Chapter 1: Sets

**Example 1**

**Write the solution set of the equation $\mathbf{x^2 + x - 2 = 0}$ in roster form.**

**Solution**

The given equation can be written as

$x^2 + 2x - x - 2 = 0$

$(x+2)(x-1) = 0$

$x(x+2) - 1(x+2) = 0$

So, $x = -2$ or $x = 1$

Thus, the solution set of the equation can be written in roster form as $\{1, -2\}$.

**Example 2**

**Write the set $\{x : x$ is a positive integer and $x^2 < 40\}$ in the roster form.**

**Solution**

Given,

$\{x : x$ is a positive integer and $x^2 < 40\}$

$1^2 = 1 < 40$

$2^2 = 4 < 40$

$3^2 = 9 < 40$

$4^2 = 16 < 40$

$5^2 = 25 < 40$

$6^2 = 36 < 40$

$7^2 = 49 > 40$

The required numbers are $\mathbf{1, 2, 3, 4, 5, 6}$.

So, we have the given set in the roster form is $\{1, 2, 3, 4, 5, 6\}$

**Example 3**

Write the set $A = \{1, 4, 9, 16, 25, \dots\}$ in set-builder form.

**Solution**

Given**,** Sets $A = \{1, 4, 9, 16, 25, \dots\}$

Consider each of the elements separately.

Write them as,

$1 = (1)^2$

$4 = (2)^2$

$9 = (3)^2$

$16 = (4)^2$

$25 = (5)^2$

$36 = (6)^2$

And so on. Now, from the above, observed that all the elements are the squares of the natural numbers**.**

The range of n as natural numbers, that is $n \in N$.

$A = \{x : x$ is the square of a natural number $\}$

Alternatively, can write

$A = \left\{x : x = n^2, \text{ where } n \in \mathbf{N}\right\}$

**Example 4**

**Write the set** $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ **in the set-builder form.**

**Solution**

Given,

Set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$

Consider each of the elements separately.

$\frac{1}{2} = \frac{1}{1+1}$

$\frac{2}{3} = \frac{2}{2+1}$

$\frac{3}{4} = \frac{3}{3+1}$

$\frac{4}{5} = \frac{4}{4+1}$

$\frac{5}{6} = \frac{5}{5+1}$

$\frac{6}{7} = \frac{6}{6+1}$

Every member in the given set becomes a single numerator one less than the denominator.

Also, the numerator occurs 1 and doesn't exceed $6$.

Hence, within the set-builder form the given set is

$\left\{ x : x = \frac{n}{n+1}\right.$, where $n$ is a natural number and $\left. 1 \le n \le 6 \right\}$

**Example 5**

**Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form:**

| | |
|---|---|
| **(i)** $\{P, R, I, N, C, A, L\}$ | **(a)** $\{x : x$ **is a positive integer and is a divisor of 18**$\}$ |
| **(ii)** $\{0\}$ | **(b)** $\{x : x$ **is an integer and** $x^2 - 9 = 0\}$ |
| **(iii)** $\{1, 2, 3, 6, 9, 18\}$ | **(c)** $\{x : x$ **is an integer and** $x + 1 = 1\}$ |
| **(iv)** $\{3, -3\}$ | **(d)** $\{x : x$ **is a letter of the word PRINCIPAL** $\}$ |

**Solution**

1) Let $A = \{x : x$ is a letter of the word PRINCIPLE $\}$

Therefore, $A = \{P, R, I, N, C, L, E\}$

so (i) matches (d).

2) Let $B = \{x : x$ is an integer and $x + 1 = 1\}$

Therefore, $B = \{0\}$

(ii) matches (c) as $x+1=1$ implies $x=0$.

3) Let $C=\{x : x$ is a positive integer and divisor of $18\}$

Therefore $C=\{1,2,3,6,9,18\}$

So (iii) matches (a).

4) Let $D=\left\{x : x \text{ is an integer and } x^2-9=0\right\}$

Now, $x^2-9=0$

$x^2=9$

$x=\pm 3$

$D=\{3,-3\}$

**EXERCISE 1.1**

**1. Which of the following are sets? Justify your answer.**

**(i) The collection of all the months of a year beginning with the letter $J$.**

**(ii) The collection of ten most talented writers of India.**

**(iii) A team of eleven best-cricket batsmen of the world.**

**(iv) The collection of all boys in your class.**

**(v) The collection of all natural numbers less than 100.**

**(vi) A collection of novels written by the writer Munshi Prem Chand.**

**(vii) The collection of all even integers.**

**(viii) The collection of questions in this Chapter.**

**(ix) A collection of most dangerous animals of the world.**

**Solution**

(i) The collection consists of months January, June and July. It is well-defined and therefore, it's a group.

(ii) A writer of India could also be most talented for one person but not for an additional person. Opinion varies from person to person. So, the given collection isn't well-defined and thus, not a set.

(iii) The term 'best cricket batsman' is vague. The same batsman could also be one among the simplest for one person but not for an additional. Opinion varies from person to person. So, the given collection isn't well-defined and thus, not a set.

(iv) Any boy is either in your class or not in your class. There is no ambiguity. The given collection is well-defined and therefore, it's a group.

(v) The collection consists of first 99 natural numbers. It is well-defined and therefore, it is a set.

(vi) it's a well-defined collection and thus, it is a set.

(vii) it's a well-defined collection and thus , it is a set.

(viii) It is a well-defined collection and therefore, it is a set.

(ix) The criterion for determining an animal as most dangerous varies from person to person. For some people, even a lizard is extremely dangerous. So, the given collection isn't well defined and thus , it's not a group ..

**2. Let $A=\{1,2,3,4,5,6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:**

**(i)** $5 \ldots A$

**(ii)** $8 \ldots A$

**(iii)** $0 \ldots A$

**(iv)** $4 \ldots A$

**(v)** $2 \ldots A$

**(vi)** $10 \ldots . A$

**Solution**

Given, $A=\{1,2,3,4,5,6\}$

So, Find the symbols that matches

(i) 5 is in set A

So, $5 \in A$

(ii) 8 is not in set A

So, $8 \notin A$

(iii) 0 is not in set A

So, $0 \notin A$

(iv) 4 is in set A

Hence, $4 \in A$

(v) 2 is not in set A

Hence, $2 \in A$

(vi) 10 is not in set A

Hence, $10 \notin A$.

**3. Write the following sets in roster form:**

**(i)** $\mathrm{A}=\{x: x$ **is an integer and** $-3 \leq x<7\}$

**(ii)** $\mathrm{B}=\{x: x$ **is a natural number less than 6**$\}$

**(iii)** $\mathrm{C}=\{x: x$ **is a two-digit natural number such that the sum of its digits is 8**$\}$

**(iv)** $\mathrm{D}=\{x: x$ **is a prime number which is divisor of 60**$\}$

**(v)** $\mathrm{E}=$ **The set of all letters in the word TRIGONOMETRY**

**(vi)** $\mathrm{F}=$ **The set of all letters in the word BETTER**

**Solution**

(i) The integers are $-3,-2,-1,0,1,2,3,4,5,6$ (not 7).

$\therefore$ In roster form, $\mathrm{A}=\{-3,-2,-1,0,1,2,3,4,5,6\}$

(ii) Natural numbers less than 6 are $1,2,3,4,5$.

$\therefore$ In roster form, $B=\{1,2,3,4,5\}$

(iii) The numbers are $17,26,35,44,53,62,71,80$.

$\therefore$ In roster form, $\mathrm{C}=\{17,26,35,44,53,62,71,80\}$

(iv) Divisors of 60 are $1,2,3,4,5,6,10,12,15,20,30,60$.

Among them, prime numbers are $2,3,5$.

$\therefore$ In roster form, $\mathrm{D}=\{2,3,5\}$

(v) In the word TRIGONOMETRY, the letters T, R and O are repeat. Dropping the repetitions, in roster form $\mathrm{E}=\{\mathrm{T}, \mathrm{R}, \mathrm{I}, \mathrm{G}, \mathrm{O}, \mathrm{N}, \mathrm{M}, \mathrm{E}, \mathrm{Y}\}$

(vi) In the word BETTER, the letters $\mathrm{E}$ and $\mathrm{T}$ are repeated. Dropping the repetitions, in roster form $\mathrm{F}=\{\mathrm{B}, \mathrm{E}, \mathrm{T}, \mathrm{R}\}$

.

**4. Write the following sets in the set-builder form:**

**(i)** $(3,6,9,12\}$

**(ii)** $\{2,4,8,16,32\}$

**(iii)** $\{5,25,125,625\}$

**(iv)** $\{2,4,6, \ldots\}$

**(v)** $\{1,4,9, \ldots, 100\}$

**Solution**

(i) $\{3,6,9,12\}=\{3\times 1,3\times 2,3\times 3,3\times 4\}$

$=\{x: x=3n, n\in N \text{ and } 1\leq n\leq 4\}$

(ii) $\{2,4,8,16,32\}=\left\{2^1,2^2,2^3,2^4,2^5\right\}$

$=\left\{x: x=2^n, n\in N \text{ and } 1\leq n\leq 5\right\}$

(iii) $\{5,25,125,625\}=\left\{5^1,5^2,5^3,5^4\right\}$

$=\left\{x: x=5^n, n\in N \text{ and } 1\leq n\leq 4\right\}$

(iv) $\{2,4,6,\ldots\}=\{2\times 1,2\times 2,2\times 3,\ldots\}$

$=\{x: x=2n, n\in N\}$

Alternatively, we can write $\{x: x$ is an even natural number $\}$.

(v) $\{1,4,9,\ldots,100\}=\left\{1^2,2^2,3^2,\ldots\ldots,10^2\right\}$

$=\left\{x: x=n^2, n\in N \text{ and } 1\leq n\leq 10\right\}$

**5. List all the elements of the following sets:**

**(i)** $\mathrm{A}=\{x: x$ **is an odd natural number** $\}$

**(ii)** $\mathrm{B}=\left\{x: x \text{ is an integer, } -\frac{1}{2}<x<\frac{9}{2}\right\}$

**(iii)** $\mathrm{C}=\left\{x: x \text{ is an integer, } x^2\leq 4\right\}$

**(iv)** $\mathrm{D}=\{x: x$ **is a letter in the word "LOYAL"** $\}$

**(v)** $\mathrm{E}=\{x: x$ **is a month of a year not having 31 days** $\}$

**(vi)** $\mathrm{F}=\{x: x$ **is a consonant in the English alphabet which precedes** $k\}$.

**Solution**

(i) Odd natural numbers are $1,3,5,\ldots$

$\therefore \quad \mathrm{A}=\{1,3,5,\ldots\}$

(ii)Integers greater than $-\frac{1}{2}$ and less than $\frac{9}{2}$ are $0,1,2,3,4$

$\therefore \quad B = \{0,1,2,3,4\}$

(iii) Integers whose square is less than or equal to $4$ are $-2$,

$-1,0,1,2$

$\therefore C = \{-2,-1,0,1,2\}$

(iv) Dropping the repetition $D = \{L, O, Y, A\}$

(v) Months of a year not having 31 days are: February, April, June, September, November

$\therefore E = \{February, April, June, September, November\}$

(vi) Consonants in the English alphabet which precede $k$ are:

$b, c, d, f, g, h, j$

$\therefore F = \{b, c, d, f, g, h, j\}$

**6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:**

| | |
|---|---|
| (i) $\{1,2,3,6\}$ | (a) $\{x : x$ is a prime number and a divisor of $6\}$ |
| (ii) $\{2,3\}$ | (b) $\{x : x$ is an odd natural number less than 10 $\}$ |
| **(iii)** $\{M, A, T, H, E, I, C, S\}$ | **(c)** $\{x : x$ **is natural number and divisor of 6** $\}$ |
| **(iv)** $\{1,3,5,7,9\}$ | **(d)** $\{x : x$ **is a letter of the word MATHEMATICS** $\}$**.** |

**Solution**

(i) All the elements of the set are natural numbers similarly well-being the divisors of 6.

Hence, (i) matches with (c).

(ii) It can see the 2 and 3 are prime numbers. They include the divisors of 6.

So, (ii) matches with (a).

(iii) All the elements of the set are letters of the expression MATHEMATICS.

Hence, (iii) matches with (d).

(iv) All the elements of the set are odd natural numbers less than 10.

Hence, (iv) matches with (b).

**Example 6**

**State which of the following sets are finite or infinite:**

(i) $\{x : x \in \mathrm{N}$ **and** $(x-1)(x-2)=0\}$

(ii) $\left\{x : x \in \mathrm{N}\right.$ **and** $\left.x^{2}=4\right\}$

(iii) $\{x : x \in \mathrm{N}$ **and** $2x-1=0\}$

(iv) $\{x : x \in \mathrm{N}$ **and** $x$ **is prime** $\}$

(v) $\{x : x \in \mathrm{N}$ **and** $x$ **is odd** $\}$

**Solution**

(i) $\{x : x \in \mathrm{N}$ **and** $(x-1)(x-2)=0\}$

$(x-1)(x-2)=0$

Hence, $x=1,2$

Given set $=\{1,2\}$.

Since the elements has 2 sets

Hence, it is finite.

(ii) $\left\{x : x \in \mathrm{N}\right.$ **and** $\left.x^{2}=4\right\}$

$x^{2}=4$

$x=\pm 2$

The x be the natural numbers

Given set $=\{2\}$.

Hence, it is finite.

(iii) $\{x : x \in \mathrm{N}$ and $2x-1=0\}$

$2x-1=0$

$x=\frac{1}{2}$

But $x$ should be a natural number

It cannot be fraction.

The given set has no elements.

Given set $=\phi$.

Since the number of elements of a null set is 0 ,

Hence, it is finite.

(iv) $\{x : x \in \mathrm{N}$ and $x$ is prime $\}$

A given set is a set of all key numbers and as a set of key numbers is unlimited. So, the given set is infinite

(v) $\{x : x \in \mathrm{N}$ and $x$ is odd $\}$

As there is an infinite number of odd numbers, therefore, the set given is infinite.

**Example 7**

**Find the pairs of equal sets, if any, give reasons:**

$\mathrm{A} = \{0\}$,

$\mathrm{B} = \{x : x > 15 \text{ and } x < 5\}$

$\mathrm{C} = \{x : x - 5 = 0\}$,

$\mathrm{D} = \left\{x : x^2 = 25\right\}$

$\mathrm{E} = \left\{x : x\right.$ **is an integral positive root of the equation** $\left. x^2 - 2x - 15 = 0\right\}$

**Solution**

Since $0 \in A$ and 0 does not belong to any of the $B, C, D$ and $E$ sets, it follows, $\mathrm{A} \neq \mathrm{B}, \mathrm{A} \neq \mathrm{C}, \mathrm{A} \neq \mathrm{D}, \mathrm{A} \neq \mathrm{E}$.

From $\mathrm{B} = \phi$ but none no other sets are empty. So $\mathrm{B} \neq \mathrm{C}, \mathrm{B} \neq \mathrm{D}$ and $\mathrm{B} \neq \mathrm{E}$.

And $\mathrm{C} = \{5\}$ but $-5 \in \mathrm{D}$, hence $\mathrm{C} \neq \mathrm{D}$.

From $\mathrm{E} = \{5\}, \mathrm{C} = \mathrm{E}$. Additionally $\mathrm{D} = \{-5, 5\}$ and $\mathrm{E} = \{5\}$, we find that, $\mathrm{D} \neq \mathrm{E}$. Thus, the only pair of equal sets is $\mathrm{C}$ and $\mathrm{E}$.

**Example 8**

**Which of the following pairs of sets are equal? Justify your answer.**

**(i)** $\mathrm{X}$ **, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".**

**(ii)** $\mathrm{A} = \left\{n : n \in \mathrm{Z}\right.$ **and** $\left. n^2 \leq 4\right\}$ **and** $\mathrm{B} = \left\{x : x \in \mathrm{R}\right.$ **and** $\left. x^2 - 3x + 2 = 0\right\}$**.**

**Solution**

(i) $\mathrm{X} = \{\mathrm{A}, \mathrm{L}, \mathrm{L}, \mathrm{O}, \mathrm{Y}\}, \mathrm{B} = \{\mathrm{L}, \mathrm{O}, \mathrm{Y}, \mathrm{A}, \mathrm{L}\}$. Then $\mathrm{X}$ and $\mathrm{B}$ are equal sets as repetition of elements in a set does not change a set.

Every element of $X$ is same as every element of $B$.

So, $\mathrm{X} = \{\mathrm{A}, \mathrm{L}, \mathrm{O}, \mathrm{Y}\} = \mathrm{B}$

$(ii)$ $\mathrm{A}=\left\{n: n \in \mathrm{Z} \text{ and } n^{2} \leq 4\right\}$

Combine the terms

$n^{2}=4$

$n=\pm 2$

So, $-2 \leq n \leq 2$

$A=\{-2,-1,0,1,2\}$,

Similarly, $\mathrm{B}=\left\{x: x \in \mathrm{R} \text{ and } x^{2}-3 x+2=0\right\}$.

$x^{2}-3 x+2=0$

Solving we get

$(x-2)(x-1)=0$

$x=2$ or $x=1$

$\mathrm{B}=\{1,2\}$

Since $0 \in \mathrm{A}$ and $0 \notin \mathrm{B}, \mathrm{A}$ and $\mathrm{B}$ are not equal sets.

**EXERCISE 1.2**

**1. Which of the following are examples of the null set**

**(i) Set of odd natural numbers divisible by 2**

**(ii) Set of even prime numbers**

**(iii)** $\{x: x$ **is a natural number,** $x<5$ **and** $x>7\}$

**(iv)** $\{y: y$ **is a point common to any two parallel lines** $\}$

**Solution**

(i) No odd natural number which is divisible by 2. So, the given set is a empty set.

(ii) Set of numbers $=\{2\} \neq \phi$. So, the given set is not a empty set. It is a singleton set.

(iii) No natural number which is either than 5 and greater than 7. So, the given set is an empty set.

(iv) Two parallel do not have the common point. Therefore, the given set is a null set.

Hence $(i),(iii)$ and $(iv)$ are examples of the null set.

**2. Which of the following sets are finite or infinite?**

(i) The set of months of a year.**

**(ii)** $\{1,2,3,\ldots\}$

**(iii)** $\{1,2,3,\ldots,99,100\}$

**(iv) The set of positive integers greater than 100**

**(v) The set of prime numbers less than 99.**

**Solution**

(i) Since there are 12 months (i.e., actual number of months) per year, the given set is finite.

(ii) As per number of elements in the set is infinite, the given set is infinite.

(iii) Since the number of elements in the set is 100 (i.e., exact number), the given set is finite.

(iv) Since there are infinitely more numbers than 100, the given set is infinite.

(v) Since the number of primes less than 99 is a straight number, the given set is finite.

**3. State whether each of the following set is finite or infinite:**

**(i) The set of lines which are parallel to the $x$-axis**

**(ii) The set of letters in the English alphabet**

**(iii) The set of numbers which are multiple of 5**

**(iv) The set of animals living on the earth**

**(v) The set of circles passing through the origin** $(0,0)$

**Solution**

(i) Since there are infinite number of lines corresponding to the $x-$ axis, the given set is infinite.

(ii) Since there are 26 letters, that is, a exact number of letters, in the English alphabet, the given set is finite.

(iii) Since there are many multiples of 5, the given set is infinite.

(iv) The process of counting land animals ends. Therefore, a certain number of animals live in the world and that is why a given set is finite.

(v) There is no end to the number of circles that exceeds the origin $(0,0)$.

Hence, the given set is infinite.

**4. In the following, state whether $\mathrm{A}=\mathrm{B}$ or not:**

**(i)** $\mathrm{A}=\{a,b,c,d\};\mathrm{B}=\{d,c,b,a\}$

**(ii)** $\mathrm{A}=\{4,8,12,16\};\mathrm{B}=\{8,4,16,18\}$

**(iii)** $A = \{2, 4, 6, 8, 10\}; B = \{x : x$ **is positive even integer and** $x \le 10\}$

**(iv)** $A = \{x : x$ **is a multiple of 10**$\}, ;$ **B** $= \{10, 15, 20, 25, 30, \ldots\}$

**Solution**

(i) A and B have exactly same elements, though not in the same order.

$A = B$

(ii) $12 \in A$ but $12 \notin B$

$\therefore \quad A \ne B$

(iii)In roster form $B = \{2, 4, 6, 8, 10\}$

Since A and B have exactly same elements, therefore, $A = B$

(iv) In roster form $A = \{10, 20, 30, \ldots\}$

Since $15 \in B$ but $15 \notin A$, therefore $A \ne B$.

**5. Are the following pair of sets equal? Give reasons.**

**(i)** $A = \{2, 3\}, \quad B = \{x : x$ **is solution of** $x^2 + 5x + 6 = 0\}$

**(ii)** $A = \{x : x$ **is a letter in the word FOLLOW** $\}$

$B = \{y : y$ **is a letter in the word WOLF** $\}$

**Solution**

(i) $B = \{x : x$ is solution of $(x+2)(x+3) = 0\}$

$\left[\because x^2 + 5x + 6 = x^2 + 2x + 3x + 6\right.$

$= x(x+2) + 3(x+2)$

$= (x+2)(x+3)]$

$= \{-2, -3\}$

$2 \in A$ but $2 \notin B$

$\therefore \quad A \ne B$

(ii) Dropping repetitions $A = \{F, O, L, W\}$

$B = \{W, O, L, F\}$

Sets A and B have exactly same elements, though not in the same order.

$A = B.$

**6. From the sets given below, select equal sets:**

$A = \{2,4,8,12\}$, $B = \{1,2,3,4\}$, $C = \{4,8,12,14\}$, $D = \{3,1,4,2\}$
$E = \{-1,1\}$, $F = \{0,a\}$, $G = \{1,-1\}$, $H = \{0,1\}$

**Solution**

$A = \{2,4,8,12\}; B = \{1,2,3,4\}; C = \{4,8,12,14\}$

$D = \{3,1,4,2\}; E = \{-1,1\}; F = \{0,a\}$

$G = \{1,-1\}; A = \{0,1\}$

Set A has 4 elements,

So, $A$ is not equal to sets $E, F, G, H$

Also, elements of Set A are not in B, C, D

So, $A$ is not equal to set $B, C, D$

Set B has 4 elements,

And all elements of set $B$ are in set $D$.

Hence, B = D

Now,

$C$ is not equal to any set

And D is already equal to C

Elements of set $E$ and set $G$ are the same

So, $E = G$

And there are no other equal sets

Hence, among the given sets, $B = D$ and $E = G$

**Example 9**

**Consider the sets**

$\phi, A = \{1,3\}$, $B = \{1,5,9\}$, $C = \{1,3,5,7,9\}$

**Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets:**

**(i)** $\phi \ldots B$

(ii) $A \ldots B$

(iii) $A \ldots C$

(iv) $B \ldots C$

**Solution**

(i) $\phi \subset B$ as $\phi$ is a subset of every set.

(ii) $A \not\subset B$ as $3 \in A$ and $3 \notin B$

(iii) $A \subset C$ as $1, 3 \in A$ also belongs to $C$

(iv) $B \subset C$ as each element of $B$ is also an element of $C$.

**Example 10**

**Let $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d\}$. Is $A$ a subset of $B$ ? No. (Why?). Is B a subset of A? No. (Why?)**

**Solution**

(i) For a set to be a subset of another set, it needs to have all elements present in another set.

Set $A, \{e, i, o, u\}$ elements exists but these are not set in $B$ Hence $A$ is not a subset of $B$.

(ii)For this condition to be true, are elements of sets B should be present in set A

In set $B, \{b, c, d\}$ elements exists but these elements are not in set A

So, $B$ is not a subset of $A$

**Example 11**

**Let $A, B$ and $C$ be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$?. If not, give an example.**

**Solution**

$A \in B$ shows an element $A$ belongs to set $B$.

Let $A = 3, B = (1, 2, 3)$

$B \subset C$ shows $B$ is a proper subset of $C$.

Let $C = (1, 2, 3, 4, 5)$

So, Element A is an element of Set C .

Hence correct depiction will be $A \in C.$

$A \subset C$ is incorrect.

**EXERCISE1.3**

**1. Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces :**

(i) $\{2,3,4\} \ldots \{1,2,3,4,5\}$

(ii) $\{a,b,c\} \ldots \{b,c,d\}$

(iii) $\{x : x$ is a student of Class XI of your school $\} \ldots \{x : x$ student of your school $\}$

(iv) $\{x : x$ is a circle in the plane $\} \ldots \{x : x$ is a circle in the same plane with radius 1 unit $\}$

(v) $\{x : x$ is a triangle in a plane $\} \ldots \{x : x$ is a rectangle in the plane $\}$

(vi) $\{x : x$ is an equilateral triangle in a plane $\} \ldots \{x : x$ is a triangle in the same plane $\}$

(vii) $\{x : x$ is an even natural number $\} \ldots \{x : x$ is an integer $\}$

**Solution**

(i) Every element of the set $\{2,3,4\}$ is also an element of the set $\{1,2,3,4,5\}$

$\therefore \{2,3,4\} \subset \{1,2,3,4,5\}$ .

(ii) $a \in \{a,b,c\}$ but $a \notin b,c,d\}$ $\therefore \{a,b,c\} \subset \{b,c,d\}$ .

(iii) Every student of class XI of your school is a student of your school.

$\therefore \{x : x$ is a student of class XI of your school $\} \subset \{x : x$ is a student of your school $\}$ .

(iv) Every circle in a plane is not a circle with radius 1 unit, as it can have any radius $r,(r > 0)$ .

$\therefore \{x : x$ is a circle in the plane $\} \not\subset \{x : x$ is a circle in the same plane with radius 1 unit $\}$ .

(v) Any triangle is never a rectangle.

$\therefore \{x : x$ is a triangle in the plane $\} \not\subset \{x : x$ is a rectangle in the plane $\}$ .

(vi) Every equilateral triangle in a plane is a triangle in the plane.

$\therefore \{x : x$ is an equilateral triangle in a plane $\} \subset \{x : x$ is a triangle in the same plane 1 .

(vii) Every even natural number is an integer.

$\therefore \{x : x$ is an even natural number $\} \subset \{x : x$ is an integer $\}$ .

**2. Examine whether the following statements are true or false:**

(i) $\{a,b\} \not\subset \{b,c,a\}$

(ii) $\{a,e\} \subset \{x : x$ is a vowel in the English alphabet $\}$

(iii) $\{1,2,3\} \subset \{1,3,5\}$

(iv) $\{a\} \subset \{a,b,c\}$

(v) $\{a\} \in \{a,b,c\}$

(vi) $\{x : x$ is an even natural number less than $6\} \subset \{x : x$ is a natural number which divides $36\}$

**Solution**

(i) False, since every element of the set $\{a,b\}$ is also an element of the set $\{b,c,a\}$, therefore, $\{a,b\} \subset \{b,c,a\}$.

(ii) True, since every element of the set $\{a,e\}$ is also an element of the set of vowels $\{a,e,i,o,u\}$, therefore, $\{a,e\} \subset \{a,e,i,o,u\}$.

(iii) False, since $2 \in \{1,2,3\}$ but $2 \notin \{1,3,5\}$.

(iv) True, since $a \in \{a,b,c\}$.

(v) False, since $\{a\}$ is subset of the set $\{a,b,c\}$ but not an element of the set $\{a,b,c\}$.

(vi) True, since $\{x : x$ is an even natural number less than $6\} = \{2,4\}$ and $\{x : x$ is a natural number which divides $36\} = \{1,2,3,4,6,9,12,18,36\}$.

Clearly, $\{2,4\} \subset \{1,2,3,4,6,9,12,18,36\}$

**3. Let $A = \{1,2,\{3,4\},5\}$. Which of the following statements are incorrect and why?**

(i) $\{3,4\} \subset A$

(ii) $\{3,4\} \in A$

(iii) $\{\{3,4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1,2,5\} \subset A$

(vii) $\{1,2,5\} \in A$

(viii) $\{1,2,3\} \subset A$

(ix) $\phi \in A$

(x) $\phi \subset \text{A}$

(xi) $\{\phi\} \subset \text{A}$

**Solution**

Given $A = \{1, 2, \{3, 4\}, 5\}$

(i) The statement $\{3, 4\} \subset A$ is incorrect because $3 \in \{3, 4\}$; however, $3 \notin A$.

(ii) The statement $\{3, 4\} \in A$ is correct because $\{3, 4\}$ is an element of $A$.

(iii) The statement $\{\{3, 4\}\} \subset A$ is correct because $\{3, 4\} \in \{\{3, 4\}\}$ and, $\{3, 4\} \in A$.

(iv) The statement $1 \in \text{A}$ is correct because 1 is an element of $\text{A}$.

(v) The statement $1 \subset \text{A}$ is incorrect because an element of a set can never be a subset of itself.

(vi) The statement $\{1, 2, 5\} \subset A$ is correct because each element of $\{1, 2, 5\}$ is also an element of $A$.

(vii) The statement $\{1, 2, 5\} \in A$ is incorrect because $\{1, 2, 5\}$ is not an element of $A$.

(viii) The statement $\{1, 2, 3\} \subset A$ is incorrect because $3 \in \{1, 2, 3\}$; however, $3 \notin A$.

(ix) The statement $\Phi \in A$ is incorrect because $\Phi$ is not an element of $A$.

(x) The statement $\Phi \subset A$ is correct because $\Phi$ is a subset of every set.

(xi) The statement $\{\Phi\} \subset A$ is incorrect because $\Phi \in \{\Phi\}$; however, $\Phi \in A$.

**4. Write down all the subsets of the following sets**

(i) $\{a\}$

(ii) $\{a, b\}$

(iii) $\{1, 2, 3\}$

(iv) $\phi$

**Solution**

(i) Let $\text{A} = |a|$, then $\text{A}$ has one element.

$\therefore$ Number of subsets of the set $A = 2^n = 2^1 = 2$.

Subset of A will have either no element or one element.

Subset of A having no element is $\phi$

Subset of A having one element is $\{a\}$ $\therefore$ The subsets of $A$ are $\phi, \{a\}$.

(ii) Let $\mathrm{A}=\{a, b\}$, then $\mathrm{A}$ has 2 elements.

$\therefore$ Number of subsets of $\mathrm{A}=2^{n}=2^{2}=4$.

Subset of A having no element is $\phi$

Subsets of A having one element are $\{a|,|b|$ ,

Subset of A having two elements is $\{a, b\}$

$\therefore$ The subsets of A are $\phi,\{a\},\{b\},\{a, b\}$.

(iii) Let $\mathrm{A}=\{1,2,3\}$, then $A$ has 3 elements.

$\therefore$ Number of subsets of $\mathrm{A}=2^{n}=2^{3}=8$.

Subsets of A having no element is $\phi$

Subsets of A having one element are $\{1\},\{2\},\{3\}$

Subsets of A having two elements are $\{1,2\},\{1,3\},\{2,3\}$

Subset of A having three elements is $\{1,2,3\}$

$\therefore$ The subsets of Aare $\phi,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}$, $\{1,2,3\}$.

(iv) The only subset of the empty set $\phi$ is $\phi$ itself.

**5. How many elements has $\mathrm{P}(\mathrm{A})$, if $\mathrm{A}=\phi$ ?**

**Solution**

$A=\varnothing$

Set A does not contain any elements

Use the formula to calculate the number of Subsets,

$n(P(A))=2^{n}$

Here, Number of elements $(\mathrm{n})$ in Set A is 0

Hence,

$n(P(A))=2^{0}$

$n(P(A))=1$

Since, Set A has no elements, The number of elements is its power set is 1 .

**6. Write the following as intervals:**

(i) $\{x : x \in \mathrm{R}, -4 < x \leq 6\}$

(ii) $\{x : x \in \mathrm{R}, -12 < x < -10\}$

(iii) $\{x : x \in \mathrm{R}, 0 \leq x < 7\}$

(iv) $\{x : x \in \mathrm{R}, 3 \leq x \leq 4\}$

**Solution**

(i) The Set $\mathrm{A} = \{x : x \in R, -4 < x \leq 6\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

$\{x : x \in R, -4 < x \leq 6\}$ :

This represents that the interval is an open interval including 6 but excluding $-4$, therefore the interval will be $(-4, 6]$.

(ii) The Set $\mathrm{A} = \{x : x \in R, -12 < x \leq -10\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

This represents that the interval is an open interval excluding both the end points, that is, $-12$ and $-10$, therefore the interval will be $(-12, -10)$.

(iii) The Set $\mathrm{A} = \{x : x \in R, 0 \leq x < 7\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

$\{x : x \in R, 0 \leq x < 7\}$

This represents that the interval is an open interval including 0 but excluding the end point 7, therefore the interval will be $[0, 7)$.

(iv) The Set $\mathrm{A} = \{x : x \in R, 3 \leq x \leq 4\}$

Write the set builder form in the form of intervals after checking the type of inequality used at the end points.

$\{x : x \in R, 3 \leq x \leq 4\}$ :

This represents that the interval is a closed interval including both the end points, that is, 3 and 4, therefore the interval will be $[3, 4]$.

**7. Write the following intervals in set-builder form:**

(i) $(-3,0)$

(ii) $[\mathbf{6},\mathbf{12}]$

$(iii)$ $(\mathbf{6},\mathbf{12}]$

$(iv)$ $[-\mathbf{23},\mathbf{5})$

**Solution**

The following interval in set builder form

(i) Set builder form $\{x : x \in \mathrm{R}, -3 < x < 0\}$

(ii) Set builder form $\{x : x \in \mathrm{R}, 6 \le x \le 12\}$

(iii) Set builder form $\{x : x \in \mathrm{R}, 6 < x \le 12\}$

(iv) Set builder form $\{x : x \in \mathrm{R}, -23 \le x < 5\}$.

**8. What universal set(s) would you propose for each of the following:**

(i) The set of right triangles.

(ii) The set of isosceles triangles.

**Solution**

(i) In the right-hand triangle set, the universal set may be a set of triangles or a set of polygons.

(ii) With a set of isosceles triangles, a universal set could be a set of triangles or a set of polygons or a set of two-dimensional figures.

**9. Given the sets** $\mathrm{A} = \{1,3,5\}, \mathrm{B} = \{2,4,6\}$ **and** $\mathrm{C} = \{0,2,4,6,8\}$**, which of the following may be considered as universal set (s) for all the three sets A, B and C**

(i) $\{0,1,2,3,4,5,6\}$

(ii) $\phi$

(iii) $\{0,1,2,3,4,5,6,7,8,9,10\}$

(iv) $\{1,2,3,4,5,6,7,8\}$

**Solution**

(i) It can be seen that $A \subset \{0,1,2,3,4,5,6\}$

$B \subset \{0,1,2,3,4,5,6\}$

However, $C \not\subset \{0,1,2,3,4,5,6\}$

Hence, the set $\{0,1,2,3,4,5,6\}$ cannot be the universal set of $A, B,$ and $C$ sets.

(ii) $A \not\subset \Phi, B \not\subset \Phi, C \not\subset \Phi$

Therefore, $\Phi$ cannot be the universal set for the sets $A, B,$ and $C$.

(iii) $A \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

$\mathrm{B} \subset \{0,1,2,3,4,5,6,7,8,9,10\}$ $\mathrm{C} \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

Therefore, the set $\{0,1,2,3,4,5,6,7,8,9,10\}$ is the universal set for the sets $A, B,$ and $C$.

(iv) $A \subset \{1,2,3,4,5,6,7,8\}$

$B \subset \{1,2,3,4,5,6,7,8\}$

However, $C \not\subset \{1,2,3,4,5,6,7,8\}$

Therefore, the set $\{1,2,3,4,5,6,7,8\}$ cannot be the universal set for the sets $A, B,$ and $C$.

**Example 12**

**Let $\mathrm{A} = \{2,4,6,8\}$ and $\mathrm{B} = \{6,8,10,12\}$. Find $\mathrm{A} \cup \mathrm{B}$.**

**Solution**

**Given,**

$\mathrm{A} = \{2,4,6,8\}$

$\mathrm{B} = \{6,8,10,12\}$

The combination of two sets

$A \cup B = \{2,4,6,8\} \cup \{6,8,10,12\}$

Common elements 6,8 should be taken once

The combination of the sets the $A \cup B = \{2,4,6,8,10,12\}$

The common elements 6 and 8 have been taken only once while writing $A \cup B$

**Example 13**

Let $\mathrm{A}=\{a,e,i,o,u\}$ **and** $\mathrm{B}=\{a,i,u\}$. **Show that** $\mathrm{A}\cup\mathrm{B}=\mathrm{A}$

**Solution**

Given,

$\mathrm{A}=\{a,e,i,o,u\}$

$\mathrm{B}=\{a,i,u\}$.

The combination of sets

$\mathrm{A}\cup\mathrm{B}=\{a,e,i,o,u\}=\mathrm{A}$.

$\therefore A\cup B=A$

The example illustrates that union of sets $\mathrm{A}$ and its subset $\mathrm{B}$ is the set $\mathrm{A}$ itself,

if $\mathrm{B}\subset\mathrm{A}$, then $\mathrm{A}\cup\mathrm{B}=\mathrm{A}$

**Example 14**

**Let $X=\{$ Ram, Geeta, Akbar $\}$ be the set of students of Class XI, who are in school hockey team. Let $Y=\{$ Geeta, David, Ashok $\}$ be the set of students from Class XI who are in the school football team. Find $\mathrm{X}\cup\mathrm{Y}$ and interpret the set.**

**Solution**

Given $X=\{$ Ram, Geeta, Akbar $\}$

$\mathrm{Y}=\{$ Geeta, David, Ashok $\}$

Common elements (Geeta) should be taken once

$\mathrm{X}\cup\mathrm{Y}=\{$ Ram, Geeta, Akbar, David, Ashok $\}$.

This is the group of students from Class XI who are on the hockey team or the football team or both.

**Example 15**

**Consider the sets $\mathrm{A}$ and $\mathrm{B}$ of Example 12. Find $\mathrm{A}\cup\mathrm{B}$.**

**Solution**

From example 12

Given, $\mathrm{A}=\{2,4,6,8\}$

$\mathrm{B}=\{6,8,10,12\}$

6,8 are the only elements which are common to both $\mathrm{A}$ and $\mathrm{B}$.

Intersection of common sets

Hence $A \cap B = \{6,8\}$.

**Example 16**

**Consider the sets $\mathrm{X}$ and $\mathrm{Y}$ of Example 14. Find $\mathrm{X} \cap \mathrm{Y}$.**

**Solution**

Given $X = \{$ Ram, Geeta, Akbar $\}$

$\mathrm{Y} = \{$ Geeta, David, Ashok $\}$

So, the intersection of sets

$\mathrm{X} \cap \mathrm{Y} = \{$ Ram, Geeta, Akbar $\} \cap \{$ Geeta, David, Ashok $\}$

'Geeta' is the only element common to both.

Hence, $\mathrm{X} \cap \mathrm{Y} = \{$ Geeta $\}$.

**Example 17**

**Let $\mathrm{A} = \{1,2,3,4,5,6,7,8,9,10\}$ and $\mathrm{B} = \{2,3,5,7\}$. Find $\mathrm{A} \cap \mathrm{B}$ and hence show that $\mathrm{A} \cap \mathrm{B} = \mathrm{B}$.**

**Solution**

**Given,** $\mathrm{A} = \{1,2,3,4,5,6,7,8,9,10\}$

$\mathrm{B} = \{2,3,5,7\}$

The intersection of sets

The sets of $A \cap B = \{1,2,3,4,5,6,7,8,9,10\} \cap \{2,3,5,7\}$

$A \cap B = \{2,3,5,7\} = B$.

So $\mathrm{B} \subset \mathrm{A}$ and that $\mathrm{A} \cap \mathrm{B} = \mathrm{B}$ is proved.

**Example 18**

**Let $\mathrm{A} = \{1,2,3,4,5,6\}, \mathrm{B} = \{2,4,6,8\}$. Find $\mathrm{A} - \mathrm{B}$ and $\mathrm{B} - \mathrm{A}$.**

**Solution**

Given,

$\mathrm{A} = \{1,2,3,4,5,6\}, \mathrm{B} = \{2,4,6,8\}$

Intersection of common sets

$A-B=A-(A\cap B)$

The intersection of sets

$A\cap B=\{1,2,3,4,5,6\}\cap\{2,4,6,8\}$

$=\{2,4,6\}$

Substitute the values

$A-B=A-(A\cap B)$

$=\{1,2,3,4,5,6\}-\{2,4,6\}$

$=\{1,3,5\}$

Finally,

$B-A=B-(B\cap A)$

$=B-(A\cap B)$

$=\{2,4,6,8\}-\{2,4,6\}$

$=\{8\}$

So, $\mathrm{A}-\mathrm{B}=\{1,3,5\}$, since the elements $1,3,5$ belong to $\mathrm{A}$ but not to $\mathrm{B}$ and $\mathrm{B}-\mathrm{A}=\{8\}$,

Hence the element 8 belongs to $\mathrm{B}$ and not to $\mathrm{A}$.

Therefore, $\mathrm{A}-\mathrm{B}\neq\mathrm{B}-\mathrm{A}$.

**Example 19**

**Let** $\mathrm{V}=\{a,e,i,o,u\}$ **and** $\mathrm{B}=\{a,i,k,u\}$. **Find** $\mathrm{V}-\mathrm{B}$ **and** $\mathrm{B}-\mathrm{V}$

**Solution**

Given, $\mathrm{V}=\{\mathrm{a,e,i,o,u}\}$ and $\mathrm{B}=\{\mathrm{a,i,k,u}\}$

$\mathrm{V}-\mathrm{B}=$ is a set which contain elements which are only present in $\mathrm{V}$

$\therefore \mathrm{V}-\mathrm{B}=\{\mathrm{e,o}\}$

$\mathrm{B}-\mathrm{V}=$ is a set which contain elements which are only present in $\mathrm{B}$ $\therefore \mathrm{B}-\mathrm{V}=\{\mathrm{k}\}$

Hence, $\mathrm{V}-\mathrm{B}\neq\mathrm{B}-\mathrm{V}$

Since the elements $e$, $o$ belong to $\mathrm{V}$ but not to $\mathrm{B}$ and $\mathrm{B}-\mathrm{V}=\{k\}$, since the element $k$ belongs to $\mathrm{B}$ but not to $\mathrm{V}$.

**EXERCISE 1.4**

**1. Find the union of each of the following pairs of sets:**

(i) $X=\{1,3,5\}\ \ Y=\{1,2,3\}$

(ii) $A=[a,e,i,o,u\}\quad B=\{a,b,c\}$

(iii) $A=\{x:x$ is a natural number and multiple of $3\}$ $B=\{x:x$ is a natural number less than $6\}$

(iv) $A=\{x:x$ is a natural number and $1<x\le 6\}$ $B=\{x:x$ is a natural number and $6<x<10\}$

(v) $A=\{1,2,3\},B=\phi$

**Solution**

(i) $X=\{1,3,5\}\ Y=\{1,2,3\}$

The combination of two sets

$X\cup Y=\{1,3,5\}\cup\{1,2,3\}$

$=\{1,2,3,5\}$

(ii) $A=[a,e,i,o,u\}\quad B=\{a,b,c\}$

Combination of two sets

$A\cup B=\{a,e,i,o,u\}\cup\{a,b,c\}$

$=\{a,b,c,e,i,o,u\}$

(iii) $A=\{3,6,9,12,\ldots\}B=\{1,2,3,4,5\}$

$A\cup B\ =\{3,6,9,12,\ldots\}\cup\{1,2,3,4,5\}$

$=\{1,2,3,4,5,6,9,12,\ldots\}$

$=\{x:x=1,2,4,5$ or a multiple of $3\}$

(iv)

$A=\{2,3,4,5,6\}$
$B=\{7,8,9\}$

$A\cup B=\{2,3,4,5,6\}\cup|\ 7,8,9\}$

$=\{2,3,4,5,6,7,8,9\}$

$=\{x:1<x<10$ and $x\in N\}$

(v)

$A\cup B=\{1,2,3\}\cup\phi$
$=\{1,2,3\}$
$=A$

The result of this is true in general also.

$A \cup \phi = A$ for every set $A$.

**2. Let $\mathrm{A} = \{a, b\}, \mathrm{B} = \{a, b, c\}$. Is $\mathrm{A} \subset \mathrm{B}$ ? What is $\mathrm{A} \cup \mathrm{B}$ ?**

**Solution**

Given, $A = \{a, b\}$

$B = \{a, b, c\}$

Every element of $A$ is in $B$.

So, $A$ is a subset of $B$

That is $A \subset B$.

$A \cup B = \{a, b\} \cup \{a, b, c\}$

$= \{a, b, c\}$

Yes. $\mathrm{A} \cup \mathrm{B} = \{a, b, c\} = \mathrm{B}$

**3. If $\mathrm{A}$ and $\mathrm{B}$ are two sets such that $\mathrm{A} \subset \mathrm{B}$, then what is $\mathrm{A} \cup \mathrm{B}$?**

**Solution**

Let's take an example

$A = \{1, 2\}$

$B = \{1, 2, 3\}$

Every element of $A$ is in $B$.

Hence, $A$ is a subset of $B$

That is $A \subset B$.

$\mathrm{A} \cup \mathrm{B} = \{x : x \in \mathrm{A} \text{ or } x \in \mathrm{B}\}$

$= \{x : x \in \mathrm{B}\}$

**4. If $\mathrm{A} = \{1, 2, 3, 4\}, \mathrm{B} = \{3, 4, 5, 6\}, \mathrm{C} = \{5, 6, 7, 8\}$ and $\mathrm{D} = \{7, 8, 9, 10\}$; find**

**(i)** $\mathrm{A} \cup \mathrm{B}$

**(ii)** $\mathrm{A} \cup \mathrm{C}$

**(iii)** $B \cup C$

**(iv)** $B \cup D$

**(v)** $A \cup B \cup C$

**(vi)** $A \cup B \cup D$

**(vii)** $B \cup C \cup D$

**Solution**

Given, $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\};$

(i) The combination of sets A and B

$A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

(ii) The combination of sets A and C

$A \cup C = \{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) The combination of sets B and C

$B \cup C = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$

$= \{3, 4, 5, 6, 7, 8\}$

(iv) The combination of sets B and D

$B \cup D = \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) The combination of sets

$A \cup B \cup C = (A \cup B) \cup C$

$= \{1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) The combination of sets

$A \cup B \cup D = (A \cup B) \cup D$

$= \{1, 2, 3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) The combination of sets

$\mathrm{B}\cup\mathrm{C}\cup\mathrm{D}=(\mathrm{B}\cup\mathrm{C})\cup\mathrm{D}=\{3,4,5,6,7,8\}\cup\{7,8,9,10\}$

$=\{3,4,5,6,7,8,9,10\}$

**5. Find the intersection of each pair of sets of question 1 above.**

**Solution**

From exercise 1

(i) The intersection of sets

$\mathrm{X}\cap\mathrm{Y}=\{1,3,5\}\cap\{1,2,3\}=\{1,3\}$.

(ii) The intersection of sets

$\mathrm{A}\cap\mathrm{B}=\{a,e,i,o,u\}\cap|a,b,c|=|a|$.

(iii) $\mathrm{A}=\{3,6,9,12,\ldots\}$

$\mathrm{B}=\{1,2,3,4,5\}$

The intersection of sets

$\mathrm{A}\cap\mathrm{B}=\{3,6,9,12,\ldots\}\cap\{1,2,3,4,5\}=\{3\}$

(iv) $\mathrm{A}=\{2,3,4,5,6\}$

$\mathrm{B}=\{7,8,9\}$

The intersection of sets

$\mathrm{A}\cap\mathrm{B}=\{2,3,4,5,6\}\cap|\,7,8,9\}=\phi$

(v) The intersection of sets

$\mathrm{A}\cap\mathrm{B}=\{1,2,3|\cap\phi=\phi$

The result of this part is true in general also.

$\mathrm{A}\cap\phi=\phi$ for every set $\mathrm{A}$.

**6. If $\mathrm{A}=\{3,5,7,9,11\},\mathrm{B}=\{7,9,11,13\},\mathrm{C}=\{11,13,15\}$ and $\mathrm{D}=\{15,17\}$; find**

(i) $\mathrm{A}\cap\mathrm{B}$

(ii) $\mathrm{B}\cap\mathrm{C}$

(iii) $\mathrm{A}\cap\mathrm{C}\cap\mathrm{D}$

(iv) $A\cap C$

(v) $B \cap D$

(vi) $A \cap (B \cup C)$

(vii) $A \cap D$

(viii) $A \cap (B \cup D)$

(ix) $(A \cap B) \cap (B \cup C)$

**Solution**

Given $A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}$ and $D = \{15,17\}$

Intersection of sets

(i) $A \cap B = \{3,5,7,9,11\} \cap \{7,9,11,13\}$

$= \{7,9,11\}$

(ii) Intersection of sets

$B \cap C = \{7,9,11,13\} \cap \{11,13,15\} = \{11,13\}$

(iii) Intersection of sets

$A \cap C \cap D = (A \cap C) \cap D$

$= (|3,5,7,9,11| \cap |11,13,15|) \cap |15,17|$

$= \{11\} \cap |15,17\} = \phi$

(iv) Intersection of sets

$A \cap C = \{3,5,7,9,11\} \cap |11,13,15| = |11|$

(v) Intersection of sets

$B \cap D = \{7,9,11,13\} \cap \{15,17\} = \phi$

(vi) Intersection of sets

$A \cap (B \cup C) = \{3,5,7,9,11\} \cap (\{7,9,11,13\} \cup \{11,13,15\})$

$= \{3,5,7,9,11\} \cap \{7,9,11,13,15\}$

$= \{7,9,11\}$

(vii) Intersection of sets

$A \cap D = \{3,5,7,9,11\} \cap \{15,17\} = \phi$

(viii) Intersection of sets

$A \cap (B \cup D) = \{3,5,7,9,11\} \cap (\{7,9,11,13\} \cup \{15,17\})$

$= \{3,5,7,9,11\} \cap |7,9,11,13,15,17|$

$=\{7,9,11\}$

(ix) Intersection of sets

$(A\cap B)\cap(B\cup C)=(\{3,5,7,9,11\}\cap\{7,9,11,13\})\cap(\{7,9,11,13\}\cup\{11,13,15\})$

$=\{7,9,11\}\cap\{7,9,11,13,15\}=\{7,9,11\}$

(x) Intersection of sets

$(A\cup D)\cap(B\cup C)=(\{3,5,7,9,11\}\cup\{15,17\})\cap(\{7,9,11,13\}\cup\{11,13,15\})$

$=\{3,5,7,9,11,15,17\}\cap|\ 7,9,11,13,15\}$

$=\{7,9,11,15\}$

**7. If $A=\{x:x$ is a natural number $\},B=\{x:x$ is an even natural number $\}$ $C=\{x:x$ is an odd natural number $\}$ and D $=\{x:x$ is a prime number $\}$, find**

(i) $A\cap B$

(ii) $A\cap C$

(iii) $A\cap D$

(iv) $B\cap C$

(v) $B\cap D$

(vi) $C\cap D$

**Solution**

Given

$A=\{x:x$ is a natural number $\}=\{1,2,3,4,5\ldots\}$

$B=\{x:x$ is an even natural number $\}=\{2,4,6,8\ldots\}$

$C=\{x:x$ is an odd natural number $\}=\{1,3,5,7,9\ldots\}$

$D=\{x:x$ is a prime number $\}=\{2,3,5,7\ldots\}$

(i) $A\cap B=B$

Because, $B\subset A$

Every even natural number is a natural number

(ii) $A\cap C=C$

Because, $C\subset A$

Every odd natural number is a natural number

(iii) $A\cap D=D$

Because $D \subset A$

Every prime number is a natural number

(iv) $B \cap C = \phi$ There is no natural number which is both even and odd

(v) $B \cap D = \{2\}, 2$ is only even prime number.

(vi) $C \cap D = \{x : x$ is an odd prime number $\}$.

**8. Which of the following pairs of sets are disjoint**

(i) $\{1,2,3,4\}$ and $\{x : x$ is a natural number and $4 \le x \le 6\}$

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x : x$ is an even integer $\}$ and $\{x : x$ is an odd integer $\}$

**Solution**

(i) Let $A = \{1,2,3,4\}$ and

$B = \{x : x \in N \text{ and } 4 \le x \le 6\} = \{4,5,6\}$

$A \cap B = \{4\} \ne \phi \therefore$ Sets $A$ and $B$ are not disjoint.

(ii) $e$ is a common element of the two sets.

$\therefore$ Sets are not disjoint.

(iii) Given sets are disjoint sets because there is no natural number which is both even and odd.

**9. If** $A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$ $C = \{2,4,6,8,10,12,14,16\}$, $D = \{5,10,15,20\}$; **find**

(i) $A - B$

(ii) $A - C$

(iii) $A - D$

(iv) $B - A$

(v) $C - A$

(vi) $D - A$

(vii) $B - C$

(viii) $B - D$

(ix) $C - B$

(x) $D - B$

(xi) $C - D$

(xii) $D - C$

**Solution**

Given,

$A = \{3, 6, 9, 12, 15, 18, 21\}$,
$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$,

$D = \{5, 10, 15, 20\}$;

(i) The sets in difference

$A - B = \{3, 6, 9, 12, 15, 18, 21\} - \{4, 8, 12, 16, 20\}$

$= \{3, 6, 9, 15, 18, 21\}$

(ii) The difference of two sets

$A - C = \{3, 6, 9, 12, 15, 18, 21\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$

$= \{3, 9, 15, 18, 21\}$

(iii) The difference of two sets

$A - D = \{3, 6, 9, 12, 15, 18, 21\} - \{5, 10, 15, 20\}$

$= \{3, 6, 9, 12, 18, 21\}$

(iv) The difference of two sets

$B - A = \{4, 8, 12, 16, 20\} - \{3, 6, 9, 12, 15, 18, 21\}$

$= \{4, 8, 16, 20\}$

(v) The difference of two sets

$C - A = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{3, 6, 9, 12, 15, 18, 21\}$

$= \{2, 4, 8, 10, 14, 16\}$

(vi) The difference of two sets

$D - A = \{5, 10, 15, 20\} - \{3, 6, 9, 12, 15, 18, 21$

$= \{5, 10, 20\}$

(vii) The difference of two sets

$B - C = \{4, 8, 12, 16, 20\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$

$=\{20\}$

(viii) The difference of two sets

$B-D=\{4,8,12,16,20\}-\{5,10,15,20\}$

$=\{4,8,12,16\}$

(ix) The difference of two sets

$C-B=\{2,4,6,8,10,12,14,16\}-\{4,8,12,16,20\}$

$=\{2,6,10,14\}$

(x) The difference of two sets

$D-B=\{5,10,15,20\}-\{4,8,12,16,20\}$

$=\{5,10,15\}$

(xi) The difference of two sets

$C-D=\{2,4,6,8,10,12,14,16\}-\{5,10,15,20\}$

$=\{2,4,6,8,12,14,16\}$

(xii) The difference of two sets

$D-C=\{5,10,15,20\}-\{2,4,6,8,10,12,14,16\}$

$=\{5,15,20\}$

**10. If $X=\{a,b,c,d\}$ and $Y=\{f,b,d,g\}$, find**

(i) $X-Y$

(ii) $Y-X$

(iii) $X\cap Y$

**Solution**

Given

$X=\{a,b,c,d\}$

$Y=\{f,b,d,g\}$

(i) The difference of two sets

$X-Y=\{a,b,c,d\}-\{f,b,d,g\}$

$=\{a,c\}$

(ii) The difference of two sets

$Y - X = \{f, b, d, g\} - \{a, b, c, d\}$

$= \{f, g\}$

(iii) The intersection of two sets

$X \cap Y = \{a, b, c, d\} \cap | f, b, d, g |$

$= \{b, d\}.$

**11. If $\mathbf{R}$ is the set of real numbers and $\mathbf{Q}$ is the set of rational numbers, then what is $\mathbf{R} - \mathbf{Q}$?**

**Solution**

The Set $\mathbf{R} = \{x : x$ is a real number $\}$

The Set $Q\{x : x$ is a rational number $\}$

$R - Q = \{x : x \in R \text{ and } x \notin Q\}$

$= \{x : x$ is a real number and $x$ is not a rational

$= \{x : x$ is an irrational number $\}$

$= T$

Each real number is either rational or irrational but not both

**12. State whether each of the following statement is true or false. Justify your answer.**

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

(iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.

(iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

**Solution**

(i) Given sets is

$\{2, 3, 4, 5\}$ and $\{3, 6\}$

Intersection sets are:

$\{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

Hence these are not disjoint sets.

The given disjoint sets are false.

(ii) Given sets is

$\{a, e, i, o, u\}$ and $\{a, b, c, d\}$

Intersection sets are:

$\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

Hence these are not disjoint sets.

The given disjoint sets are false

(iii) Given sets is

$\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$

Intersection sets are:

$\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \varnothing$

Then, $A \cap B$ is an empty set.

Hence these are disjoint sets.

The given disjoint sets are True.

(iv) Given sets is

$\{2, 6, 10\}$ and $\{3, 7, 11\}$.

Intersection sets are:

$\{2, 6, 10\} \cap \{3, 7, 11\} = \varnothing$

Then, $A \cap B$ is an empty set.

Hence these are disjoint sets.

The given disjoint sets are True.

**Example 20**

**Let** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **and** $A = \{1, 3, 5, 7, 9\}$. **Find** $A'$.

**Solution**

**Given,**

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}$

The value of $A' = U - A = \{2, 4, 6, 8, 10\}$

$2, 4, 6, 8, 10$ are the elements of $U$ which does not belong to A.

**Example 21**

**Let U be universal set of all the students of Class XI of a coeducational school and $A$ be the set of all girls in Class XI. Find $A'$.**

**Solution**

In a coeducational school, there can be only boys and girls in a school.

$A$ is the set of all girls,

$A'$ = Set of all Students − Set of all girls

$A'$ = Set of all boys in class XI.

Since $A$ is the set of all girls, $A'$ is clearly the set of all boys in the class.

**Example 22**

**Let** $U=\{1,2,3,4,5,6\}, A=\{2,3\}$ **and** $B=\{3,4,5\}$

**Find** $A', B', A'\cap B', A\cup B$ **and hence show that** $(A\cup B)' = A'\cap B'$**.**

**Solution**

**Given**

$U=\{1,2,3,4,5,6\}, A=\{2,3\}$

$B=\{3,4,5\}$

The complement of Set A

$A' = U - A = \{1,2,3,4,5,6\}-\{2,3\}$

$=\{1,4,5,6\}$

Clearly $A' = \{1,4,5,6\}$,

Similarly

$B' = U - B = \{1,2,3,4,5,6\}-\{3,4,5\}$

$=\{1,2,6\}$

$B' = \{1,2,6\}$

Now,

$A'\cap B' = \{1,4,5,6\}\cap\{1,2,6\}$

$=\{1,6\}$

Hence $A' \cap B' = \{1, 6\}$

Also,

$A \cup B = \{2, 3\} \cup \{3, 4, 5\}$

$= \{2, 3, 4, 5\}$

Also $A \cup B = \{2, 3, 4, 5\}$,

So that $(A \cup B)' = \{1, 6\}$

Now, to prove $(A \cup B)' = A' \cap B'$

$(A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6\} - \{2, 3, 4, 5\}$

$= \{1, 6\}$

Now, $A' \cap B' = \{1, 6\}$

$(A \cup B)' = \{1, 6\}$

Thus, $(A \cup B)' = A' \cap B'$

Hence proved

**EXERCISE 1.5**

1. **Let** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ **and** $C = \{3, 4, 5, 6\}$**. Find**

(i) $A'$

(ii) $B'$

(iii) $(A \cup C)'$

(iv) $(A \cup B)'$

(v) $\left(A'\right)'$

(vi) $(B - C)'$

**Solution**

Given,

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$A = \{1, 2, 3, 4\}$,

$B = \{2, 4, 6, 8\}$

$C = \{3, 4, 5, 6\}$

(i) The complement of set

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$

$= \{5, 6, 7, 8, 9\}$

(ii) The complement of set

$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$

$= \{1, 3, 5, 7, 9\}$

(iii) The complement of set

$A \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

$(A \cup C)' = U - (A \cup C)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 5, 6\}$

$= \{7, 8, 9\}$

(iv) The complement of set

$A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6, 8\} = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = U - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 6, 8\}$

$= \{5, 7, 9\}$

(v) The complement of set

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4\}$

$= \{5, 6, 7, 8, 9\}$

So,

$(A')' = U - A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{5, 6, 7, 8, 9\}$

$= \{1, 2, 3, 4\} = A$

(vi) The difference of two sets

$B - C = \{2, 4, 6, 8\} - \{3, 4, 5, 6\} = \{2, 8\}$

$(B - C)' = U - (B - C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 8\}$

$= \{1, 3, 4, 5, 6, 7, 9\}.$

**2. If $U=\{a,b,c,d,e,f,g,h\}$, find the complements of the following sets:**

(i) $A=\{a,b,c\}$

(ii) $B=\{d,e,f,g\}$

(iii) $C=\{a,c,e,g\}$

(iv) $D=\{f,g,h,a\}$

**Solution**

Given,

$U=\{a,b,c,d,e,f,g,h\}$

(i) The complement of sets

$A'=U-A=\{a,b,c,d,e,f,g,h\}-\{a,b,c\}$

$=\{d,e,f,g,h\}$

(ii) The complement of sets

$B'=U-B=\{a,b,c,d,e,f,g,h\}-\{d,e,f,g\}$

$=\{a,b,c,h\}$

(iii) The complement of sets

$C'=U-C=\{a,b,c,d,e,f,g,h\}-\{a,c,e,g\}$

$=\{b,d,f,h\}$

(iv) The difference of universal sets ad set D

$D'=U-D=\{a,b,c,d,e,f,g,h\}-\{f,g,h,a\}$

$=\{b,c,d,e\}$

**3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:**

(i) $\{x : x$ is an even natural number $\}$

(ii) $\{x : x$ is an odd natural number $\}$

(iii) $\{x : x$ is a positive multiple of $3\}$

(iv) $\{x : x$ is a prime number $\}$

(v) $\{x: x$ is a natural number divisible by 3 and 5$\}$

(vi) $\{x: x$ is a perfect square $\}$

(vii) $\{x: x$ is a perfect cube $\}$

(viii) $\{x: x+5=8\}$

(ix) $\{x: 2x+5=9\}$

(x) $\{x: x \geq 7\}$

(xi) $\{x: x \in \mathrm{N}$ and $2x+1>10\}$

**Solution**

(i) Complement of the set = Universal set (set of all natural numbers) - Set of even natural numbers.

The complement of $\{x: x$ is an even natural number $\}$ is $\{x: x$ is an odd natural number $\}$

(ii) Complement of the set = Universal set (set of all natural numbers) - Set of odd natural numbers

The complement of $\{x: x$ is an odd natural number $\}$ is $\{x: x$ is an even natural number $\}$

(iii) Complement of the set $=$ Universal set (set of all natural numbers) - Set of all positive multiples of 3

The complement of $\{x: x$ is a positive multiple of 3$\}$ is $\{x: x \in \mathrm{N}$ and $x$ is not a multiple of 3$\}$

(iv) Complement of the set = Universal set (set of all natural numbers) - Set of all prime numbers

The complement of $\{x: x$ is a prime number $\}$ is $\{x: x$ is positive composite number and $x=1\}$

(v) Complement of the set = Universal set (set of all natural numbers) - Set of all numbers divisible by 3 and 5

The complement of $\{x: x$ is a natural number divisible by 3 and 5$\}$ is $\{x: x \in \mathrm{N}$ and $x$ is not divisible by 3 or not divisible by 5$\}$

(vi) Complement of the set = Universal set (set of all natural numbers) - Set of all perfect squares

The complement of $\{x: x$ **is a perfect square** $\}$ is $\{x: x \in \mathrm{N}$ and $x$ is not a perfect square $\}$

(vii) Complement of the set = Universal set (set of all natural numbers) - Set of all perfect cubes

The complement of $\{x: x$ **is a perfect cube** $\}$ **is** $\{x: x \in \mathrm{N}$ and $x$ is not a perfect cube $\}$

(viii) The complement of $\{x: x+5=8\}$ **is** $\{x: x=3\}' = \{x: x \in \mathrm{N}$ and $x \neq 3\}$

(ix) The complement of $\{x: 2x+5=9\}$ **is** $\{x: x \in \mathrm{N}$ and $x \neq 2\}$

(x) The complement of $\{x: x \geq 7\}$ **is** $\{x: x \in \mathrm{N}$ and $x<7\}=\{1,2,3,4,5,6\}$

(xi) The complement of $\{x : x \in \mathrm{N}$ **and** $2x+1>10\}$ **is** $\left\{x : x \in \mathrm{N} \text{ and } x>\frac{9}{2}\right\}=\left\{x : x \in \mathrm{N} \text{ and } x \leq \frac{9}{2}\right\}=\{1,2,3,4\}$.

**4.** If $\mathrm{U}=\{1,2,3,4,5,6,7,8,9\}, \mathrm{A}=\{2,4,6,8\}$ and $\mathrm{B}=\{2,3,5,7\}$. Verify that

(i) $(\mathrm{A} \cup \mathrm{B})'=\mathrm{A}' \cap \mathrm{B}'$

(ii) $(\mathrm{A} \cap \mathrm{B})'=\mathrm{A}' \cup \mathrm{B}'$

**Solution**

Given

$\mathrm{U}=\{1,2,3,4,5,6,7,8,9\}$,

$\mathrm{A}=\{2,4,6,8\}$

$\mathrm{B}=\{2,3,5,7\}$

Here $\mathrm{A}'=\mathrm{U}-\mathrm{A}=\{1,3,5,7,9\}$ and $\mathrm{B}'=\mathrm{U}-\mathrm{B}=\{1,4,6,8,9\}$

(i) The union of the sets

$\mathrm{A} \cup \mathrm{B}=\{2,3,4,5,6,7,8\}$

$(\mathrm{A} \cup \mathrm{B})'=\mathrm{U}-(\mathrm{A} \cup \mathrm{B})=\{1,9\}$

$\mathrm{A}' \cap \mathrm{B}'=\{1,3,5,7,9\} \cap\{1,4,6,8,9\}=\{1,9\}$

Therefore the $(\mathrm{A} \cup \mathrm{B})'=\mathrm{A}' \cap \mathrm{B}'$.

(ii) The union of sets

$\mathrm{A} \cap \mathrm{B}=\{2\}$

$(\mathrm{A} \cap \mathrm{B})'=\mathrm{U}-(\mathrm{A} \cap \mathrm{B})=\{1,3,4,5,6,7,8,9\}$

$\mathrm{A}' \cup \mathrm{B}'=\{1,3,5,7,9\} \cup\{1,4,6,8,9\}$

$=\{1,3,4,5,6,7,8,9\}$

$\therefore(\mathrm{A} \cap \mathrm{B})'=\mathrm{A}' \cup \mathrm{B}'$

**5. Draw appropriate Venn diagram for each of the following:**

(i) $(A \cup B)'$,

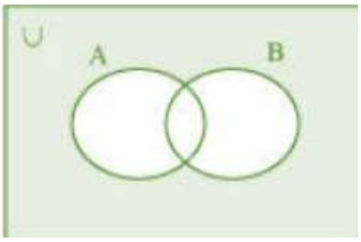
(ii) $A' \cap B'$,

(iii) $(A \cap B)'$,

(iv) $A' \cup B'$

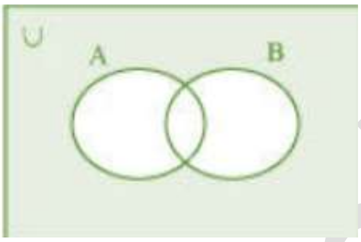
**Solution**

The venn diagram of $(A \cup B)'$



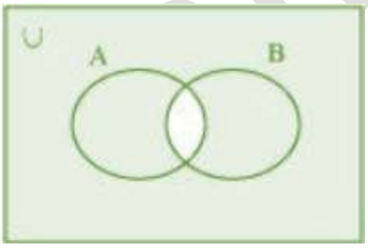

The green region is $(A \cup B)'$

**(ii) The venn diagram $A' \cap B'$,**

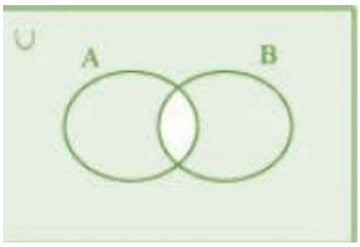



The required region is the green region

**(iii) The venn diagram $(A \cap B)'$,**




**(iv) The venn diagram $A' \cup B'$**



**6. Let $U$ be the set of all triangles in a plane. If $A$ is the set of all triangles with at least one angle different from $60^{\circ}$, what is $A'$ ?**

**Solution**

$U =$ Set of all the triangles in a plane (Universal Set)

$A =$ Set of all the triangles with at least one angle different from $\angle 60$

Use $A' = U - A$

$=$ Set of all triangles with no different angles to $60^{\circ}$

$=$ Set of all triangles per angle $60^{\circ}$

$=$ Set of all equal triangles.

$A'$ is the set of all equal triangles.

**7. Fill in the blanks to make each of the following a true statement :**

(i) $A \cup A' = \ldots$

(ii) $\phi' \cap A = \ldots$

(iii) $A \cap A' = \ldots$

(iv) $U' \cap A = \ldots$

**Solution**

(i) $A \cup A'$ (elements of set $A$) $+$ (elements of universal set that does not belong to set $A =$ Universal Set

The statement of $A \cup A' = U$ [Property of complement sets]

(ii) $\phi' \cap A = U \cap A = A$

The statement of $\phi' \cap A = U \cap A = A$

(iii) No common element between the two sets

The statement of $A \cap A' = \phi$

(iv) The complement of the universal set.

$\phi$ is the null set.

The statement of $U' \cap A = \phi \cap A = \phi$

**Example 23**

If $X$ and $Y$ are two sets such that $X \cup Y$ has 50 elements, $X$ has 28 elements and $Y$ has 32 elements, how many elements does $X \cap Y$ have ?

**Solution**

Given that

$n(X \cup Y) = 50,$

$n(X) = 28,$

$n(Y) = 32,$

$n(X \cap Y) = ?$

By using the formula

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$$

Substitute the values

$= 28 + 32 - 50$

$= 10$

**Example 24**

In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics?

**Solution**

Let M mean a set of math teachers and P mean a set of physics teachers

In the problem statement, the word 'or' gives us a clue as well as the word 'and' gives us a clue.

Number of teachers teaching Maths or Physics $= n(M \cup P) = 20$

Number of teachers teaching Maths $= n(M) = 12$

Number of teachers teaching Maths and physics $= n(M \cap P) = 4$

Number of teachers teaching physics $= n(P) = ?$

Using the result

$$n(M \cup P) = n(M) + n(P) - n(M \cap P)$$

$$20 = 12 + n(P) - 4 \quad n(P) = 12$$

So, $n(\mathrm{P})=12$

Hence 12 teachers teach physics.

**Example 25**

In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

**Solution**

Let $\mathrm{X}$ be the set of students who like to play cricket and $\mathrm{Y}$ be the set of students who like to play football.

Then $X \cup Y$ is the set of students who like to play at least one game, and $X \cap Y$ is the set of students who like to play both games.

Given

$$n(\mathrm{X})=24,$$
$$n(\mathrm{Y})=16,$$
$$n(\mathrm{X} \cup \mathrm{Y})=35,$$
$$n(\mathrm{X} \cap \mathrm{Y})=?$$

Using the formula

$$n(\mathrm{X} \cup \mathrm{Y})=n(\mathrm{X})+n(\mathrm{Y})-n(\mathrm{X} \cap \mathrm{Y}),$$

$$35=24+16-n(\mathrm{X} \cap \mathrm{Y})$$

Thus, $n(\mathrm{X} \cap Y)=5$

5 students like to play both games.

**Example 26**

In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

**Solution**

Let $U$ mean a set of surveyed students and A mean a set of students taking apple juice and B means a set of students taking orange juice. Then

Number of students taking apple juice $n(\mathrm{A})=100$

Number of students taking orange juice $n(\mathrm{B})=150$

Number of students taking both orange juice and apple juice $n(\mathrm{A} \cap \mathrm{B})=75$.

Total students $n(\mathrm{U})=400$,

Now $n\left(\mathrm{A}' \cap \mathrm{B}'\right)=n(\mathrm{A} \cup \mathrm{B})'$

$=n(\mathrm{U})-n(\mathrm{A} \cup \mathrm{B})$

$=n(\mathrm{U})-n(\mathrm{A})-n(\mathrm{B})+n(\mathrm{A} \cap \mathrm{B})$

$=400-100-150+75=225$

So, 225 students were not taking apple juice or orange juice.

**Example 27**

There are 200 individuals with a skin disorder, 120 had been exposed to the chemical $\mathrm{C}_1, 50$ to chemical $\mathrm{C}_2$, and 30 to both the chemicals $\mathrm{C}_1$ and $\mathrm{C}_2$. Find the number of individuals exposed to

(i) Chemical $\mathrm{C}_1$ but not chemical $\mathrm{C}_2$

(ii) Chemical $\mathrm{C}_2$ but not chemical $\mathrm{C}_1$

(iii) Chemical $\mathrm{C}_1$ or chemical $\mathrm{C}_2$

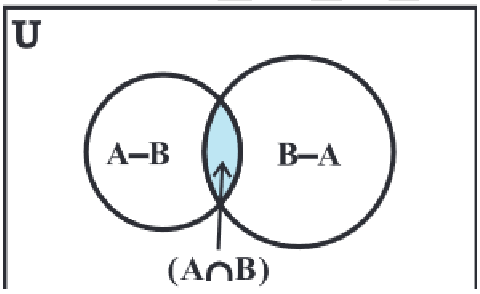
**Solution**

Let $U$ refers universal set that includes people with skin disorders,

A mean set of people exposed to the chemical $\mathrm{C}_1$

B means a set of people exposed to the chemical $\mathrm{C}_2$.

Then, $n(u)=200, n(A)=120, n(B)=50$ and $n(A \cap B)=30$




(i) From the Venn diagram, we have $A=(A-B) \cup(A \cap B)$ $\therefore n(A)=n(A-B)+n(A \cap B) A-B$ and $A \cap B$ are disjoint

$\therefore n(A-B)=n(A)-n(A \cap B)$

$=120-30=90$

Thus, the number of people exposed to the chemical $\mathrm{C}_1$ but not to chemical $C_2$ is 90 .

(ii) Similarly, $B=(B-A)\cup(A\cap B)$

$n(B)=n(B-A)+n(A\cap B)$

$\because (B-A)\cap(A\cap B)=\varphi$

$\therefore n(B-A)=n(B)-n(A\cap B)$

$=50-30=20$

Thus, the number of people exposed to the chemical $C_2$ and not to chemical $C_1$ is 20

(iii) Number of people exposed to both chemicals $C_1$ and $C_2=n(A\cap B)$

$=30$

Number of people exposed to the chemical $C_1$ or to chemical $C_2=n(A\cup B)$

We know that

$n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$=120+50-30$

$=140$

Thus, 140 people are exposed to the chemical $C_1$ or to chemical $\mathrm{C}_2$

**EXERCISE 1.6**

1. If $X$ and $Y$ are two sets such that $n(X)=17, n(Y)=23$ and $n(X\cup Y)=38$, find $n(\mathrm{X}\cap\mathrm{Y})$.

**Solution**

Given that

$n(X)=17,$
$n(Y)=23$

$n(X\cup Y)=38$

Using $n(\mathrm{X}\cup\mathrm{Y})=n(\mathrm{X})+n(\mathrm{Y})-n(\mathrm{X}\cap\mathrm{Y}),$

Substitute the values

$38=17+23-n(\mathrm{X}\cap\mathrm{Y})$

$n(\mathrm{X}\cap\mathrm{Y})=40-38$

$n(\mathrm{X}\cap\mathrm{Y})=2$

**2.** If $X$ and $Y$ are two sets such that $X \cup Y$ has 18 elements, $X$ has 8 elements and $Y$ has 15 elements ; how many elements does $X \cap Y$ have?

**Solution**

According to given information,

$n(\mathrm{X} \cup \mathrm{Y}) = 18,$
$n(\mathrm{X}) = 8$
$n(\mathrm{Y}) = 15$

Putting values in $n(\mathrm{X} \cup \mathrm{Y}) = n(\mathrm{X}) + n(\mathrm{Y}) - n(\mathrm{X} \cap \mathrm{Y})$,

we have $18 = 8 + 15 - n(\mathrm{X} \cap \mathrm{Y})$

$\therefore n(\mathrm{X} \cap \mathrm{Y}) = 23 - 18$

$n(\mathrm{X} \cap \mathrm{Y}) = 5.$

**3.** In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

**Solution**

Let $\mathrm{H}$ be the set of people who can speak Hindi and $\mathrm{E}$ be the set of people who can speak English. Then,

$n(\mathrm{H} \cup \mathrm{E}) = 400,$
$n(\mathrm{H}) = 250,$
$n(\mathrm{E}) = 200$

Have to find $n(\mathrm{H} \cap \mathrm{E})$.

Using

$n(\mathrm{H} \cup \mathrm{E}) = n(\mathrm{H}) + n(\mathrm{E}) - n(\mathrm{H} \cap \mathrm{E}),$

Substitute the values

$400 = 250 + 200 - n(\mathrm{H} \cap \mathrm{E})$

$n(\mathrm{H} \cap \mathrm{E}) = 450 - 400$

$n(\mathrm{H} \cap \mathrm{E}) = 50$

**4.** If $\mathrm{S}$ and $\mathrm{T}$ are two sets such that $\mathrm{S}$ has 21 elements, $\mathrm{T}$ has 32 elements, and $\mathrm{S} \cap \mathrm{T}$ has 11 elements, how many elements does $\mathrm{S} \cup \mathrm{T}$ have?

**Solution**

According to given information,

$n(\mathrm{S}) = 21,$
$n(\mathrm{T}) = 32$
$n(\mathrm{S} \cap \mathrm{T}) = 11$

Using $n(\mathrm{S} \cup \mathrm{T}) = n(\mathrm{S}) + n(\mathrm{T}) - n(\mathrm{S} \cap \mathrm{T})$, we get

Substitute the given data

$n(\mathrm{S} \cup \mathrm{T}) \ = 21 + 32 - 11$

$= 53 - 11 = 42$

$\therefore \mathrm{S} \cup \mathrm{T}$ has 42 elements.

**5.** If $X$ and $Y$ are two sets such that $X$ has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does $Y$ have?

**Solution**

Here, given

$n(\mathrm{X}) = 40,$
$n(\mathrm{X} \cup \mathrm{Y}) = 60,$
$n(\mathrm{X} \cap \mathrm{Y}) = 10$

Have to find $n(\mathrm{Y})$.

Using $n(\mathrm{X} \cup \mathrm{Y}) = n(\mathrm{X}) + n(\mathrm{Y}) - n(\mathrm{X} \cap \mathrm{Y})$,

Substitute the values

$60 = 40 + n(\mathrm{Y}) - 10$

$60 = 30 + n(\mathrm{Y})$

$n(\mathrm{Y}) \ = 30$

**6.** In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

**Solution**

Let $\mathrm{C}$ be the set of people who like coffee and $\mathrm{T}$ be the set of people who like tea.

Given data

$n(\mathrm{C} \cup \mathrm{T}) = 70,$
$n(\mathrm{C}) = 37,$
$n(\mathrm{T}) = 52$

Using $n(C \cup T) = n(C) + n(T) - n(C \cap T)$, we get

Substitute the values

$$70 = 37 + 52 - n(C \cap T)$$

$$n(C \cap T) = 89 - 70 = 19$$

19 people like both coffee and tea.

**7.** In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

**Solution**

Let $C$ mean a set of people liked cricket and $T$ mean a set of people liked tennis.

Given information

$$n(C \cup T) = 65,$$
$$n(C) = 40,$$
$$n(C \cap T) = 10$$

Using $n(C \cup T) = n(C) + n(T) - n(C \cap T)$,

Substitute the values

$$65 = 40 + n(T) - 10$$

$$65 = 30 + n(T)$$

$$n(T) = 35$$

$\therefore$ 35 people like tennis.

Now, $n(T - C) = n(T) - n(C \cap T)$

Substitute the values

$$= 35 - 10$$

$$= 25$$

25 people like tennis only and not cricket.

**8.** In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

**Solution**

Let $F$ means a set of people speaking French and $S$ means a set of people speaking Spanish.

Given information

$n(\mathrm{F})=50,$

$n(\mathrm{S})=20,$

$n(\mathrm{F}\cap\mathrm{S})=10$

To find the number of people speaking at least one of the two languages,

That is $n(\mathrm{F}\cup\mathrm{S})$

$n(\mathrm{F}\cup\mathrm{S})=n(\mathrm{F})+n(\mathrm{S})-n(\mathrm{F}\cap\mathrm{S})$

Substitute the values

$=50+20-10=60$

The required number of people speaking at least one of the two languages is $60$.

**Example 28**

Show that the set of letters needed to spell " CATARACT " and the set of letters needed to spell " TRACT" are equal.

**Solution**

Let $X$ be the set of letters in "CATARACT".

Then $X=\{C,A,T,R\}$

Let $\mathrm{Y}$ be the set of letters in "TRACT". Then

$Y=\{T,R,A,C,T\}=\{T,R,A,C\}$

Since every element in $\mathrm{X}$ is in $\mathrm{Y}$ and every element in $\mathrm{Y}$ is in $\mathrm{X}$.

It follows that $\mathrm{X}=\mathrm{Y}$.

**Example 29**

List all the subsets of the set $\{-1,0,1\}$.

**Solution**

Given Set $\mathrm{A}=\{-1,0,1\}$.

The element A-free element is an empty set of $\phi$. The sub-sets of A have one item in $\{-1\},\{0\},\{1\}$. The sub-sets of A have two element $\{-1,0\},\{-1,1\},\{0,1\}$. A subset of A with three elements of A is A itself.

Subsets of given set $=\phi,\{-1\},\{0\},\{1\},\{-1,0\},\{0,1\},\{1,-1\},\{-1,0,1\}$

Thus, all sets under A are $\phi,\{-1\},\{0\},\{1\},\{-1,0\},\{-1,1\}$, $\{0,1\}$ and $\{-1,0,1\}$

**Example 30**

Show that $A \cup B = A \cap B$ implies $A = B$

**Solution**

Let $a \in$ A. Then $a \in A \cup B$. Since $A \cup B = A \cap B, a \in A \cap$ B. So $a \in B$.

Therefore, $A \subset B$. Similarly, if $b \in B$, then $b \in A \cup B$.

Since $A \cup B = A \cap B, b \in A \cap$ B. So, $b \in$ A.

Therefore, $B \subset$ A. Thus, $A = B$

**Example 31**

**For any sets A and B, show that** $P(A \cap B) = P(A) \cap P(B)$

**Solution**

Let a set $X$ belong to Power set $P(A \cap B)$

$X \in P(A \cap B)$

As set $X$ is in the power set of $A \cap B, X$ is a subset of $A \cap B$ because power set is the set of all subsets

Thus, $X$ is a subset of $A \cap B$

$X \subset A \cap B$.

So, $X \subset A$ and $X \subset B$.

Therefore, Since $X$ is a subset of $A$ & $B, X$ is in power set of $A$ and $X$ is in power set of $B$

$X \in P(A)$ and $X \in P(B)$

$X \in P(A)$ and $X \in P(B)$

$\Rightarrow X \in P(A) \cap P(B)$

So, if $X \in P(A \cap B)$, then $X \in P(A) \cap P(B)$

All elements of set $P(A \cap B)$ are in set $P(A) \cap P(B)$

Thus, gives $\mathbf{P(A \cap B) \subset P(A) \cap P(B)}$.

Similarly,

Let a set $Y$ belong to Power set $P(A) \cap P(B)$

i.e. $Y \in P(A) \cap P(B)$

Then $Y \in P(A)$ and $Y \in P(B)$

As set $Y$ is in the power set of $A \& B, Y$ is a subset of $A \& Y$ is $a$ subset of B because power set is the set of all subsets

Thus, $\mathrm{Y} \subset \mathrm{A}$ and $\mathrm{Y} \subset \mathrm{B}$

$\therefore Y \subset A \cap B$

Therefore, since $Y$ is a subset of $A \cap B, Y$ is in power set of $A \cap B$

$\Rightarrow Y \in P(A \cap B)$

So, if $Y \in P(A) \cap P(B)$, then $Y \in P(A \cap B)$

This gives $\mathbf{P(A) \cap P(B) \subset P(A \cap B)}$

Now,

Since $P(A \cap B) \subset P(A) \cap P(B)$ & $P(A) \cap P(B) \subset P(A \cap B)$

Hence, $P(A \cap B) = P(A) \cap P(B)$.

**Example 32**

A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product $\mathrm{A}$ and 450 consumers like product $\mathrm{B}$, what is the least number that must have liked both products?

**Solution**

Set of buyers who liked the product $A$ & product $B$ to $A$ & $B$ respectively

Number of buyers who liked product $A = n(A) = 720$,

Number of buyers who liked product $B = n(B) = 450$

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 720 + 450 - n(A \cap B)$

$n(A \cup B) = 1170 - n(A \cap B)$

$n(A \cap B) = 1170 - n(A \cup B)$

Therefore, $\mathrm{n(A \cap B)}$ is less than $\mathrm{n(A \cup B)}$ is maximum

Number of people who liked product A or B (i.e. n (A U B)) is higher than the number of people surveyed

So, maximum value of $n(A \cup B)$ is 1000.

Thus, the least value of $n(A \cap B)$ is

$n(A \cap B) = 1170 - n(A \cup B)$

$= 1170 - 1000$

$= 170$

Hence, the minimum number of consumers who prefer both products is 170.

**Example 33**

Out of 500 car owners investigated, 400 owned car A and 200 owned car $\mathrm{B}, 50$ owned both $\mathrm{A}$ and $\mathrm{B}$ cars. Is this data correct?

**Solution**

Let the set of owners of car A & car B became A & B respectively

Numbers of car owned by A $= n(A) = 400$

Numbers of car owned by B $= n(B) = 200$

Number of car owned by A and B $= n(A \cap B) = 50$

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 200 + 400 - 50$

$= 550$

Thus, number of people who own car A or car B $= 550$

But total number of car owners is only 500 (given in question)

Number of persons owning car A or car B is less than total number of car owners investigated

But, here $550 > 500$

This is a contradiction.

So, the data provided is incorrect.

**Example 34**

A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports?

**Solution**

Let $F, B, C$ means a set of people received medals in football, basketball & cricket respectively

Given,

Number of medals won in football $= n(F) = 38$,

Number of medals won in basketball $= n(B) = 15$,

Number of medals won in cricket $= n(C) = 20$

Number of medals won in either football, basketball or cricket $= n(F \cup B \cup C) = 58$

Number of medals won in all football, basketball and cricket $= n(F \cap B \cap C) = 3$

We know

$$n(F \cup B \cup C) = n(F) + n(B) + n(C) - n(F \cap B) - n(F \cap C) - n(B \cap C) + n(F \cap B \cap C)$$

Substitute the values

$$58 = 38 + 15 + 20 - n(F \cap B) - n(F \cap C) - n(B \cap C) + 3$$

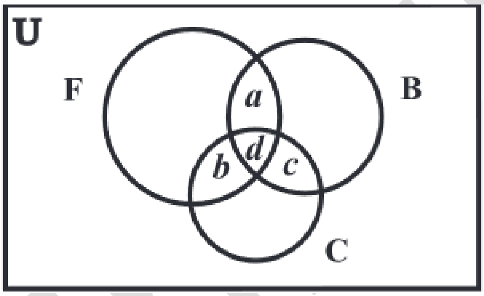
$$58 = 38 + 15 + 20 + 3 - n(F \cap B) - n(F \cap C) - n(B \cap C)$$

$$58 = 76 - n(F \cap B) - n(F \cap C) - n(B \cap C)$$

Rearrange the equation

$$n(F \cap B) + n(F \cap C) + n(B \cap C) = 76 - 58$$

$$n(F \cap B) + n(F \cap C) + n(B \cap C) = 18$$




Also, In the Venn diagram,

Let **a** means the number of men who win medals in football & basketball but not cricket

Let **b** means the number of men win medals in football & cricket but not basketball

Let **c** means the number of men win got medals in cricket & basketball but not football

Let **d** means the number of men who win medals in football & basketball & cricket

Here $d = n(F \cap B \cap C)$

$\therefore d = 3$

Number of people who received exactly two medals $= a + b + c$

From equation

$n(F \cap B)+n(F \cap C)+n(B \cap C)=18$

Rearrange the equation as per diagram

$(a+d)+(b+d)+(c+d)=18$

$a+b+c+d+d+d=18$

Rearrange the equation

$a+b+c+3d=18$

$a+b+c+3\times 3=18$

$a+b+c+9=18$

Arrange in common terms

$a+b+c=18-9$

$a+b+c=9$

$\therefore 9$ people who won medals in two of the three sports

**Miscellaneous Exercise on Chapter 1**

1**.** Decide, among the following sets, which sets are subsets of one and another: $\mathrm{A}=\{x : x \in \mathbf{R}$ and $x$ satisfy $x^2-8x+12=0\}$

$\mathrm{B}=\{2,4,6\}, \quad \mathrm{C}=\{2,4,6,8,\ldots\}, \mathrm{D}=\{6\}$

**Solution**

**Given**

$\mathrm{A}=\{x : x \in \mathrm{R}$ and $(x-2)(x-6)=0\}=\{2,6\}$

$\mathrm{B}=\{2,4,6\}, \mathrm{C}=\{2,4,6,8,\ldots\}, \mathrm{D}=\{6\}$

Here,

So, all elements of $A$ are in $B$, $A$ is a subset of $B$,

$A \subset B$

Hence all elements of $A$ are in $C$, $A$ is a subset of $C$,

$A \subset C$

$A=\{2,6\}, B=\{2,4,6\}, C=\{2,4,6,8\ldots\}, D=\{6\}$

$C$ is not a subset of any set.

So, all elements of $D$ is in $A$, $D$ is a subset of $A$,

That is $D \subset A$

Since all elements of $D$ are in $B$, $D$ is a subset of B,

That is $D \subset B$

**2.** In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x \in A$ and $A \in B$, then $x \in B$

(ii) If $A \subset B$ and $B \in C$, then $A \in C$

(iii) If $A \subset B$ and $B \subset C$, then $A \subset C$

(iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$

(v) If $x \in A$ and $A \not\subset B$, then $x \in B$

(vi) If $A \subset B$ and $x \notin B$, then $x \notin A$

**Solution**

(i) False. Let $A = \{1\}$ and $B = \{1\}, 2\}$. Clearly, $1 \in A$ and $A \in B$ but $1 \notin B$.

Thus, $x \in A$ and $A \in B$ need not imply $x \in B$.

(ii) False. Let $A = \{1\}, B = \{1, 2\}$ and $C = \{\{1, 2\}, 3\}$. Clearly, $A \subset B$ and $B \in C$ but $A \notin C$.

Thus, $A \subset B$ and $B \in C$ need not imply $A \in C$.

(iii) True. Let $x$ be any element of $A$. Then

$x \in A \Rightarrow x \in B$

$\Rightarrow x \in Cz$

Thus, $x \in A \Rightarrow x \in C$ for all $x \in A$, therefore, $A \subset C$.

Hence, $A \subset B$ and $B \subset C \Rightarrow A \subset C$.

(iv) False. Let $A = \{1, 2\}, B = \{2, 3\}$ and $C = \{1, 2, 5\}$. Clearly, $A \not\subset B$ since $1 \in A$ and $1 \notin B$. Also $B \not\subset C$ since $3 \in B$ and $3 \notin$ C.

But $A \subset C$.

Thus, $A \not\subset B$ and $B \not\subset C$ need not imply $A \not\subset C$.

(v) False. Let $A = \{1, 2\}$ and $B = \{2, 3\}$. Clearly, $1 \in A$ and AB but $1 \notin B$.

Thus, $x \in A$ and $A \not\subset B$ need not imply $x \in B$.

(vi) True. Let $A \subset B$ and $x \notin B$. If possible, suppose $x \in A$.

Now, $x \in A$ and $A \subset B \Rightarrow x \in B$ which is a contradiction to given.

Therefore, our supposition is wrong. Hence $x \notin A$.

Thus, $A \subset B$ and $x \notin B \Rightarrow x \notin A$.

**3.** Let $A, B$, and $C$ be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

**Solution**

Let $x \in B$

The subsets is combination of $A \cup B$

$\Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cup C$

$[\because A \cup B = A \cup C$ (given)$]$

$\Rightarrow x \in A$ or $x \in C$

For Case I

Let $x \in A$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$

$[\because A \cap B = A \cap C$ (given)$]$

$\Rightarrow x \in A \cap C$

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in C$ also

For Case II

Let $x \in C$

$\therefore$ In each case $x \in B \Rightarrow x \in C$

$\because B \subset C$

Similarly, $C \subset B$

So, $B = C$.

**4.** Show that the following four conditions are equivalent:

(i) $A \subset B$

(ii) $A-B=\phi$

(iii) $A\cup B=B$

(iv) $A\cap B=A$

**Solution**

Showing Condition (i) is equals to Condition (ii).

Let $A\subset B$

It means all elements of $A$ are in $B$,

hence, A has no element different from B

$\Rightarrow A-B=\Phi$

Showing Condition (ii) is equals to Condition (iii).

$A-B=\varnothing$

It means $A$ has no elements different from $B$

Therefore, all elements of $A$ are in $B$

Since, $A\cup B=B$

Showing Condition (iii) is equals to Condition (iv).

$A\cup B=B$

It means all elements of $A$ are in $B$,

So, the same elements of $A$ and $B$ must be the elements of $A$

So, $A\cap B=A$

Thus, all the four conditions are equals.

**5.** Show that if $A\subset B$, then $C-B\subset C-A$.

**Solution**

Given $A\subset B$

Let $x\in(C-B)\Rightarrow x\in C$ and $x\notin B$

$x\in C$ and $x\notin A$

$x\in(C-A)$

It is proven that $(C-B)\subset(C-A)$

**6.** Assume that $P(A)=P(B)$. Show that $A=B$

**Solution**

Let assume $P(A) = P(B)$,

i.e., Power set of $A =$ Power set of $B$

To prove $A = B$.

Let $x \in A$.

$\therefore |x| \subset A$

Subsets of $P(A)$

$\therefore \{x\} \in P(A)$

$\{x| \in P(B)$

$\{x\} \subset B$

$x \in B$

$A \subset B$

Similarly, $B \subset A$

Hence it $A = B$ is shown

**7.** Is it true that for any sets $A$ and $B, P(A) \cup P(B) = P(A \cup B)$ ? Justify your answer.

**Solution**

Let

$A = \{1\}, B = \{2\}$, then

$P(A) = \{\phi, \{1\}\}$ and $P(B) = \{\phi, \{2\}\}$

$P(A) \cup P(B) = \{\phi, \{1\}\} \cup \{\phi, \{2\}|$

$= \{\phi, \{1\}, \{2\}\}$

Also $A \cup B = \{1, 2\}$, then

$P(A \cup B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Now, $\{1, 2\} \in P(A \cup B)$ but $\{1, 2\} \notin P(A) \cup P(B)$

Therefore, $P(A) \cup P(B) \neq P(A \cup B)$

**8.** Show that for any sets A and B,

$A = (A \cap B) \cup (A - B)$ **and** $A \cup (B - A) = (A \cup B)$

**Solution**

Let consider $U=\{1,2,3,4,5\}$

$A=\{1,2\}$

$B=\{2,3,4\}$

$A-B=A-(A\cap B)=\{1,2\}-\{2\}$

$=\{1\}$

Also,

$B'=U-B=\{1,2,3,4,5\}-\{2,3,4\}$

$=\{1,5\}$

$A-B=A\cap B'=\{1,2\}\cap\{1,5\}$

$=\{1\}$

$(A\cap B)\cup(A-B)\ =(A\cap B)\cup\left(A\cap B'\right)$

Applying the Distributive Law

$=A\cap\left(B\cup B'\right)$

$=A\cap U=A$

Also $A\cup(B-A)=A\cup\left(B\cap A'\right)$

Applying the Distributive Law

$=(A\cup B)\cap\left(A\cup A'\right)$

$=(A\cup B)\cap U$

$=A\cup B$

**9.** Using properties of sets, show that

(i) $A\cup(A\cap B)=A$

(ii) $A\cap(A\cup B)=A$.

**Solution**

In order to prove

**(i)** $A\cup(A\cap B)=A$

we should prove

$A\cup(A\cap B)$ is a subset of $A$

That is $A\cup(A\cap B)\subset A$

$A$ is a subset of $A\cup(A\cap B)$

Then $A\subset A\cup(A\cap B)$

As set is a subset of itself, $A\subset A$

Also, $A$ is a subset of $A\cap B$,

So, $A\subset A\cap B$

as all elements of set $A$ are in $A\cap B$

Now,

$A\cup(A\cap B)$

Using distributive law: $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$=(A)\cap(A\cup B)$

$=A$

Thus, $A\cup(A\cap B)=A$

Hence its proved

**(ii)** $A\cap(A\cup B)=A$

$A\cup(A\cap B)$

Using distributive law: $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$=(A\cup A)\cap(A\cup B)$

$=(A)\cap(A\cup B)$

$=A$

Thus, $A\cup(A\cap B)=A$

Hence its proved

**10.** Show that $A\cap B=A\cap C$ need not imply $B=C$.

**Solution**

Let $A=\{1,2,3\}, B=\{2,4\}, C=\{2,5\}$

Then $A\cap B=\{2\}=A\cap C$

But $B\neq C$.

**11.** Let $A$ and $B$ be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set $X$, show that $A = B$.

**Solution**

Given:

Let $A$ and $B$ be two sets such that $A \cap X = B \cap X = \varnothing$ and $A \cup X = B \cup X$ for some set $X$

To prove: $A = B$

Let $A = A \cap (A \cup X)$

Given $A \cup X = B \cup X$

$A = A \cap (B \cup X)$

Using distributive law:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$= (A \cap B) \cup (A \cap X)$

As $A \cap X = \varnothing$ given

$= (A \cap B) \cup \varnothing$

So, $A = A \cap B$

Again,

Let $B = B \cap (B \cup X)$

Given $A \cup X = B \cup X$

$B = B \cap (A \cup X)$

Using distributive law:

$A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

$= (B \cap A) \cup (B \cap X)$

As $B \cap X = \Phi$

$= (B \cap A) \cup \Phi$

$B = B \cap A$

$B = A \cap B$

$A = A \cap B$ & $B = A \cap B$

$A = B$

Hence the both condition is proved.

**12.** Find sets $\mathrm{A}, \mathrm{B}$ and $\mathrm{C}$ such that $\mathrm{A} \cap \mathrm{B}, \mathrm{B} \cap \mathrm{C}$ and $\mathrm{A} \cap \mathrm{C}$ are non-empty sets and $\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}=\phi$.

**Solution**

Let consider $\mathrm{A}=\{x, y\}, \mathrm{B}=\{x, z\}, \mathrm{C}=\{y, z\}$

So the sets will be $\mathrm{A} \cap \mathrm{B}=\{x\} \neq \phi, \mathrm{B} \cap \mathrm{C}=\{z| \neq \phi, \mathrm{A} \cap \mathrm{C}=\{y| \neq \phi$

and $A \cap B \cap C=\phi$.

**13.** In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

**Solution**

Given,

Total students $=600$

$n(\mathrm{T})=$ Number of students drinking tea $=150$

$n(\mathrm{C})=$ Number of students drinking coffee $=225$

$n(\mathrm{T} \cap \mathrm{C})=n($ both tea and coffee $)=100$

Using the formula

$n(\mathrm{T} \cup \mathrm{C})=n(\mathrm{T})+n(\mathrm{C})-n(\mathrm{T} \cap \mathrm{C})$

Substitute the values

$=150+225-100=275$

i.e., number of students taking at least one of the two drinks

$=275$

$\therefore$ Number of students drinking tea or coffee

$=$Total number of students $-n(\mathrm{T} \cup \mathrm{C})$

$=600-275=325$

$325$ students were taking tea or coffee.

**14.** In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

**Solution**

Let $H$ mean a set of students who knows Hindi and $E$ mean a set of students who knows English.

Number of students knows Hindi $= n(H) = 100$

Number of students knows English $= n(E) = 50$

Number of students knowing Hindi and English $= n(H \cap E) = 25$

So, each of the students knows Hindi or English

Number of students in group

= Number of students knows Hindi or English

$= n(H \cup E)$

Using the formula

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$
$$= 100 + 50 - 25$$
$$= 125$$

Hence, 125 students in the group.

**15.** In a survey of 60 people, it was found that 25 people read newspaper $H, 26$ read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both $T$ and $I, 3$ read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

**Solution**

Number of people reads newspaper $H = n(H) = 25$,

Number of people reads newspaper $T = n(T) = 26$,

Number of people reads newspaper $I = n(I) = 26$,

Number of people who reading both $H \ \& \ I = n(H \cap I) = 9$,

Number of people who reading both $H \ \& \ T = n(H \cap T) = 11$

Number of people who reading both $T \ \& \ I = n(T \cap I) = 8$

Number of people who reading all $H, T \ \& \ I = n(H \cap T \cap I) = 3$

(i) Number of persons who reading one of the newspapers

$$= n(H \cup T \cup I) = n(H) + n(T) + n(I) - n(H \cap T) - n(H \cap I) - n(I \cap T) + n(H \cap T \cap I)$$

Substitute the values

$= 25 + 26 + 26 - 9 - 11 - 8 + 3 = 52$.

(ii) Number of people who reads exactly one newspaper.

$n[\text{only(H)}] = n(\text{H}) - n(\text{H} \cap \text{T}) - n(\text{H} \cap \text{I}) + n(\text{H} \cap \text{T} \cap \text{I})$

Substitute the values

$= 25 - 11 - 9 + 3 = 8$

$n[\text{only(T)}] = n(\text{T}) - n(\text{T} \cap \text{H}) - n(\text{T} \cap \text{I}) + n(\text{T} \cap \text{H} \cap \text{I})$

Substitute the values

$= 26 - 11 - 8 + 3 = 10$

$n[\text{only (I)}] = n(\text{I}) - n(\text{I} \cap \text{H}) - n(\text{I} \cap \text{T}) + n(\text{T} \cap \text{H} \cap \text{I})$

Substitute the values

$= 26 - 8 - 9 + 3 = 12$

Number of persons who read exactly one newspaper

$= n(\text{ only H}) + n(\text{ only T}) + n(\text{ only I})$

Substitute the values

$= 8 + 10 + 12 = 30$

Hence, 30 people reading one newspaper exactly.

**16.** In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

**Solution**

Let A, B, C set of people who liked product A, product B & product C respectively

Number of people liked product $A = n(A) = 21$

Number of people liked product $B = n(B) = 26$,

Number of people liked product $C = n(C) = 29$,

Number of people liked product $A$ and $B = n(A \cap B) = 14$,

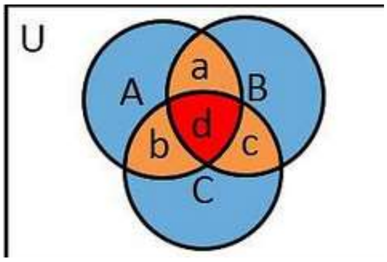
Number of people liked product $C$ and $A = n(C \cap A) = 12$,

Number of people liked product $B$ and $C = n(B \cap C) = 14$,

Number of people liked all three products A, B and C

$= n(A \cap B \cap C) = 8$

Let us draw a Venn diagram




Let a be the number of people who liked product A & B but not C.

Let **b** be the number of people who liked product A & C but not B.

Let **c** be the number of people who liked product **B** & C but not **A**.

Let **d** be the the number of people who liked all three products.

Number of people liked product C only

$= n(C) - b - d - c$

Now, $d = n(A \cap B \cap C) = 8$

Given $n(A \cap C) = 12$

$b + d = 12$

Putting $d = 8$

$b + 8 = 12$

$b = 12 - 8$

$b = 4$

Similarly, $n(B \cap C) = 14$

$c + d = 14$

Putting $d = 8$

$c + 8 = 14$

$c = 14 - 8$

$c = 6$

Number of people who liked product C only $= n(C) - b - d - c$

$= 29 - 4 - 8 - 6$

$= 11$

Therefore, number of people liked product C only is 11.