^x\right)$$

$$\Rightarrow y'' = e^x$$

Using result from equation (1)

$$y'' - y' = 0$$

Thus, the given function is solution of differential equation $y'' - y' = 0$

2. Verify the function $y = x^2 + 2x + C$ is solution of differential equation $y' - 2x - 2 = 0$

Solution:

The given function is $y = x^2 + 2x + C$

Take its derivative

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + 2x + C\right)$$

$$\Rightarrow y' = 2x + 2$$

Thus, the given function is solution of differential equation $y'-2x-2=0$

3. Verify the function $y=\cos x+C$ is solution of differential equation $y'+\sin x=0$

Solution:

The given function is $y=\cos x+C$

Take its derivative

$$\frac{dy}{dx}=\frac{d}{dx}(\cos x+c)$$

$$\Rightarrow y'=-\sin x$$

$$\Rightarrow y'+\sin x=0$$

Thus the given function is solution of differential equation $y'+\sin x=0$

4. Verify the function $y=\sqrt{1+x^2}$ is solution of differential equation $y'=\frac{xy}{1+x^2}$

Solution:

The given function is $y=\sqrt{1+x^2}$

Take its derivative

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{1+x^2}\right)$$

$$y'=\frac{1}{2\sqrt{1+x^2}}\times\frac{d}{dx}\left(1+x^2\right)$$

$$y'=\frac{2}{2\sqrt{1+x^2}}$$

$$\Rightarrow y'=\frac{1}{\sqrt{1+x^2}}$$

Multiply numerator and denominator by $\sqrt{1+x^2}$

$$y'=\frac{1}{\sqrt{1+x^2}}\times\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}$$

Substitute $y=\sqrt{1+x^2}$

$$\Rightarrow y'=\frac{xy}{1+x^2}$$

Thus the given function is solution of differential equation $y'=\frac{xy}{1+x^2}$

5. Verify the function $y=Ax$ is solution of differential equation $xy'=y\left(x\neq 0\right)$

Solution:

The given function is $y=Ax$

Take its derivative

$$\frac{dy}{dx}=\frac{d}{dx}\left(Ax\right)$$

$$\Rightarrow y'=A$$

Multiply by x on both side

$$xy'=Ax$$

Substitute $y=Ax$

$$\Rightarrow xy'=y$$

Thus the given function is solution of differential equation $xy'=y\left(x\neq 0\right)$

6. Verify the function $y=x\sin x$ is solution of differential equation $xy'=y+x\sqrt{x^2-y^2}$ ($x\neq 0$ and $x>y$ or $x<-y$)

Solution:

The given function is $y = x\sin x$

Take its derivative

$$\frac{dy}{dx} = \frac{d}{dx}(x\sin x)$$

$$\Rightarrow y' = \sin x\frac{d}{dx}(x) + x\frac{d}{dx}(\sin x)$$

$$\Rightarrow y' = \sin x + x\cos x$$

Multiply by x on both side

$$xy' = x(\sin x + x\cos x)$$

$$xy' = x\sin x + x^2\cos x$$

Substitute $y = x\sin x$

$$\Rightarrow xy' = y + x^2\cos x$$

Use $\sin x = \frac{y}{x}$ and substitute $\cos x$

$$xy' = y + x^2\sqrt{1-\sin^2 x}$$

$$xy' = y + x^2\sqrt{1-\left(\frac{y}{x}\right)^2}$$

$$\Rightarrow xy' = y + x\sqrt{y^2 - x^2}$$

Thus the given function is solution of differential equation $\Rightarrow xy' = y + x\sqrt{y^2 - x^2}$

7. Verify the function $xy = \log y + c$ is solution of differential equation

$$y' = \frac{y^2}{1-xy}(xy \neq 1)$$

Solution:

The given function is $xy = \log y + c$

Take derivative on both sides

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y\frac{d}{dx}(x) = x\frac{d}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow y + xy' = \frac{1}{y}y'$$

$$\Rightarrow y^2 + xyy' = y'$$

Shift the y' term on one side and take it common

$$\Rightarrow (xy-1)y' = -y^2$$

$$\Rightarrow y' = \frac{y^2}{1-xy}$$

Thus the given function is solution of differential equation $y' = \dfrac{y^2}{1-xy}$

8. Verify the function $y - \cos y = x$ is solution of differential equation $(y\sin y + \cos y + x)y' = 1$

Solution:

The given function is $y - \cos y = x$

Take derivative on both side

$$\frac{dy}{dx} - \frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$$

$$\Rightarrow y' + y'\sin y = 1$$

$\Rightarrow y'(1+\sin y)=1$

$\Rightarrow y'=\dfrac{1}{1+\sin y}$

Multiply by $(y\sin y+\cos y+x)$ on both side

$(y\sin y+\cos y+x)y'=\dfrac{(y\sin y+\cos y+x)}{1+\sin y}$

Substitute $y=\text{cosy}+\text{x}$ in the numerator

$(y\sin y+\cos y+x)y'=\dfrac{(y\sin y+y)}{1+\sin y}$

$(y\sin y+\cos y+x)y'=\dfrac{y(\sin y+y)}{1+\sin y}$

$\Rightarrow(y\sin y+\cos y+x)y'=y$

Thus the given function is solution of differential equation $(y\sin y+\cos y+x)y'=y$

9. Verify the function $x+y=\tan^{-1}y$ is solution of differential equation $y^2y'+y^2+1=0$

Solution:

The given function is $x+y=\tan^{-1}y$

Take derivative on both side

$\dfrac{d}{dx}(x+y)=\dfrac{d}{dx}\left(\tan^{-1}y\right)$

$1+y'=\left(\dfrac{1}{1+y^2}\right)y'$

$\Rightarrow y'\left[\dfrac{1}{1+y^2}-1\right]=1$

$$\Rightarrow y'\left[\frac{1-\left(1+y^2\right)}{1+y^2}\right]=1$$

$$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right]=1$$

$$\Rightarrow -y^2y'=1+y^2$$

$$\Rightarrow y^2y'+y^2+1=0$$

Thus the given function is solution of differential equation $y^2y'+y^2+1=0$

10. Verify the function $y=\sqrt{a^2-x^2}\;\; x\in(-a,a)$ is solution of differential equation

$$x+y\frac{dy}{dx}=0\left(y\neq 0\right)$$

Solution:

The given function is $y=\sqrt{a^2-x^2}\;\; x\in(-a,a)$

Take derivative on both side

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{a^2-x^2}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{a^2-x^2}}\frac{d}{dx}\left(a^2-x^2\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{-2x}{2\sqrt{a^2-x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-x}{\sqrt{a^2-x^2}}$$

Substitute $y=\sqrt{a^2-x^2}$

$$\Rightarrow \frac{dy}{dx}=\frac{-x}{y}$$

$$\Rightarrow x+y\frac{dy}{dx}=0$$

Thus the given function is solution of differential equation $x+y\frac{dy}{dx}=0\left(y\neq 0\right)$

11. Find the numbers of arbitrary constants in the general solution of a differential equation of fourth order

Solution:

The number of arbitrary constants in the general solution of a differential equation is equal to its order. As the given differential equation is of fourth order, thus it has four arbitrary constants in its solution.

The correct answer is (D).

12. Find the numbers of arbitrary constants in the particular solution of a differential equation of third order

Solution:

The particular solution of any differential equation does not have any arbitrary constants.

Thus it has zero constants in its solution.

The correct answer is (D).