Infinity Learn by Sri Chaitanya Educational Institutions

Chapter – 9: Sequences and Series

Examples

1. Write the first three terms in each of the following sequences defined by the following

(i) $a_n = 2n+5$

Answer: $a_n = 2n+5$

Substitute n = 1, 2, 3 we get

$a_1 = 2(1)+5 = 7$

$a_2 = 2(2)+5 = 9$

$a_3 = 2(3)+5 = 11$

So, terms are 7, 9, 11

(ii) $a_n = \frac{n-3}{4}$

Answer: Substitute n = 1, 2, 3 we get

$a_1 = \frac{1-3}{4} = -\frac{1}{2}$

$a_2 = \frac{2-3}{4} = -\frac{1}{4}$

$a_3 = \frac{3-3}{4} = 0$

So, terms are $-\frac{1}{2}, -\frac{1}{4}, 0$

2. What is the 20^{th} **term of the sequence defined by** $a_n = (n-1)(2-n)(3+n)$

Answer: Putting n = 20, we get

$a_{20} = (20-1)(2-20)(3+20)$

$= 19\times(-18)\times(23)$

$= -7866$

3. Let the sequence a_n **be defined as follows** $a_1 = 1,\ a_n = a_{n-1}+2$ for $n \geq 2$. **Find first five terms and write corresponding series**

Answer: $a_1 = 1$

$a_2 = a_1+2 = 1+2 = 3$
$a_3 = a_2+2 = 3+2 = 5$
$a_4 = a_3+2 = 5+2 = 7$
$a_5 = a_4+2 = 7+2 = 9$

So, first five terms = 1, 3, 5, 7, 9
Corresponding series = 1+3+5+7+9+…

4. In an AP if m^{th} term is n and the n^{th} term is m where $m \neq n$, find the p^{th} term

Answer: $a_m = a+(m-1)d = n$

$a_n = a+(n-1)d = m$
From above two equations we get
$(m-n)d = n-m$ or $d = -1$
$a = n+m-1$
So, $a_p = a+(p-1)d$
$= n+m-1+(p-1)(-1) = n+m-p$

5. If the sum of n terms of an AP is $nP+\frac{1}{2}n(n-1)Q$ where P, Q are constants, find the common difference

Answer: Given that $a_1, a_2, \ldots a_n$ be an A.P.

Then $S_n = a_1+a_2+a_3+\ldots+a_{n-1}+a_n = nP+\frac{1}{2}n(n-1)Q$
$S_1 = a_1 = P$
$S_2 = a_1+a_2 = 2P+Q$
So, $a_2 = S_2-S_1 = P+Q$
Hence, $d = a_2-a_1 = (P+Q)-P = Q$

6. The sum of n terms of two arithmetic progression are in the ratio $(3n+8):(7n+15)$. Find the ratio of their 12^{th} term

Answer: For the first arithmetic progression, first term = a_1 and common difference = d_1

For the second arithmetic progression, first term = a_2 and common difference = d_2

$$\frac{\text{Sum to n terms of first A.P.}}{\text{Sum to n terms of second A.P.}} = \frac{3n+8}{7n+15}$$

$$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{3n+8}{7n+15}$$

$$\frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{3n+8}{7n+15}$$

$$\frac{12^{th}\ \text{term of first A.P.}}{12^{th}\ \text{term of second A.P}} = \frac{a_1+11d_1}{a_2+11d_2}$$

$$\frac{2a_1+22d_1}{2a_2+22d_2} = \frac{3\times23+8}{7\times23+15}$$

$$\frac{a_1+11d_1}{a_2+11d_2} = \frac{12^{th}\ \text{term of first A.P.}}{12^{th}\ \text{term of second A.P.}} = \frac{7}{16}$$

7. The income of a person is Rs. 300000 in the first year and he receives an increase of Rs. 10000 to his income per year for the next 19 years. Find the total amount he received in 20 years

Answer: In this AP

$a = 300000,\ d = 10000,\ n = 20$

$$S_{20} = \frac{20}{2}[600000+19\times10000]$$

$= 10(790000) = 79{,}00{,}000$

8. Insert six numbers between 3 and 24 such that the resulting sequence is an AP

Answer: Let six numbers = $A_1, A_2, A_3, A_4, A_5, A_6$ between 3 and 24

So, $a = 3,\ b = 24,\ n = 8$

$24 = 3+(8-1)d$

so that $d = 3$

Now, $A_1 = a+d = 3+3 = 6$

$A_2 = a+2d = 3+2\times3 = 9$

$A_3 = a+3d = 3+3\times3 = 12$

$A_4 = a+4d = 3+4\times3 = 15$

$A_5 = a+5d = 3+5\times3 = 18$

$A_6 = a+6d = 3+6\times3 = 21$

Thus, six numbers between 3 and 24 = 6, 9, 12, 15, 18, 21

9. Find the 10^{th} and n^{th} terms of the G.P. 5, 25, 125,…

Answer: a = 5, r = 5

So, $a_{10} = 5(5)^{10-1} = 5(5)^9 = 5^{10}$

$a_n = ar^{n-1} = 5(5)^{n-1} = 5^n$

10. Which term of the G.P. 2, 8, 32,…upto n terms is 131072

Answer: Here 131072 = n^{th} term of G.P.

where a = 2, r = 4

So, $131072 = a_n = 2(4)^{n-1}$

$65536 = 4^{n-1}$

$4^8 = 4^{n-1}$

comparying we get n-1 = 8 i.e. n = 9

So, 131072 is 9^{th} term of GP

11. In a G.P., the 3^{rd} term is 24 and the 6^{th} term is 192. Find the 10^{th} term.

Answer: Given that $a_3 = ar^2 = 24$ and $a_6 = ar^5 = 192$

Now, $\frac{ar^5}{ar^2} = \frac{192}{24}$

$r^3 = 8$

r = 2

Substitute r = 2 in $ar^2 = 24$, we get

$a = \frac{24}{4} = 6$

So, $a_{10} = 6(2)^9 = 3072$

12. Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+\ldots$

Answer: From the above given series a = 1, $r = \frac{2}{3}$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}} = 3\left[1-\left(\frac{2}{3}\right)^n\right]$$

$$\text{Now, } S_5 = 3\left[1-\left(\frac{2}{3}\right)^5\right] = 3\times\frac{211}{243} = \frac{211}{81}$$

13. How many terms of the GP $3, \frac{3}{2}, \frac{3}{4}, \ldots$ are needed to give the sum $\frac{3069}{512}$

Answer: Let no. of terms = n

From given series a = 3, $r = \frac{1}{2}$, $S_n = \frac{3069}{512}$

$$\text{As } S_n = \frac{a(1-r^n)}{1-r}$$

$$\frac{3069}{512} = \frac{3\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} = 6\left(1-\frac{1}{2^n}\right)$$

$$\frac{3069}{3072} = 1-\frac{1}{2^n}$$

$$\frac{1}{2^n} = 1-\frac{3069}{3072} = \frac{3}{3072} = \frac{1}{1024}$$

$$2^n = 1024 = 2^{10}$$

after comparing we get n = 10

14. The sum of first three terms of a GP is $\frac{13}{12}$ and their product is -1. Find the common ratio and the terms

Answer: Let first three terms of GP be $\frac{a}{r}$, a, ar

Then From given conditions $\frac{a}{r}+ar+a = \frac{13}{12}$ and $\left(\frac{a}{r}\right)(a)(ar) = -1$

So, $a^3 = -1$ i.e. a = -1

Now, put value of a in first condition we get

$-\frac{1}{r}-1-r=\frac{13}{12}$

$\frac{-1-r-r^2}{r}=\frac{13}{12}$

$12r^2+25r+12=0$

$12r^2+16r+9r+12=0$

$4r(3r+4)+9r+12=0$

$4r(3r+r)+3(3r+4)=0$

$(3r+4)(4r+3)=0$

$r=-\frac{4}{3},-\frac{3}{4}$

three terms of G.P. are $\frac{4}{3}$, -1, $\frac{3}{4}$ for $r=\frac{-3}{4}$

and $\frac{3}{4}$, -1, $\frac{4}{3}$ for $r=\frac{-4}{3}$

15. Find the sum of the sequence 7,77,777,7777,…to n terms

Answer: $S_n=7+77+777+7777+\ldots$ to n terms

$=\frac{7}{9}[9+99+999+9999+\ldots \text{ to n term}]$

$=\frac{7}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\left(10^4-1\right)+\ldots\text{n terms}\right]$

$=\frac{7}{9}\left[\left(10+10^2+10^3+\ldots\text{n terms}\right)-(1+1+1+\ldots\text{n terms})\right]$

$=\frac{7}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$

$=\frac{7}{9}\left[\frac{10\left(10^n-1\right)}{9}-n\right]$

16. A person has 2 parents, 4 grandparents, 8 great grandparents and so on. Find the number of his ancestors during ten generations preceding his own

Answer: $a=2$, $r=2$, $n=10$

As, $S_n = \frac{a(r^n-1)}{r-1}$

So, $S_{10} = 2(2^{10}-1) = 2(1023) = 2046$

17. Insert three numbers between 1 and 256 so that the resulting sequence is a GP

Answer: Let three nos.between 1 and 256 be g_1, g_2, g_3

i.e. $1, g_1, g_2, g_3, 256$ are in G.P

$a = 1$, 5^{th} term $= 256$

$ar^4 = 256$

$(1)r^4 = 256$

$r^4 = 4^4$

$r = 4$

Now, $g_1 = ar = 1\times4 = 4$

$g_2 = ar^2 = 1\times4^2 = 16$

$g_3 = ar^3 = 1\times4^3 = 64$

So, nos. are 4, 16, 64

18. If AM and GM be two positive numbers a, b are $10, 8$ respectively. Find numbers

Answer: $\frac{a+b}{2} = 10 \Rightarrow a+b = 20$ (i)

$\sqrt{ab} = 8 \Rightarrow ab = 8^2 = 64$

Put values in the identity $(a-b)^2 = (a+b)^2-4ab$, we get

$(a-b)^2 = 400-256 = 144$

$a-b = \pm12$ (ii)

From (i) and (ii)

$a = 4, b = 16$ or $a = 16, b = 4$

Hence, nos. a and b are 4, 16 or 16, 4 respectively.

19. Find the sum to n terms of the series $5+11+19+29+41+\ldots$

Answer: $S_n = 5+11+19+29+\ldots+a_{n-1}+a_n$

$S_n = 5+11+19+\ldots+a_{n-2}+a_{n-1}+a_n$

Now, subtract these equations we get

$0 = 5+[6+8+10+12+\ldots(n-1) \text{ terms}]-a_n$

$$a_n = 5+\frac{(n-1)[12+(n-2)\times 2]}{2} = 5+(n-1)(n+4) = n^2+3n+1$$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n}\left(k^2+3k+1\right)$$

$$= \sum_{k=1}^{n} k^2+3\sum_{1}^{n} k+n$$

$$= \frac{n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}+n$$

$$= \frac{n(n+2)(n+4)}{3}$$

20. Find the sum to n terms of the series whose n^{th} term is $n(n+3)$

Answer: As it is given that

$$a_n = n(n+3) = n^2+3n$$

Sum to n terms i.e. $S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2+3\sum_{k=1}^{n} k$

$$= \frac{n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2} = \frac{n(n+1)(n+5)}{3}$$

21. If $p^{th}, q^{th}, r^{th}, s^{th}$ terms of an A.P. are in G.P, then show that $(p-q), (q-r), (r-s)$ are also in G.P.

Answer: From given conditions:

$a_p = a+(p-1)d$

$a_q = a+(q-1)d$

$a_r = a+(r-1)d$

$a_s = a+(s-1)d$

As a_p, a_q, a_r, a_s are in G.P

So, $\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_s}{a_r}$

$$\frac{a_q}{a_p} = \frac{a_r}{a_q}$$

$$\frac{a_q}{a_p}-1 = \frac{a_r}{a_q}-1$$

$$\frac{a_q-a_p}{a_r-a_q}=\frac{a_p}{a_q}$$

$$\frac{a+(q-1)d-[a+(p-1)d]}{a+(r-1)d-[a+(q-1)d]}=\frac{a+(p-1)d}{a+(q-1)d}$$

$$\frac{q-r}{p-q}=\frac{a_p}{a_q} \qquad \text{(i)}$$

$$\frac{a_r}{a_q}=\frac{a_s}{a_r}$$

$$\frac{a_r}{a_q}-1=\frac{a_s}{a_r}-1$$

$$\frac{a_r-a_s}{a_q-a_r}=\frac{a_r}{a_q}$$

$$\frac{a+(r-1)d-[a+(s-1)d]}{a+(q-1)d-[a+(r-1)d]}=\frac{a+(r-1)d}{a+(q-1)d}$$

$$\frac{r-s}{q-r}=\frac{a_q}{a_p} \qquad \text{(ii)}$$

From (i) and (ii), we get

$$\frac{a_p}{a_q}=\frac{a_q}{a_p}$$

$$\frac{q-r}{p-q}=\frac{r-s}{q-r}$$

Thus, p-q, q-r, r-s are in GP

22. If a, b, c are in G.P. and $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}$, prove that x,y,z are in A.P.

Answer: Here $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}=k$

Then, $a=k^x$, $b=k^y$, $c=k^z$ (i)

As a, b, c are in G.P.

So, $b^2=ac$ (ii)

By using (i) in (ii), we get

$k^{2y}=k^{x+z}$

Now, after comparing we get, $2y=x+z$

Thus, x, y, z are in A.P.

23. If a, b, c, d, p are different real numbers such that $\left(a^2+b^2+c^2\right)p^2-2(ab+bc+cd)p+\left(b^2+c^2+d^2\right)\le 0,$ then show that a, b, c, d are in G.P.

Answer: As it is given that

$$\left(a^2+b^2+c^2\right)p^2-2(ab+bc+cd)p+\left(b^2+c^2+d^2\right)\le 0 \qquad (i)$$

From (i) we get

$$\left(a^2p^2-2abp+b^2\right)+\left(b^2p^2-2bcp+c^2\right)+\left(c^2p^2-2cdp+d^2\right)\le 0$$

$$(ap-b)^2+(bp-c)^2+(cp-d)^2\le 0$$

As Sum of squares for real numbers cannot be negative

Hence ap = b, bp = c, cp = d

We get $\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=p$

Thus a, b, c, d are in G.P.

24. If p, q, r are in G.P. and the equations $px^2+2qx+r=0$ and $dx^2+2ex+f=0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

Answer: From given equation $px^2+2qx+r=0$

here a = p, b = 2q, c = r

So, roots are $x=\frac{-2q\pm\sqrt{4q^2-4rp}}{2p}$

As $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Now, given that p, q, r are in G.P.

We get, $q^2=pr$

Hence $x=\frac{-q}{p}$ but $\frac{-q}{p}$ is also root of $dx^2+2ex+f=0$

$$d\left(\frac{-q}{p}\right)^2+2e\left(\frac{-q}{p}\right)+f=0$$

$$dq^2-2eqp+fp^2=0$$

Now, divide above equation by pq^2, we get

$$\frac{dq^2}{pq^2} - \frac{2eqp}{pq^2} + \frac{fp^2}{pq^2} = 0$$

$$\frac{d}{p} - \frac{2e}{q} + \frac{fp}{q^2} = 0$$

$$\frac{d}{p} - \frac{2e}{q} + \frac{fp}{pr} = 0 \qquad (\text{put } q^2 = pr)$$

$$\frac{d}{p} - \frac{2e}{q} + \frac{f}{r} = 0$$

$$\frac{2e}{q} = \frac{d}{p} + \frac{fp}{pr}$$

Thus, $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

Exercise 9.1

1. Write the first five terms of the sequence whose n^{th} term is $a_n = n(n+2)$

Answer: $a_n = n(n+2)$

Substituting n = 1, 2, 3, 4 and 5

$a_1 = 1(1+2) = 3$

$a_2 = 2(2+2) = 8$

$a_3 = 3(3+2) = 15$

$a_4 = 4(4+2) = 24$

$a_5 = 5(5+2) = 35$

Thus, first five terms are 3, 8, 15, 24 and 35

2. Write the first five terms of the sequence whose n^{th} term is $a_n = \frac{n}{n+1}$

Answer: $a_n = \frac{n}{n+1}$

Substituting n = 1, 2, 3, 4, 5

$a_1 = \frac{1}{1+1} = \frac{1}{2}$

$a_2 = \frac{2}{2+1} = \frac{2}{3}$

$a_3 = \frac{3}{3+1} = \frac{3}{4}$

$a_4 = \frac{4}{4+1} = \frac{4}{5}$

$a_5 = \frac{5}{5+1} = \frac{5}{6}$

Thus, first five terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ and $\frac{5}{6}$

3. Write the first five terms of the sequence whose n^{th} term is $a_n = 2^n$

Answer: $a_n = 2^n$

Substituting n = 1, 2, 3, 4, 5

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

Thus, first five terms are 2, 4, 8, 16 and 32

4. Write the first five terms of the sequence whose n^{th} term is $a_n = \frac{2n-3}{6}$

Answer: $a_n = \frac{2n-3}{6}$

Substituting n = 1, 2, 3, 4, 5

$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$

$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$

$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$

$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$

$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$

Thus, first five terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$ and $\frac{7}{6}$

5. Write the first five terms of the sequence whose n^{th} term is $a_n = (-1)^{n-1}5^{n+1}$

Answer: $a_n = (-1)^{n-1}5^{n+1}$

Substituting n = 1, 2, 3, 4, 5

$a_1 = (-1)^{1-1}5^{1+1} = 5^2 = 25$

$a_2 = (-1)^{2-1}5^{2+1} = -5^3 = -125$

$a_3 = (-1)^{3-1}5^{3+1} = 5^4 = 625$

$a_4 = (-1)^{4-1}5^{4+1} = -5^5 = -3125$

$a^5 = (-1)^{5-1}5^{5+1} = 5^6 = 15625$

Thus, first five terms 25, -125, 625, -3125 and 15625

6. Write the first five terms of the sequence whose n^{th} term is $a_n = n\frac{n^2+5}{4}$

Answer: $a_n = n\frac{n^2+5}{4}$

Substituting n = 1, 2, 3, 4, 5

$a_1 = 1\times\frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$

$a_2 = 2\times\frac{2^2+5}{4} = 2\times\frac{9}{4} = \frac{9}{2}$

$a_3 = 3\times\frac{3^2+5}{4} = 3\times\frac{14}{4} = \frac{21}{2}$

$a_4 = 4\times\frac{4^2+5}{4} = 21$

$a_5 = 5\times\frac{5^2+5}{4} = 5\times\frac{30}{4} = \frac{75}{2}$

Thus, first five terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}$, 21 and $\frac{75}{2}$

7. Write the first 17^{th} term in the following sequence whose n^{th} term is $a_n = 4n-3$; a_{17}, a_{24}

Answer: Substituting n = 17

$a_{17} = 4(17)-3 = 68-3 = 65$

Substituting n = 24

$a_{24} = 4(24)-3 = 96-3 = 93$

8. Write the 7^{th} term of the following sequence whose n^{th} term is $a_n = \frac{n^2}{2n}$; a_7

Answer: $a_n = \frac{n^2}{2n}$

Substituting n = 7

$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$

9. Write the 9^{th} term of the following sequence whose n^{th} term is $a_n = (-1)^{n-1}n^3$; a_9

Answer: $a_n = (-1)^{n-1}n^3$

Substituting n = 9

$a_9 = (-1)^{9-1}(9)^3 = (9)^3 = 729$

10. Write the 20^{th} term of the following sequence whose n^{th} term is $a_n = \frac{n(n-2)}{n+3}$; a_{20}

Answer: $a_n = \frac{n(n-2)}{n+3}$

Substituting n = 20

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

11. Write the first five term of the following sequence and obtain the corresponding series

$a_1 = 3,\ a_n = 3a_{n-1}+2$ for all $n > 1$

Answer: $a_1 = 3,\ a_n = 3a_{n-1}+2$ for all $n > 1$

$a_2 = 3a_1+2 = 3(3)+2 = 11$

$a_3 = 3a_2+2 = 3(11)+2 = 35$

$a_4 = 3a_3+2 = 3(35)+2 = 107$

$a_5 = 3a_4+2 = 3(107)+2 = 323$

Thus, first five terms are 3, 11, 35, 107 and 323

Corresponding series is 3+11+35+107+323+…

12. Write the first five term of the following sequence and obtain the corresponding series

$a_1 = -1,\ a_n = \frac{a_{n-1}}{n},\ n \geq 2$

Answer: $a_1 = -1,\ a_n = \frac{a_{n-1}}{n},\ n \geq 2$

$a_2 = \frac{a_1}{2} = \frac{-1}{2}$

$a_3 = \frac{a_2}{3} = \frac{-1}{6}$

$a_4 = \frac{a_3}{4} = \frac{-1}{24}$

$a_5 = \frac{a_4}{4} = \frac{-1}{120}$

Thus, first five terms of the sequence are -1, $\frac{-1}{2}$, $\frac{-1}{6}$, $\frac{-1}{24}$ and $\frac{-1}{120}$

Corresponding series is $(-1)+\left(\frac{-1}{2}\right)+\left(\frac{-1}{6}\right)+\left(\frac{-1}{24}\right)+\left(\frac{-1}{120}\right)+\ldots$

13. Write the first five term of the following sequence and obtain the corresponding series

$a_1 = a_2 = 2, a_n = a_{n-1} - 1$, $n>2$

Answer: $a_1 = a_2 = 2, a_n = a_{n-1} - 1$, $n>2$

$a_3 = a_2 - 1 = 2 - 1 = 1$

$a_4 = a_3 - 1 = 1 - 1 = 0$

$a_5 = a_4 - 1 = 0 - 1 = -1$

Thus, first five terms of the sequence are 2, 2, 1, 0 and -1

Corresponding series is 2+2+1+0+(-1)+…

14. The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}$, $n > 2$ **. find**

$\frac{a_{n-1}}{a_n}$, for n = 1, 2, 3, 4, 5

Answer: $1 = a_1 = a_2$

$a_n = a_{n-1} + a_{n-2}, n > 2$

$a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

$a_6 = a_5 + a_4 = 5 + 3 = 8$

For n = 1, $\frac{a_n+1}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$

For n = 2, $\frac{a_n+1}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2$

For n = 3, $\frac{a_n+1}{a_n} = \frac{a_4}{a_3} = \frac{3}{2}$

For n = 4, $\frac{a_n+1}{a_n} = \frac{a_5}{a_4} = \frac{5}{3}$

For n = 5, $\frac{a_n+1}{a_n} = \frac{a_6}{a_5} = \frac{8}{5}$

Exercise 9.2

1.Find the sum of odd integers from 1 to 2001.

Answer: The odd integers from 1 *to* 2001 are $1, 3, 5 ...1999, 2001$.

This sequence forms an A.P.

Here, first term, $a = 1$

Common difference, $d = 2$

Here, $a+(n-1)d = 2001$

$\Rightarrow 1+(n-1)(2) = 2001$

$\Rightarrow 2n-2 = 2000$

$\Rightarrow n = 1001$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\therefore S_n = \frac{1001}{2}[2\times1+(1001-1)\times2]$$

$$= \frac{1001}{2}[2+1000\times2]$$

$$= \frac{1001}{2}\times2002$$

$= 1001\times1001$

$= 1002001$

As a result, the sum of odd numbers between 1 *and* 2001 *is* 1002001.

2.Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Answer: $105, 110, ... 995$ are natural numbers between 100 *and* 1000 that are multiples of 5.

$$Here, a = 105 and$$
$$d = 5$$
$$a + (n-1)d = 995$$
$$\Rightarrow 105 + (n-1)5 = 995$$
$$\Rightarrow (n-1)5 = 995 - 105 = 890$$
$$\Rightarrow n - 1 = 178$$
$$\Rightarrow n = 179$$
$$\therefore S_n = \frac{179}{2}[2(105) + (179-1)(5)]$$
$$= \frac{179}{2}[2(105) + (178)(5)]$$
$$= 179[105 + (89)5]$$
$$= (179)(105 + 445)$$
$$= (179)(550)$$
$$= 98450$$

As a result, the sum of all natural numbers in the range of 100 *to* 1000 that are multiples of 5 is 98450.

3. In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.

Answer: First term = 2

Let d be the common difference of the A.P.

Therefore, the A.P. is $2,\ 2 + d,\ 2 + 2d,\ 2 + 3d\ ...$

Sum of first five terms $= 10 + 10d$

Sum of next five terms $= 10 + 35d$

According to the given condition,

$10+10d=\frac{1}{4}(10+35d)$

$\Rightarrow 40+40d=10+35d$

$\Rightarrow 30=-5d$

$\Rightarrow d=-6$

$\therefore a_{20}=a+(20-1)d$

$=2+(19)(-6)$

$=2-114$

$=-112$

As a result, the A.P.'s $20th$ term is -112.

4.How many terms of the A.P. $-6,-\frac{11}{2},-5,\ldots$ **are needed to give the sum** -25 **?**

Answer: Let the total of the provided A.P.'s n terms be -25.

It is well knowledge that,

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

Where n = *number of terms*, a = *first term, and* d = *common difference*

Here, $a = -6$

$$d=-\frac{11}{2}+6=\frac{-11+12}{2}=\frac{1}{2}$$

Therefore, We obtain,

$$-25=\frac{n}{2}\left[2\times(-6)+(n-1)\left(\frac{1}{2}\right)\right]$$

$$\Rightarrow -50=n\left[-12+\frac{n}{2}-\frac{1}{2}\right]$$

$$\Rightarrow -50=n\left[-\frac{25}{2}+\frac{n}{2}\right]n$$

$$\Rightarrow -100=n(-25+n)n$$

$$\Rightarrow n^2-25n+100=0n$$

$$\Rightarrow n^2-5n-20n+100=0$$

$$\Rightarrow n(n-5)-20(n-5)=0$$

$$\Rightarrow n=20 \text{ or } 5$$

5. In an A.P., if p^{th} term is $1/q$ and q^{th} term is $1/p$, prove that the sum of first pq terms is $1/2\,(pq+1)$, where $p \neq q$.

Answer: The standard phrase for an $A.P.$ is known as $a_n = a + (n-1)d$.

According to the information provided,

$$p^{th} \text{ term } = a_p = a + (p-1)d = \frac{1}{q} \ldots (1)$$

$$q^{th} \text{ term } = a_q = a + (q-1)d = \frac{1}{p} \ldots (2)$$

When we subtract (2) from (1), we get,

$$(p-1)d - (q-1)d = \frac{1}{q} - \frac{1}{p}$$

$$\Rightarrow (p-1-q+1)d = \frac{p-q}{pq}$$

$$\Rightarrow (p-q)d = \frac{p-q}{pq}$$

$$\Rightarrow d = \frac{1}{pq}$$

We get (1) by plugging in the value of d.

$$a + (p-1)\frac{1}{pq} = \frac{1}{q}$$

$$\Rightarrow a = \frac{1}{q} - \frac{1}{q} + \frac{1}{pq} = \frac{1}{pq}$$

$$\therefore S_{pq} = \frac{pq}{2}[2a + (pq-1)d]$$

$$= \frac{pq}{2}\left[\frac{2}{pq} + (pq-1)\frac{1}{pq}\right]$$

$$= 1 + \frac{1}{2}(pq-1)$$

$$= \frac{1}{2}pq + 1 - \frac{1}{2}$$

$$= \frac{1}{2}pq + \frac{1}{2}$$

$$= \frac{1}{2}(pq+1)$$

As a result, the A.P.'s initial pq words add up to $\frac{1}{2}(pq+1)$.

6.If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116. Find the last term.

Answer: Let the sum of n terms of the given $A.P.$ be 116.

$Here, a = 25 \ and \ d = 22 - 25 = -3$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore S_n = \frac{n}{2}[2 \times 25 + (n-1)(-3)]$$

$$\Rightarrow 116 = \frac{n}{2}[50 - 3n + 3]$$

$$\Rightarrow 232 = n(53 - 3n) = 53n - 3n^2$$

$$\Rightarrow 3n^2 - 53n + 232 = 0$$

$$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$$

$$\Rightarrow 3n(n-8) - 29(n-8) = 0$$

$$\Rightarrow (n-8)(3n-29) = 0$$

$$\Rightarrow n = 8 \text{ or } n = \frac{29}{3}$$

So, n cannot be $\neq \frac{29}{3}$

So, $n = 8$

$$\therefore a_8 = \text{ Last term } = a + (n-1)d = 25 + (8-1)(-3)$$

$$= 25 + (7)(-3) = 25 - 21$$

$$= 4$$

Hence, The Last Term of the $A.P.$ is 4.

7.Find the sum to n terms of the A.P., whose k^{th} term is $5k + 1$.

Answer: It is given that the k^{th} term of the $A.P.$ is $5k + 1$.

k^{th} term $= a_k = a + (k-1)d$

therefore $a + (k-1)\ d = 5\ k + 1\ a + k\ d - d = 5\ k + 1$

therefore, Comparing the coefficient of k, we obtain

$d = 5a - d = 1$

$\Rightarrow a - 5 = 1$

$\Rightarrow a = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(6) + (n-1)(5)]$

$= \frac{n}{2}[12 + 5n - 5]$

$= \frac{n}{2}(5n + 7)$

8. If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants,find the common difference.

Answer: As, $S_n = \frac{n}{2}[2a + (n-1)d]$

So, Given in the question,

$\frac{n}{2}[2a + (n-1)d] = pn + qn^2$

$\Rightarrow \frac{n}{2}[2a + nd - d] = pn + qn^2$

$\Rightarrow na + n^2 \frac{d}{2} - n \cdot \frac{d}{2} = pn + qn^2$

Comparing the coefficients of n^2 on both sides, we obtain

$\frac{d}{2} = q$

$\therefore d = 2q$

Thus, the common difference of the $A.P.$ is $2q$.

9. The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18^{th} terms.

Answer: Let a_1, a_2, and d_1, d_2 be the first terms and the common difference of the first and second arithmetic progression respectively.

According to the given condition,

$$\frac{\text{Sum of } n \text{ terms of first A.P.}}{\text{Sum of } n \text{ terms of second A.P.}}=\frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{n}{\frac{n}{2}[2a_1+(n-1)d_1]}=\frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}=\frac{5n+4}{9n+6}\ldots(1)$$

Substituting $n=35$ in (1), we obtain

$$\frac{2a_1+34d_1}{2a_2+34d_2}=\frac{5(35)+4}{9(35)+6}\Rightarrow \frac{a_1+17d_1}{a_2+17d_2}=\frac{179}{321}\ldots(2)$$

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P}}=\frac{a_1+17d_1}{a_2+17d_2}\ldots(3)$$

From (2) and (3), we obtain

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P.}}=\frac{179}{321}$$

Thus, the ratio of 18^{th} term of both the A.P.s is $179:321$.

10. If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p+q)$ terms.

Answer: Let a and d represent the $A.P.$'s initial term and common difference, respectively.

$$S_p=\frac{p}{2}[2a+(p-1)d]S_q$$

$$=\frac{q}{2}[2a+(q-1)d]$$

As Given in the question,

$\frac{p}{2}[2a+(p-1)d]=\frac{q}{2}[2a+(q-1)d]$

$\Rightarrow p[2a+(p-1)d]=q[2a+(q-1)d]$

$\Rightarrow 2ap+pd(p-1)=2aq+qd(q-1)$

$\Rightarrow 2a(p-q)+d[p(p-1)-q(q-1)]=0$

$\Rightarrow 2a(p-q)+d\left[p^2-p-q^2+q\right]=0$

$\Rightarrow 2a(p-q)+d[(p-q)(p+q)-(p-q)]=0$

$\Rightarrow 2a(p-q)+d[(p-q)(p+q-1)]=0$

$\Rightarrow 2a+d(p+q-1)=0$

$\Rightarrow d=\frac{-2a}{p+q-1}...(1)$

$\therefore S_{p+q}=\frac{p+q}{2}[2a+(p+q-1)\cdot d]$

$\Rightarrow S_{p+q}=\frac{p+q}{2}\left[2a+(p+q-1)\left(\frac{-2a}{p+q-1}\right)\right]\quad [From(1)]$

$=\frac{p+q}{2}[2a-2a]$

$=0$

As a result, the $A.P.'s$ initial $(p+q)$ terms add up to 0.

11. Sum of the first p,q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Answer: Let $a_1 and d$ be the first term and the common difference of the $A.P$ respectively.

According to the given information,

$S_p=\frac{p}{2}\left[2a_1+(p-1)d\right]=a$

$\Rightarrow 2a_1+(p-1)d=\frac{2a}{p}...(1)$

$$S_q = \frac{q}{2}\left[2a_1 + (q-1)d\right] = b$$

$$\Rightarrow 2a_1 + (q-1)d = \frac{2b}{q} \quad ...(2)$$

$$S_r = \frac{r}{2}\left[2a_1 + (r-1)d\right] = c$$

$$\Rightarrow 2a_1 + (r-1)d = \frac{2c}{r} \quad ...(3)$$

Subtracting (2) from (1), we obtain

$$(p-1)d - (q-1)d = \frac{2a}{p} - \frac{2b}{q}$$

$$\Rightarrow d(p-1-q+1) = \frac{2aq-2bq}{pq}$$

$$\Rightarrow d(p-q) = \frac{2aq-2bp}{pq}$$

$$\Rightarrow d = \frac{2(aq-bp)}{pq(p-q)} \quad ...(4)$$

Subtracting (3) from (2), we obtain

$$(q-1)d - (r-1)d = \frac{2b}{q} - \frac{2c}{r}$$

$$\Rightarrow d(q-1-r+1) = \frac{2b}{q} - \frac{2c}{r}$$

$$\Rightarrow d(q-r) = \frac{2br-2qc}{qr}$$

$$\Rightarrow d = \frac{2(br-qc)}{qr(q-r)} \quad ...(5)$$

Equating both the values of d obtained in (4) and (5), we obtain

$$\frac{aq-bp}{pq(p-q)} = \frac{br-qc}{qr(q-r)}$$

$$\Rightarrow qr(q-r)(aq-bq) = pq(p-q)(br-qc)$$

$$\Rightarrow r(aq-bp)(q-r) = p(br-qc)(p-q)$$

$$\Rightarrow (aqr-bpr)(q-r) = (bpr-pqc)(p-q)$$

Dividing both sides by pqr, we obtain

$$\left(\frac{a}{p}-\frac{b}{q}\right)(q-r)=\left(\frac{b}{q}-\frac{c}{r}\right)(p-q)$$

$$\Rightarrow \frac{a}{p}(q-r)-\frac{b}{q}(q-r+p-q)+\frac{c}{r}(p-q)=0$$

$$\Rightarrow \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

As a consequence, the stated outcome is established.

12.The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} term is $(2m-1):(2n-1)$.

Answer: Let a and b represent the $A.P.'s$ initial term and common difference, respectively.

In light of the circumstances,

$$\frac{\text{Sum of } m \text{ terms}}{\text{Sum of } n \text{ terms}}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]}=\frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n}\ldots(1)$$

Putting $m=2m-1$ and $n=2n-1$ in (1), we obtain

$$\frac{2a+(2m-2)d}{2a+(2n-2)d}=\frac{2m-1}{2n-1}$$

$$\Rightarrow \frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1}$$

$$\frac{m^{\text{th}}}{n^{\text{th}} \text{ term of A.P.}}=\frac{a+(m-1)d}{a+(n-1)d}$$

From (2) and (3), we obtain

$$\frac{m^{\text{th}} \text{ term of A.P}}{n^{\text{th}} \text{ term of A.P}}=\frac{2m-1}{2n-1}$$

Thus, the given result is proved.

13.If the sum of n terms of an A.P. is $2n^2+5n$ and its m^{th} term is 164, find the value of m.

Answer: Let a and b be the first term and the common difference of the A.P. respectively.

$a_m = a+(m-1)d = 164\ldots(1)$

Sum of n terms: $S_n = \frac{n}{2}[2a+(n-1)d]$

Here,

$$\frac{n}{2}[2a+nd-d] = 3n^2+5n$$
$$\Rightarrow na+n^2\cdot\frac{d}{2} = 3n^2+5n$$

Comparing the coefficient of n^2 on both sides, we obtain

$$\frac{d}{2}=3$$
$$\Rightarrow d=6$$

Comparing the coefficient of n on both sides, we obtain

$$a-\frac{d}{2}=5$$
$$\Rightarrow a-3=5$$
$$\Rightarrow a=8$$

Therefore, from (1), we obtain

$$8+(m-1)6=164$$
$$\Rightarrow (m-1)6=164-8=156$$
$$\Rightarrow m-1=26$$
$$\Rightarrow m=27$$

Thus, the value of m is 27 .

14.Insert five numbers between 8 and 26 such that the resulting sequence is anA.P.

Answer: Let $A1,\ A2,\ A3,\ A4,\ and\ A5$ be five numbers between 8 and 26

such that $8,\ A1,\ A2,\ A3,\ A4,\ A5,\ 26$ is an A.P.

Here, $a=8, b=26, n=7$

Therefore,

$26 = 8 + (7-1)d$

$\Rightarrow 6d = 26 - 8 = 18$

$\Rightarrow d = 3$

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2\times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3\times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4\times 3 = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 5\times 3 = 8 + 15 = 23$

Thus, the required five numbers between 8 and 26 are $11, 14, 17, 20, and\ 23$.

15. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ **is the A.M. between** a and b, **then find the value of** n.

Answer: A.M. of a and $b = \frac{a+b}{2}$

According to the question,

$\frac{a+b}{2} = \frac{a^n + b^n}{a^{n-1} + b^{n-1}}$

$\Rightarrow (a+b)\left(a^{n-1} + b^{n-1}\right) = 2\left(a^n + b^n\right)$

$\Rightarrow a^n + ab^{n-1} + ba^{n-1} + b^n = 2a^n + 2b^n$

$\Rightarrow ab^{n-1} + a^{n-1}b = a^n + b^n \Rightarrow ab^{n-1} - b^n = a^n - a^{n-1}$

$\Rightarrow b^{n-1}(a-b) = a^{n-1}(a-b) \Rightarrow b^{n-1} = a^{n-1}$

$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0$

$\Rightarrow n - 1 = 0$

$\Rightarrow n = 1$

16.Between 1 and 31, m numbers have been inserted in such a way that theresulting sequence is an A.P. and the ratio of 7^{th} and $(m-1)^{th}$ numbers is $5:9$.Find the value of m .

Answer: Let $A_1, A_2, \ldots A_m$ be m numbers such that $1, A_1, A_2, \ldots A_m$, 31 is an A.P. Here,

$a=1, b=31, n=m+2$

$\therefore 31=1+(m+2-1)(d)$

$\Rightarrow 30=(m+1)d$

$\Rightarrow d=\dfrac{30}{m+1}n \ldots(1)$

$A_1=a+d$

$A_2=a+2d$

$A_3=a+3d \ldots$

$\therefore A_7=a+7d$

$A_{m-1}=a+(m-1)d$

According to the given condition,

$$\frac{a+7d}{a+(m-1)d}=\frac{5}{9}$$

$$\Rightarrow \frac{1+7\left(\frac{30}{(m+1)}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)}=\frac{5}{9}$$

$$\Rightarrow \frac{m+1+7(30)}{m+1+30(m-1)}=\frac{5}{9}$$

$$\Rightarrow \frac{m+1+210}{m+1+30m-30}=\frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29}=\frac{5}{9}$$

$\Rightarrow 9m+1899=155m-145$

$\Rightarrow 155m-9m=1899+145$

$\Rightarrow 146m=2044$

$\Rightarrow m=14$

Thus, the value of m is 14.

17.A man starts repaying a loan as first installment of Rs. 100. If he increasesthe installment by Rs 5 every month, what amount he will pay in the 30^{th} installment?

Answer: The loan's initial instalment is $Rs.100$.

The loan's second instalment is $Rs.105$, and so on.

The amount the man repays each month is called an A.P.
The A.P. values are $100, 105, and\ 110 \ldots$

First term, $a = 100$
Common difference, $d = 5$
$A_{30} = a + (30-1)d$
$= 100 + (29)(5)$
$= 100 + 145$
$= 245$
Thus, the amount to be paid in the 30^{th} installment is $Rs.245$.

18.The difference between any two consecutive interior angles of a polygon is 5° . If the smallest angle is 120° , find the number of the sides of the polygon.

Answer: The angles of the polygon will form an A.P. with common difference d as 5° and first term a as 120°.

It is known that the sum of all angles of a polygon with n sides is $180^\circ(n-2)$.

$\therefore S_n = 180^\circ(n-2)$
$\Rightarrow \frac{n}{2}[2a+(n-1)d] = 180^\circ(n-2)$
$\Rightarrow \frac{n}{2}\left[240^\circ + (n-1)5^\circ\right] = 180(n-2)$
$\Rightarrow n[240+(n-1)5] = 360(n-2)$
$\Rightarrow 240n + 5n^2 - 5n = 360n - 720$
$\Rightarrow 5n^2 + 235n - 360n + 720 = 0$
$\Rightarrow 5n^2 - 125n + 720 = 0$
$\Rightarrow n^2 - 25n + 144 = 0$
$\Rightarrow n^2 - 16n - 9n + 144 = 0$
$\Rightarrow n(n-16) - 9(n-16) = 0$
$\Rightarrow (n-9)(n-16) = 0$
$\Rightarrow n = 9$ or 16

Exercise 9.3

1. Find the 20^{th} and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Answer: The G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

$a = \text{first term} = \frac{5}{2}$

$r = \text{common ratio} = \frac{\frac{5}{4}}{5}$

$= \frac{1}{2} a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$

$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$

2. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

Answer: Common ratio, $r = 2$

Let a be the first term of the $G.P.$

$\therefore a_8 = ar^{8-1} = ar^7$

$\Rightarrow ar^7 = 192 a(2)^7 = 192 a(2)^7 = (2)^6(3)$

$\Rightarrow a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$

$\therefore a_{12} = ar^{12-1}$

$= \left(\frac{3}{2}\right)(2)^{11}$

$= (3)(2)^{10}$

$= 3072$

3. The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

Answer: Let a be the first term and r be the common ratio of the $G.P.$

According to the given condition,

$a_5 = ar^{5-1} = ar^4 = p \ldots(1)$

$a_8 = ar^{8-1} = ar^7 = q \ldots(2)$

$a_{11} = ar^{11-1} = ar^{10} = s \ldots(3)$

Dividing equation (2) by (1), we obtain

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \ldots(4)$$

Dividing equation (3) by (2), we obtain

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$\Rightarrow r^3 = \frac{s}{q} \ldots(5)$$

Equating the values of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow q^2 = ps$$

Hence Proved.

4.The 4^{th} term of a G.P. is square of its second term, and the first term is -3.

Determine its 7^{th} term.

Answer: Let a be the first term and r be the common ratio of the $G.P.$

therefore $a = -3$

It is known that, $a_n = ar^{n-1}$

therefore $a_4 = ar^3 = (-3)r^3$

$a_2 = ar^1 = (-3)r$

According to the given condition,

$$(-3)r^3 = [(-3)r]^2$$

$$\Rightarrow -3r^3 = 9r^2$$

$$\Rightarrow r = -3 a_7 = ar^{7-1} = a$$

$$r^6 = (-3)(-3)^6$$

$$= -(3)^7$$

$$= -2187$$

Thus, the seventh term of the G.P. is -2187.

5.Which term of the following sequences:

a) $2, 2\sqrt{2}, 4, \ldots is 128?$

Answer: Here, $a = 2$ and $r = (2\sqrt{2})/2 = \sqrt{2}$

Let the n^{th} term of the given sequence be 128.

$$a_n = ar^{n-1}$$
$$\Rightarrow (2)(\sqrt{2})^{n-1} = 128$$
$$\Rightarrow (2)(2)^{\frac{n-1}{2}} = (2)^7$$
$$\Rightarrow (2)^{\frac{n-1}{2}+1} = (2)^7$$
$$\therefore \frac{n-1}{2} + 1 = 7$$
$$\Rightarrow \frac{n-1}{2} = 6$$
$$\Rightarrow n - 1 = 12$$
$$\Rightarrow n = 13$$

Thus, the 13^{th} term of the given sequence is 128 .

(b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots is 729?$

Answer:

$$a = \sqrt{3} \text{ and } r = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Let the n^{th} term of the given sequence be 729.

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1} = 729$$
$$\Rightarrow (\sqrt{3})(\sqrt{3})^{n-1} = 729$$
$$\Rightarrow (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$$
$$\Rightarrow (3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)^6$$
$$\therefore \frac{1}{2} + \frac{n-1}{2} = 6$$
$$\Rightarrow \frac{1+n-1}{2} = 6$$
$$\Rightarrow n = 12$$

Thus, the 12^{th} term of the given sequence is 729.

c) $\frac{1}{3},\frac{1}{9},\frac{1}{27},\ldots is \frac{1}{19683}$?

Answer: Here, $a=\frac{1}{3} and$
$r=\frac{1}{9}\div\frac{1}{3}=\frac{1}{3}$

Let the n^{th} term of the given sequence be $\frac{1}{19683}$.

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1}=\frac{1}{19683}$$
$$\Rightarrow\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1}=\frac{1}{19683}$$
$$\Rightarrow\left(\frac{1}{3}\right)^{n}=\left(\frac{1}{3}\right)^{9}$$
$$\Rightarrow n=9$$

Thus, the 9^{th} term of the given sequence is $\frac{1}{19683}$.

6.For what values of x, the numbers $\frac{2}{7},x,-\frac{7}{2}$ are in G.P ?

Answer: The given numbers are $\frac{-2}{7},x,\frac{-7}{2}$

Common ratio $=\frac{x}{\frac{-2}{7}}=\frac{-7x}{2}$

Also, common ratio $=\frac{\frac{-7}{2}}{x}=\frac{-7}{2x}$

$$\therefore \frac{-7x}{2} = \frac{-7}{2x}$$

$$\Rightarrow x^2 = \frac{-2\times 7}{-2\times 7} = 1$$

$$\Rightarrow x = \sqrt{1}$$

$$\Rightarrow x = \pm 1$$

Thus, for $x = \pm 1$, the given numbers will be in $G.P.$

7.Find the sum to 20 terms in the geometric progression $0.15,\ 0.015,\ 0.0015$ ...

Answer:

$Here, a = 0.15 and$

$$r = \frac{0.015}{0.15} = 0.1$$

$$S_n = \frac{a\left(1-r^a\right)}{1-r}$$

$$S_n = \frac{a\left(1-r^a\right)}{1-r}$$

$$\therefore S_{20} = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$

$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$

$$= \frac{15}{90}\left[1-(0.1)^{20}\right]$$

$$= \frac{1}{6}\left[1-(0.1)^{20}\right]$$

8.Find the sum to n terms in the geometric progression $\sqrt{7}$, $\sqrt{21}$, $3\sqrt{7}$, ...

Answer:

$Here, a=\sqrt{7} and$

$r=\frac{\sqrt{21}}{7}=\sqrt{3}$

$S_n=\frac{a\left(1-r^n\right)}{1-r}$

$\Rightarrow \quad S_n=\frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}}$

$\Rightarrow \quad S_n=\frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow \quad S_n=\frac{\sqrt{7}(\sqrt{3}+1)\left[1-(\sqrt{3})^n\right]}{1-3}$

$\Rightarrow \quad S_n=\frac{-\sqrt{7}(\sqrt{3}+1)\left[1-(\sqrt{3})^n\right]}{2}$

9.Find the sum to n terms in the geometric progression $1,-a, a^2,-a^3 \ldots(if a \neq-1)$

Answer:

Here, first term $=a_1=1$

Common ratio $=r=-a$

$S_n=\frac{a_1\left(1-r^n\right)}{1-r}$

$\therefore S_n=\frac{1\left[1-(-a)^n\right]}{1-(-a)}$

$=\frac{\left[1-(-a)^n\right]}{1+a}$

10. Find the sum to n terms in the geometric progression $x^3, x^5, x^7 \ldots(if x \neq \pm 1)$

Answer: Here, $a=x^3 and$
$r=x^2$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$= \frac{x^3\left[1-\left(x^2\right)^n\right]}{1-x^2}$$

$$= \frac{x^3\left(1-x^{2n}\right)}{1-x^2}$$

11.Evaluate $\sum_{k=1}^{11}\left(2+3^k\right)$

Answer:

$$\sum_{k=1}^{11}\left(2+3^k\right) = \sum_{k=1}^{11}(2) + \sum_{k=1}^{11}3^k = 2(11) + \sum_{k=1}^{11}3^k = 22 + \sum_{k=1}^{11}3^k \ldots(1)$$

$$\sum_{k=1}^{11}3^k = 3^1 + 3^2 + 3^3 + \ldots + 3^{''}$$

The terms of this sequence $3, 3^2, 3^3 \ldots$ forms a $G.P.$

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$

$$\Rightarrow S_{11} = \frac{3\left[(3)^{11} - 1\right]}{3-1}$$

$$\Rightarrow S_{11} = \frac{3}{2}\left(3^{11} - 1\right)$$

Substituting this value in equation (1), we obtain

$$\sum_{k=1}^{11}\left(2+3^k\right) = 22 + \frac{3}{2}\left(3^{11} - 1\right)$$

12.The sum of first three terms of a G.P. is $\frac{39}{10}$ **and their product is** $1.$ **Find the common ratio and the terms.**

Answer: Let $\frac{a}{r}, a, ar$ be the first three terms of the G.P.

$$\frac{a}{r}+a+ar=\frac{39}{10}$$

$$\left(\frac{a}{r}\right)(a)(ar)=1$$

From (2), weobtain $a^3=1$

$\Rightarrow a=1$ (Considering real roots only)

Substituting $\Rightarrow a=1$ in equation (1), we obtain

$$\frac{1}{r}+1+r=\frac{39}{10}n$$

$$\Rightarrow 1+r+r^2=\frac{39}{10}rn$$

$$\Rightarrow 10+10r+10r^2-39r=0$$

$$\Rightarrow 10r^2-29r+10=0$$

$$\Rightarrow 10r^2-25r-4r+10=0$$

$$\Rightarrow 5r(2r-5)-2(2r-5)=0$$

$$\Rightarrow (5r-2)(2r-5)=0$$

$$\Rightarrow r=\frac{2}{5} \text{ or } \frac{5}{2}$$

Thus, the three terms of G.P. are $\frac{5}{2},1,\frac{2}{5}$

13.How many terms of $G.P. 3,3^2,3^3\ldots$ **are needed to give the sum** 120?

Answer: The given $G.P.$ is $3,3^2,3^3\ldots$

Let n terms of this $G.P.$ be required to obtain the sum as 120 .

$$S_n=\frac{a\left(1-r^n\right)}{1-r}$$

Here, $a=3$ and $r=3$

$$\therefore S_n = 120 = \frac{3(3^n - 1)}{3-1}$$
$$\Rightarrow 120 = \frac{3(3^n - 1)}{2}$$
$$\Rightarrow \frac{120 \times 2}{3} = 3^n - 1$$
$$\Rightarrow 3^n - 1 = 80$$
$$\Rightarrow 3^n = 81$$
$$\Rightarrow 3^n = 3^4$$
$$\therefore n = 4$$

Thus, four terms of the given $G.P.$ are required to obtain the sum as 120 .

14.The sum of first three terms of a G.P. is 16 and the sum of the next threeterms is 128. Determine the first term, the common ratio and the sum to n terms of theG.P.

Answer: Let the G.P. be $a, ar, ar^2, ar^3, \ldots$

According to the given condition

$$a + ar + ar^2 = 16 and$$
$$ar^3 + ar^4 + ar^5 = 128$$
$$\Rightarrow a(1 + r + r^2) = 16 \ldots (1)$$
$$ar^3(1 + r + r^2) = 128 \ldots (2)$$

Dividing equation (2) by (1), we obtain

$$\frac{ar^3(1 + r + r^2)}{a(1 + r + r^2)} = \frac{128}{16}$$
$$\Rightarrow r^3 = 8$$
$$\therefore r = 2$$

Substituting $r = 2$ in (1), we

obtain $a(1 + 2 + 4) = 16$

$\Rightarrow a(7) = 16$

$\Rightarrow a = \frac{16}{7}$

$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$

$\Rightarrow S_n = \frac{16}{7}\frac{\left(2^n - 1\right)}{2 - 1}$

$= \frac{16}{7}\left(2^n - 1\right)$

15. Given a G.P. with a = 729 and 7th term 64, determine S_7.

Answer:

$a = 729$

$a_7 = 64$

Let r be the common ratio of the G.P. It is known that,

$a_n = ar^{n-1} a_7 = ar^{7-1} = (729)r^6$

$\Rightarrow 64 = 729r^6$

$\Rightarrow r^6 = \frac{64}{729}$

$\Rightarrow r^6 = \left(\frac{2}{3}\right)^6$

$\Rightarrow r = \frac{2}{3}$

Also, it is known that,

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\therefore S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$$

$$= 3\times 729\left[1-\left(\frac{2}{3}\right)^7\right]$$

$$= (3)^7\left[\frac{(3)^7-(2)^7}{(3)^7}\right]$$

$$= (3)^7-(2)^7$$

$$= 2187-128$$

$$= 2059$$

16.Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

Answer: Let a be the first term and r be the copanwin ratio of the G.P. According to the given conditions,

$$S_2 = -4 = \frac{a\left(1-r^2\right)}{1-r} a_5 = 4\times a_3$$

$$\Rightarrow ar^4 = 4ar^2$$

$$\Rightarrow r^2 = 4$$

$$\therefore r = \pm 2$$

From (1), we obtain

$$-4 = \frac{a\left[1-(2)^2\right]}{1-2} \text{ for } r = 2$$

$$\Rightarrow -4 = \frac{a(1-4)}{-1}$$

$$\Rightarrow -4 = a(3)$$

$$\Rightarrow a = \frac{-4}{3}$$

Also, $-4 = \dfrac{a\left[1-(-2)^2\right]}{1-(-2)}$ for $r = -2$

$\Rightarrow -4 = \frac{a(1-4)}{1+2}$

$\Rightarrow -4 = \frac{a(-3)}{3}$

$\Rightarrow a = 4$

$G.P.$ is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$ or $4, -8, 16, -32\ldots$

17.If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z, respectively. Provethat x, y, z are in G.P.

Answer: Let a be the first term and r be the common ratio of the $G.P.$

According to the given condition,

$a_4 = ar^3 = x \ldots\ldots\ldots\ldots\ldots\ldots(1)$

$a_{10} = ar^9 = y \ldots\ldots\ldots\ldots\ldots\ldots(2)$

$a_{16} = ar^{15} = z \ldots\ldots\ldots\ldots\ldots(3)$

Dividing (2) by (1), we obtain

$\frac{y}{x} = \frac{ar^9}{ar^3} \Rightarrow \frac{y}{x} = r^6$

Dividing (3) by (2), we obtain

$\frac{z}{y} = \frac{ar^{15}}{ar^9}$

$\Rightarrow \frac{z}{y} = r^6$

$\frac{y}{x} = \frac{z}{y}$

Thus, x, y, z are in $G.P.$

18.Find the sum to n terms of the sequence, $8, 88, 888, 8888\ldots$

Answer: The given sequence is $8, 88, 888, 8888\ldots$

This sequence is not a G.P. However, it can be changed to G.P. by writing the terms as

$S_n = 8 + 88 + 888 + 8888 + \ldots\ldots$ to n terms.

$=\frac{8}{9}[9+99+999+9999+\ldots\ldots\ldots \text{ to } n \text{ terms }]$

$=\frac{8}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\left(10^4-1\right)+\ldots\ldots \text{ to } n \text{ terms }\right]n$

$=\frac{8}{9}\left[\left(10+10^2+\ldots..n \text{ terms }\right)-(1+1+1+\ldots n \text{ terms })\right]$

$=\frac{8}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$

$=\frac{8}{9}\left[\frac{10\left(10^n-1\right)}{9}-n\right]$

$=\frac{80}{81}\left(10^n-1\right)-\frac{8}{9}n$

19.Find the sum of the products of the corresponding terms of the sequences $\mathbf{2,4,8,16,32}$ and $\mathbf{128,32,8,2,1/2}$.

Answer:

Required sum $=2\times128+4\times32+8\times8+16\times2+32\times\frac{1}{2}$

$=64\left[4+2+1+\frac{1}{2}+\frac{1}{2^2}\right]$

Here, $4,2,1,\frac{1}{2},\frac{1}{2^2}$ is a $G.P.$

First term, $a=4$

Common ratio, $r=\frac{1}{2}$

It is known that, $S_n=\frac{a\left(1-r^n\right)}{1-r}$

$\therefore S_5=\frac{4\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}}=\frac{4\left[1-\frac{1}{32}\right]}{\frac{1}{2}}=8\left(\frac{32-1}{32}\right)=\frac{31}{4}$

$\therefore$ Required sum $=64\left(\frac{31}{4}\right)=(16)(31)=496$

20.Show that the products of the corresponding terms of the sequences form

$a, ar, ar^2, \ldots ar^{n-1} and$

$A, AR, AR^2, \ldots AR^{n-1}$

a G.P, and find the common ratio.

Answer: It has to be proved that the sequence: $aA, arAR, ar^2AR^2, \ldots ar^{n-1}AR^{n-1}$, forms a G.P.

$$\frac{\text{Second term}}{\text{First term}} = \frac{arAR}{aA} = rR$$
$$\frac{\text{Third term}}{\text{Second term}} = \frac{ar^2AR^2}{arAR} = rR$$

Thus, the above sequence forms a G.P. and the common ratio is $r\ R$.

21.Find four numbers forming a geometric progression in which third term greater than the first term by 9, and the second term is greater than 4^{th} by 18.

Answer:

Let a be the first term and r be the common ratio of the $G.P.$

$a_1 = a, \quad a_2 = ar, \quad a_3 = ar^2, \quad a_4 = ar^3$

By the given condition,

$a_3 = a_1 + 9 \Rightarrow ar^2 = a + 9 \ldots (1)$

$a_2 = a_4 + 18 \Rightarrow ar = ar^3 + 18 \ldots (2)$

From (1) and (2), we obtain

$a\left(r^2 - 1\right) = 9 \ldots (3)$

$ar\left(1 - r^2\right) = 18 \ldots (4)$

Dividing (4) by (3), we obtain

$$\frac{ar\left(1-r^2\right)}{a\left(r^2-1\right)} = \frac{18}{9}$$
$\Rightarrow -r = 2$

$\Rightarrow r = -2$

Substituting the value of r in (1), we obtain

$4a = a + 9$

$\Rightarrow 3a = 9$

$\therefore a = 3$

Thus, the first four numbers of the G.P. are $3, 3(-2), 3(-2)^2$, and $3(-2)^3$ *i.e.*, $3, -6, 12$, and -24.

22.If $\mathrm{p^{th}}, \mathrm{q^{th}}$ and $\mathrm{r^{th}}$ terms of a G.P. are a, b and c, respectively. Prove that $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$

Answer:

Let A be the first term and R be the common ratio of the **G.P.**

According to the given information,

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

$\mathrm{a}^{q-r} \cdot \mathrm{b}^{r-p} \cdot \mathrm{C}^{-p-q} = A^{q-r} \times R^{(p-1)}(q-r) \times \mathrm{A}^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Thus, the given result is proved.

23.If the first and the n^{th} term of a G.P. are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Answer: The first term of the $G.P$ is a and the last term is b.

Therefore, the $G.P$ is $a, ar, ar^2, ar^3 \ldots ar^{n-1}$, where r is the common ratio.

$b = ar^{n-1}$

$P =$ Product of n terms

$= (a)(ar)\left(ar^2\right)\ldots\left(ar^{n-1}\right)$

$= (a \times a \times \ldots a)\left(r \times r^2 \times \ldots r^{n-1}\right)$

$= anr1 + 2 + \ldots(n-1)\ldots(2)$

Here, $1, 2, \ldots(n-1)$ is an $A.P.$

$1+2+\dots\dots\dots+(n-1)$

$=\frac{n-1}{2}[2+(n-1-1)\times 1]$

$=\frac{n-1}{2}[2+n-2]$

$=\frac{n(n-1)}{2}\ P=a^n r^{n(n-1)n}$

$\therefore P^2=a^{2n}r^{n(n-1)}$

$=\left[a^2 r^{(n-1)}\right]^n$

$=\left[a\times ar^{n-1}\right]^n$

$=(ab)^n$

Thus, the given result is proved.

24.Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}\ to(2n)^{th}$ term is $1/r^n$.

Answer: Let a be the first term and r be the common ratio of the $G.P$.

Sum of first n terms $=\frac{a\left(1-r^n\right)}{(1-r)}$

Since there are n terms from $(n+1)^{th}\ to(2n)^{th}\ term,$

Sum of terms from $(n+1)^{th}$ to $(2n)^{th}\ term$

$$S_n=\frac{a_{n+1}\left(1-r^n\right)}{1-r}$$

Thus, required ratio $=\frac{a\left(1-r^n\right)}{(1-r)}\times\frac{(1-r)}{ar^n\left(1-r^r\right)}=\frac{1}{r^n}$

Thus, the ratio of the sum of first n terms of a $G.P$ to the sum of terms from $(n+1)^{th}\ to(2n)^{th}$ term is $1/r^n$.

25.If a,b,c and d are in G.P. show that:

$\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=(ab+bc-cd)^2$

Answer: a,b,c,d are in $G.P.$

Therefore,

$bc = ad$ ……………..(1)

$b^2 = ac$ ……………..(2)

$c^2 = bd$ ……………(3)

It has to be proved that,

$$\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=(ab+bc-cd)^2$$

R.H.S.

$$=(ab+bc+cd)^2$$
$$=(ab+ad+cd)^2$$
$$=[ab+d(a+c)]^2$$
$$=a^2b^2+2abd(a+c)+d^2(a+c)^2$$
$$=a^2b^2+2a^2bd+2acbd+d^2\left(a^2+2ac+c^2\right)$$

$$=a^2b^2+2a^2c^2+2b^2c^2+d^2a^2+2d^2b^2+d^2c^2 [Using (1) and (2)]$$
$$=a^2b^2+a^2c^2+a^2c^2+b^2c^2+b^2c^2+d^2a^2+d^2b^2+d^2b^2+d^2c^2$$
$$=a^2b^2+a^2c^2+a^2d^2+b^2\times b^2+b^2c^2+b^2d^2+c^2b^2+c^2\times c^2+c^2d^2$$

[Using (2) and (3) and rearranging terms]

$$=a^2\left(b^2+c^2+d^2\right)+b^2\left(b^2+c^2+d^2\right)+c^2\left(b^2+c^2+d^2\right)$$
$$=\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$$

= L.H.S.

$\therefore$ L.H.S. = R.H.S.

$$\therefore\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=(ab+bc-cd)^2$$

26.Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Answer: Let $G_1 and G_2$ be two numbers between 3 and 81 such that the series, 3 , G_1, G_2, 81, forms a $G.P.$

Let a be the first term and r be the common ratio of the $G.P.$

$$\therefore 81=(3)(r)^3$$
$$\Rightarrow r^3=27$$

therefore $r=3$(Taking real roots only)

For $r=3$,

$G_1 = ar = (3)(3) = 9$

$G_2 = ar^2 = (3)(3)^2 = 27$

Thus, the required two numbers are 9 and 27 .

27.Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Answer:M. of a and b. is $\sqrt{ab}$

By the given condition: $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

Squaring both sides, we get,

$$\frac{\left(a^{n+1}+b^{n+1}\right)^2}{\left(a^n+b^n\right)^2} = ab$$

$$\Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} = (ab)\left(a^{2n} + 2a^n b^n + b^{2n}\right)$$

$$\Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} = a^{2n+1}b + 2a^{n+1}b^{n+1} + ab^{2n+1}$$

$$\Rightarrow a^{2n+2} + b^{2n+2} = a^{2n+1}b + ab^{2n+1}$$

$$\Rightarrow a^{2n+2} - a^{2n+1}b = ab^{2n+1} - b^{2n+2}$$

$$\Rightarrow a^{2n+1}(a-b) = b^{2n+1}(a-b)$$

$$\Rightarrow \left(\frac{a}{b}\right)^{2n+1} = 1 = \left(\frac{a}{b}\right)^0$$

$$\Rightarrow 2n+1 = 0$$

$$\Rightarrow n = \frac{-1}{2}$$

28.The sum of two numbers is 6 times their geometric mean, show thatnumbers are in the rationumbers are in the ratio $(3+2\sqrt{2}):(3-2\sqrt{2})$

Answer: Let the two numbers be a and b.

G.M. $= \sqrt{ab}$

According to the given condition,

$$a + b = 6\sqrt{ab}$$

$$\Rightarrow (a+b)^2 = 36(ab) \ldots (1)$$

Also,

$$(a-b)^2=(a+b)^2-4ab=36ab-4ab=32ab$$
$$\Rightarrow a-b=\sqrt{32}\sqrt{ab}$$
$$=4\sqrt{2}\sqrt{ab}$$

Adding (1) and (2), we obtain

$$2a=(6+4\sqrt{2})\sqrt{ab}$$
$$\Rightarrow a=(3+2\sqrt{2})\sqrt{ab}$$

Substituting the value of a in (1), we obtain

$$b=6\sqrt{ab}-(3+2\sqrt{2})\sqrt{ab}$$
$$\Rightarrow b=(3-2\sqrt{2})\sqrt{ab}$$
$$\frac{a}{b}=\frac{(3+2\sqrt{2})\sqrt{ab}}{(3-2\sqrt{2})\sqrt{ab}}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Thus, the required ratio is $(3+2\sqrt{2}):(3-2\sqrt{2})$.

29.If A and G be $A.M.$ and $G.M.$, respectively between two positive numbers, prove that the numbers are $A\pm\sqrt{(A+G)(A-G)}$

Answer: It is given that A and G are $A.M.$ and $G.M.$ between two positive numbers. Let these two positive numbers be a and b.

$$\therefore \mathrm{AM}=\mathrm{A}=\frac{\mathrm{a}+\mathrm{b}}{2}\ldots(1)$$
$$\mathrm{GM}=\mathrm{G}=\sqrt{\mathrm{ab}}\ldots(2)$$

From (1) and (2), we obtain

$$a+b=2A\ldots(3)$$
$$ab=G^2\ldots(4)$$

Substituting the value of a and b. from (3) and (4) in the identity $(a-b)^2=(a+b)^2-4ab$

we obtain

$(a-b)^2 = 4A^2 - 4G^2 = 4\left(A^2 - G^2\right)$

$(a-b)^2 = 4(A+G)(A-G)$

$(a-b) = 2\sqrt{(A+G)(A-G)}$...(5)

From (3) and (5), we obtain

$2a = 2A + 2\sqrt{(A+G)(A-G)}$

$\Rightarrow a = A + \sqrt{(A+G)(A-G)}$

Substituting the value of a in (3), we obtain

$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$

Thus, the two numbers are $\mathrm{A} \pm \sqrt{(\mathrm{A}+\mathrm{G})(\mathrm{A}-\mathrm{G})}$.

30. The number of bacteria in a certain culture doubles every hour. If therewere 30 bacteria present in the culture originally, how many bacteria willbe present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?

Answer: It is given that the number of bacteria doubles every hour. Therefore, the number of bacteria after every hour will form a G.P.

Here, $a = 30$ *and* $r = 2$

therefore $a_3 = ar^2 = (30)(2)^2 = 120$

Therefore, the number of bacteria at the end of 2^{nd} hour will be 120.

$a_5 = ar^4 = (30)(2)^4 = 480$

The number of bacteria at the end of 4^{th} hour will be 480.

$a_{n+1} = ar^n = (30)2^n$

Thus, number of bacteria at the end of n^{th} hour will be $30(2)^n$.

31.What will Rs. 500 amounts to in 10 years after its deposit in a bank which

pays annual interest rate of 10% compounded annually?

Answer: The amount deposited in the bank is $Rs\ 500$.

At the end of first year, amount $= Rs500\left(1+\frac{1}{10}\right) = Rs500(1.1)$

At the end of 2^{nd} $year, amount = Rs500(1.1)(1.1)$

At the end of 3^{rd} $year, amount = Rs500(1.1)(1.1)(1.1)$

and so on

Amount at the end of 10 years $= Rs500(1.1)(1.1)...(10 times) = Rs500(1.1)^{10}$

32.If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Answer: Let a and b be the roots of the quadratic equation.

According to the given condition,

$$A.M. = \frac{a+b}{2} = 8$$
$$\Rightarrow a+b = 16\ldots(1)$$
$$G.M. = \sqrt{ab} = 5$$
$$\Rightarrow ab = 25...(2)$$

The quadratic equation is given by,

$$x^2 - x\text{ (sum of roots) } + \text{(product of roots)} = 0$$

$$x^2 - x(a+b) + (ab) = 0$$
$$x^2 - 16x + 25 = 0$$

Thus, the required quadratic equation is $x^2 - 16x + 25 = 0$

Exercise 9.4

1. Find the sum to n terms of the series $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \ldots$

Answer: The given series is $1\times2+2\times3+3\times4+4\times5+\ldots n^{\text{th}}\ term, a_n=n(n+1)$

$$\therefore S_n=\sum_{k=1}^{n}a_k$$

$$=\sum_{k=1}^{n}k(k+1)$$

$$=\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}k$$

$$=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}n$$

$$=\frac{n(n+1)}{2}\left(\frac{2n+1}{3}+1\right)$$

$$=\frac{n(n+1)}{2}\left(\frac{2n+4}{3}\right)$$

$$=\frac{n(n+1)(n+2)}{3}$$

2.Find the sum to n terms of the series $1\times2\times3+2\times3\times4+3\times4\times5+\ldots$

Answer:

The given series is $1\times2\times3+2\times3\times4+3\times4\times5+\ldots n^{\text{th}}$ term,

$$a_n=n(n+1)(n+2)$$

$$=\left(n^2+n\right)(n+2)$$

$$=n^3+3n^2+2n$$

$$\therefore S_n = \sum_{k=1} a_k$$

$$= \sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{2} + n(n+1)n$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2n + 1 + 2\right]n$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2 + n + 4n + 6}{2}\right]n$$

$$= \frac{n(n+1)}{4}\left(n^2 + 5n + 6\right)n$$

$$= \frac{n(n+1)}{4}\left(n^2 + 2n + 3n + 6\right)n$$

$$= \frac{n(n+1)[n(n+2) + 3(n+2)]}{4}$$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

3.Find the sum to n terms of the series $3\times3\times1^2 + 5\times2^2 + 7\times3^2 + \ldots 1^2 + 5\times2^2 + 7\times3^2 + \ldots$

Answer: The given series is

$3\times1^2 + 5\times2^2 + 7\times3^2 + \ldots n^{\text{th}}$ *term,*

$a_n = (2n+1)n^2$

$= 2n^3 + n^2$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} = \left(2k^3 + k^2\right) = 2\sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left[n(n+1) + \frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+5n+1}{3}\right]$$

$$= \frac{n(n+1)\left(3n^2+5n+1\right)}{6}$$

4.Find the sum to n terms of the series $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \dots$

Answer: The given series is

$$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \dots$$

(By Partial Fractions)

$$n^{th}\ term, a_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$a_1 = \frac{1}{1} - \frac{1}{2}$$

$$a_2 = \frac{1}{2} - \frac{1}{3}$$

$$a_3 = \frac{1}{3} - \frac{1}{4} \dots$$

$$a_n = \frac{1}{n} - \frac{1}{n+1}$$

Now, Add These:

$$a_1+a_2+\ldots+a_n$$
$$=\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n}\right]-\left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots\frac{1}{n+1}\right]$$
$$\therefore S_n=1-\frac{1}{n+1}$$
$$=\frac{n+1-1}{n+1}$$
$$=\frac{n}{n+1}$$

5.Find the sum to n terms of the series $5^2+6^2+7^2+\ldots+20^2$

Answer: The given series is $5^2+6^2+7^2+\ldots+20^2 n^{\text{th}}$ *term,*

$$a_n=(n+4)^2$$
$$=n^2+8n+16$$
$$\therefore S_n=\sum_{k=1}^{n}a_k$$
$$=\sum_{k=1}^{n}\left(k^2+8k+16\right)$$
$$=\sum_{k=1}^{n}k^2+8\sum_{k=1}^{n}k+\sum_{k=1}^{n}16$$
$$=\frac{n(n+1)(2n+1)}{6}+\frac{8n(n+1)}{2}+16n$$

16^{th} term is $(16+4)^2=20^2$

$$\therefore S_{16}=\frac{16(16+1)(2\times16+1)}{6}+\frac{8\times16\times(16+1)}{2}+16\times16$$
$$=\frac{(16)(17)(33)}{6}+\frac{(8)\times16\times(16+1)}{2}+16\times16$$
$$=\frac{(16)(17)(33)}{6}+\frac{(8)(16)(17)}{2}+256$$
$$=1496+1088+256$$
$$=2840$$
$$\therefore 5^2+6^2+7^2+\ldots\ldots+20^2$$
$$=2840$$

6. Find the sum to n terms of the series $3\times8+6\times11+9\times14+\ldots$

Answer: The given series is:

$3\times8+6\times11+9\times14+\ldots$

$a_n=\left(n^{th}\ termof\ 3,6,9\ldots\right)\times\left(n^{th}\ termof\ 8,11,14\ldots\right)$

$=(3n)(3n+5)$

$=9n^2+15n$

$\therefore S_n=\sum_{k=1}^{n}a_k$

$=\sum_{k=1}^{n}\left(9k^2+15k\right)$

$=9\sum_{k=1}^{n}k^2+15\sum_{k=1}^{n}k$

$=9\times\frac{n(n+1)(2n+1)}{6}+15\times\frac{n(n+1)}{2}$

$=\frac{3n(n+1)(2n+1)}{2}+\frac{15n(n+1)}{2}$

$=\frac{3n(n+1)}{2}(2n+1+5)$

$=\frac{3n(n+1)}{2}(2n+6)$

$=3n(n+1)(n+3)$

7.Find the sum to n **terms of the series** $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+\ldots$

Answer: The given series is

$1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^3\right)+\ldots a_n$

$=\left(1^2+2^2+3^3+\ldots\ldots+n^2\right)$

$=\frac{n(n+1)(2n+1)}{6}$

$=\frac{n\left(2n^2+3n+1\right)}{6}$

$=\frac{2^3+3n^2+n}{6}$

$=\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n$

$$\therefore S_n = \sum_{k=1}^{w} a_k$$

$$= \sum_{k=1}^{n} \left(\frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k \right)$$

$$= \frac{1}{3}\sum_{k=1}^{n} k^3 + \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{6}\sum_{k=1}^{n} k$$

$$= \frac{1}{3}\frac{n^2(n+1)^2}{(2)^2} + \frac{1}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{6} \times \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6}\left[\frac{n(n+1)}{2} + \frac{(2n+1)}{2} + \frac{1}{2} \right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n^2 + n + 2n + 1 + 1}{2} \right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n^2 + n + 2n + 2}{2} \right]$$

$$= \frac{n(n+1)}{6}\left[\frac{n(n+1) + 2(n+1)}{2} \right]$$

$$= \frac{n(n+1)}{6}\left[\frac{(n+1)(n+2)}{2} \right]$$

8.Find the sum to n terms of the series whose n^{th} term is given by $n(n+1)(n+4)$.

Answer:

$$a_n = n(n+1)(n+4)$$

$$= n\left(n^2 + 5n + 4\right)$$

$$= n^3 + 5n^2 + 4n$$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 3n + 20n + 10 + 24}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2 + 23n + 34}{6}\right]$$

$$= \frac{n(n+1)\left(3n^2 + 23n + 34\right)}{12}$$

9.Find the sum to n terms of the series whose n^{th} terms is given by $n^2 + 2^n$

Answer: $a_n = n^2 + 2^n$

$$\therefore S_n = \sum_{k-1}^{n} k^2 + 2^k = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 2^k \ldots(1)$$

Let, $\sum_{k=1}^{n} 2^k = 2^1 + 2^2 + 2^3 + \ldots$

The above series $2, 2^2, 2^3 \ldots$ is a G.P. with both the first term and common ratio equal to 2.

$$\therefore \sum_{k=1}^{n} 2^k = \frac{(2)\left[(2)^n - 1\right]}{2-1} = 2\left(2^n - 1\right)\ldots(2)$$

Therefore, from (1) and (2), we obtain

$$S_n = \sum_{k=1}^{n} k^2 + 2\left(2^n - 1\right) = \frac{n(n+1)(2n+1)}{6} + 2\left(2^n - 1\right)$$

10.Find the sum to n terms of the series whose n^{th} terms is given by $(2n-1)^2$

Answer:

$a_n = (2n-1)^2$

$= 4n^2 - 4n + 1$

$\therefore S_n = \sum_{k=1}^{n} a_k$

$= \sum_{k=1}^{n} \left(4k^2 - 4k + 1\right)$

$= 4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$

$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$

$= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n$

$= n\left[\frac{2\left(2n^2 + 3n + 1\right)}{3} - 2(n+1) + 1\right]$

$= n\left[\frac{4n^2 + 6n + 2 - 6n - 6 + 3}{3}\right]$

$= n\left[\frac{4n^2 - 1}{3}\right]$

$= \frac{n(2n+1)(2n-1)}{3}$

Miscellaneous

1. Show that the sum of $(m+n)^{th}$ **and** $(m-n)^{th}$ **terms of an AP is equal to twice the** m^{th} **term**

Answer: Let a = first term

d = common difference

As k^{th} term of an AP is $a_k = a+(k-1)d$

$a_{m+n} = a+(m+n-1)d$

$a_{m-n} = a+(m-n-1)d$

$a_m = a+(m-1)d$

$a_{m+n}+a_{m-n} = a+(m+n-1)d+a+(m-n-1)d$

$= 2a+(m+n-1+m-n-1)d$

$= 2a+(2m-2)d$

$= 2a+2(m-1)d$

$= 2[a+(m-1)d]$

$= 2\ a_m$

Hence proved

2. If the sum of three numbers in AP is 24 and their product is 440, find the numbers

Answer: Let three nos. in AP is $a-d,\ a,\ a+d$

According to question:

$(a-d)+(a)+(a+d) = 24$

$3a = 24$

$a = 8$

$(a-d)\ a\ (a+d) = 440$

$(8-d)\ (8)\ (8+d) = 440$

$(8-d)\ (8+d) = 55$

$64-d^2 = 55$

$d^2 = 64-55 = 9$

$d = \pm 3$

So, when d = 3, numbers are 5, 8, 11

when d = -3, numbers are 11, 8, 5

Hence, three numbers are 5, 8, 11

3. Let the sum of $n,\ 2n,\ 3n$ terms of an AP be $S_1,\ S_2,\ S_3$ respectively. Show that $S_3 = 3(S_2-S_1)$

Answer: Let a = first term

d = common difference

$S_1 = \frac{n}{2}[2a+(n-1)d]$ (i)

$S_2 = \frac{2n}{2}[2a+(2n-1)d] = n[2a+(2n-1)d]$ (ii)

$S_3 = \frac{3n}{2}[2a+(3n-1)d]$ (iii)

From (i) and (ii), we get

$S_2 - S_1 = n[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]$

$= n\left\{\frac{4a+4nd-2d-2a-nd+d}{2}\right\}$

$= n\left[\frac{2a+3nd-d}{2}\right]$

$= \frac{n}{2}[2a+(3n-1)d]$

$3(S_2 - S_1) = \frac{3n}{2}[2a+(3n-1)d] = S_3$ [by (iii)]

Hence proved

4. Find the sum of all numbers between 200 and 400 which are divisible by 7

Answer: Nos. between 200 and 400 which are divisible by 7 are 203, 210, 217,......, 399

$a = 203, l = 399, d = 7$

So, $a_n = 399 = a+(n-1)d$

$399 = 203+(n-1)7$

$7(n-1) = 196$

$n-1 = 28$

$n = 29$

$S_{29} = \frac{29}{2}(203+399)$

$= \frac{29}{2}(602)$

$= (29)(301) = 8729$

5. Find the sum of integers from 1 to 100 that are divisible by 2 or 5

Answer: Here, integers from 1 to 100 that are divisible by 2 are 2, 4, 6,..., 100

$a = d = 2$

$100 = 2+(n-1)2$

$n = 50$

$2+4+6+\ldots+100 = \frac{50}{2}[2(2)+(50-1)(2)]$

$= \frac{50}{2}[4+98]$

$= (25)(102)$

$= 2550$

Now, integers from 1 to 100 that are divisible by 5 are $5, 10, 15,..., 100$

$a = d = 5$

$100 = 5+(n-1)5$

$5n = 100$

$n = 20$

$5+10+\ldots+100 = \frac{20}{2}[2(5)+(20-1)5]$

$= 10[10+(19)5]$

$= 10[10+95] = 10\times105$

$= 1050$

Integers 1 to 100 that are divisible by both 2 and 5 are $10, 20,..., 100$

$a = d = 10$

$100 = 10+(n-1)(10)$

$100 = 10n$

$n = 10$

$10+20+\ldots+100 = \frac{10}{2}[2(10)+(10-1)(10)]$

$= 5[20+90] = 5(110) = 550$

Required sum $= 2550+1050-550 = 3050$

6. Find the sum of all two-digit numbers which when divided by 4, yields 1 as remainder

Answer: Here, two-digit numbers which when divided by 4, yields 1 as remainder are $13, 17, ..., 97$

$a = 13, d = 4$

As $a_n = a+(n-1)d$

$97 = 13+(n-1)4$

$4(n-1) = 84$

$n-1 = 21$

$n = 22$

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{22} = \frac{22}{2}[22(13)+(22-1)(4)]$

$= 11[26+84]$

$= 1210$

7. If f is a function satisfying $f(x+y) = f(x).f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{1}^{n} f(x) = 120$, find value of n

Answer: Given $f(x+y) = f(x)\times f(y)$ for all $x, y \in N$ (i)

$f(1) = 3$

Put $x = y = 1$ in (1), we get

$f(1+1) = f(2) = f(1)\,f(1) = 3\times3 = 9$

Continuing in this way

$f(1+1+1) = f(3) = f(1+2) = f(1)f(2) = 3\times9 = 27$

$f(4) = f(1+3) = f(1)f(3) = 3\times27 = 81$

$f(1), f(2), f(3)$ are 3, 9, 27 and this forms a G.P.

Here, $a = d = 3$

As $S_n = \frac{a(r^n-1)}{r-1}$

Given $\sum_{x=1}^{n} f(x) = 120$

$120 = \frac{3(3^n-1)}{3-1}$

$120 = \frac{3}{2}(3^n-1)$

$3^n-1 = 80$

$3^n = 81 = 3^4$

$n = 4$

8. The sum of some terms of GP is 315 whose first term and the common difference ratio are 5 and 2 respectively. Find the last term and the number of terms

Answer: Given that sum of n terms of GP is 315

As $S_n = \frac{a(r^n-1)}{r-1}$

$a = 5, r = 2$

So, $315 = \frac{5\left(2^n-1\right)}{2-1}$

$2^n-1 = 63$

$2^n = 64 = (2)^6$

$n = 6$

Last term of GP = 6^{th} term = $a\,r^{6-1} = (5)(2)^5 = (5)(32) = 160$

9. The first term of a GP is 1 . The sum of the third term and fifth term is 90 . Find the common ratio of GP

Answer: Here a = first term and r = common ratio

$a = 1$

$a_3 = ar^2 = r^2$

$a_5 = ar^4 = r^4$

According to question

$r^2+r^4 = 90$

$r^4+r^2-90 = 0$

$r^2 = \frac{-1+\sqrt{1+360}}{2} = \frac{-1\pm\sqrt{361}}{2} = \frac{-1\pm19}{2} = -10 \text{ or } 9$

$r = \pm3$ (Taking real roots)

10. The sum of three numbers in GP is 56 . If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers

Answer: Let three nos. in GP are a, ar, ar^2

According to question:

$a+ar+ar^2 = 56$

$a\left(1+r+r^2\right) = 56$ (i)

$a-1,\ ar-7,\ ar^2-21$ forms an A.P.

$(ar-7)-(a-1) = \left(ar^2-21\right)-(ar-7)$

$ar-a-6 = ar^2-ar-14$

$ar^2-2ar+a = 8$

$ar^2-ar-ar+a = 8$

$a\left(r^2+1-2r\right) = 8$

$a(r-1)^2 = 8$ (ii)

From (i) and (ii), we get

$7\left(r^2-2r+1\right) = 1+r+r^2$

$7r^2-14r+7-1-r-r^2 = 0$

$6r^2-15r+6 = 0$

$6r^2-12r-3r+6 = 0$

$6r(r-2)-3(r-2) = 0$

$(6r-3)(r-2) = 0$

$r = 2, \frac{1}{2}$

When $r = 2$ then three nos. in GP are 8, 16, 32

When $r = \frac{1}{2}$ then nos. in GP are 32, 16, 8

So, Numbers are 8, 16, 32

11. A GP consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio

Answer: Let G.P. be $A_1, A_2, A_3, A_4 \ldots A_{2n}$

No. of terms = 2n

According to question,

$A_1+A_2+A_3+\ldots+A_{2n} = 5[A_1+A_3+\ldots+A_{2n-1}]$

$A_1+A_2+A_3+\ldots+A_{2n}-5[A_1+A_3+\ldots+A_{2n-1}] = 0$

$A_2+A_4+\ldots+A_{2n} = 4[A_1+A_3+\ldots+A_{2n-1}]$

Let G.P. be a, ar, ar^2, ar^3,…

$$\frac{ar\left(r^n-1\right)}{r-1} = \frac{4\times a\left(r^n-1\right)}{r-1}$$

ar = 4a

r = 4

12. The sum of the first four terms of an AP is 56 . The sum of the last four terms is 112 . If its first term is 11 , then find the number of terms

Answer: Let AP be $a, a+d, a+2d, \ldots, a+(n-1)d$

Sum of first four terms $= a+a+d+a+2d+a+3d = 4a+6d$

Sum of last four terms $= [a+(n-4)d]+[a+(n-3)d]+[a+(n-2)d]+[a+n-1)d]$

$= 4a+(4n-10)d$

Now, from given conditions

$4a+6d = 56$

$4(11)+6d = 56$

$6d = 12$

$d = 2$

Next, $4a+(4n-10)d = 112$

$4(11)+(4n-10)2 = 112$

$(4n-10)2 = 68$

$4n-10 = 34$

$4n = 44$

$n = 11$

13. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ $(x \neq 0)$ **then show that** a, b, c, d **are in GP**

Answer: From given conditions

$$\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}$$

$(a+bx)(b-cx) = (b+cx)(a-bx)$

$ab-acx+b^2x-bcx^2 = ab-b^2x+acx-bcx^2$

$2b^2x = 2acx$

$b^2 = ac$

$\frac{b}{a}=\frac{c}{b}$ (i)

Now, $\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}$

$(b+cx)(c-dx) = (b-cx)(c+dx)$

$bc-bdx+c^2x-cdx^2 = bc+bdx-c^2x-cdx^2$

$2c^2x = 2bdx$

$c^2 = bd$

$\frac{c}{d}=\frac{d}{c}$ (ii)

From (i) and (ii), we get

$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

Hence, a, b, c, d are in G.P.

14. Let S be the sum, P be the product and R be the sum of reciprocals of n terms in a GP. Prove that $P^2R^n = S^n$

Answer: Let terms in GP be $a, ar, ar^2, \ldots, ar^{n-1}$

According to given conditions:

$$S = \frac{a\left(r^{n}-1\right)}{r-1}$$

$$P = a^{n} \times r^{1+2+\ldots+n-1}$$

$$= a^{n} r^{\frac{a(n-1)}{2}}$$

As Sum of first n natural numbers is $n\frac{(n+1)}{2}$

$$R = \frac{1}{a}+\frac{1}{ar}+\ldots+\frac{1}{ar^{n-1}}$$

$$= \frac{r^{n-1}+r^{n-2}+\ldots r+1}{ar^{n-1}}$$

$$= \frac{1\left(r^{n}-1\right)}{(r-1)} \times \frac{1}{ar^{n-1}} = \frac{r^{n}-1}{ar^{n-1}(r-1)}$$

$$P^{2}R^{n} = a^{2n}r^{n(n-1)} \frac{(r^{n}-1)^{n}}{a^{n}r^{n(n-1)}(r-1)^{n}}$$

$$= \frac{a^{n}\left(r^{n}-1\right)^{n}}{(r-1)^{n}} = \left[\frac{a\left(r^{n}-1\right)}{(r-1)}\right]^{n}$$

$$= S^{n}$$

Hence proved

15. The p^{th}, q^{th}, r^{th} terms of an AP are a, b, c respectively. Show that $(q-r)a+(r-p)b+(p-q)c = 0$

Answer: Let t = first term and d = common difference

n^{th} term of an A.P. is $a_n = t+(n-1)d$

So, $a_p = t+(p-1)d = a$ (i)

$a_q = t+(q-1)d = b$ (ii)

$a_r = t+(r-1)d = c$ (iii)

Now, Subtract equation (ii) from (i), we get

$(p-1-q+1)d = a-b$

$(p-q)d = a-b$

$d = \frac{a-b}{p-q}$ (iv)

Subtract equation (iii) from (ii), we get

$(q-1-r+1)d = b-c$

$(q-r)d = b-c$

$d = \frac{b-c}{q-r}$ (v)

From (iv) and (v), we get

$\frac{a-b}{p-q} = \frac{b-c}{q-r}$

$(a-b)(q-r) = (b-c)(p-q)$

$aq-bq-ar+br = bp-bq-cp+cq$

$bp-cp+cq-aq+ar-br = 0$

$(-aq+ar)+(bp-br)+(-cp+cq) = 0$

$-a(q-r)-b(r-p)-c(p-q) = 0$

$a(q-r)+b(r-p)+c(p-q) = 0$

16. If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP, prove that a, b, c are in AP

Answer: Given $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP

$b\left(\frac{1}{c}+\frac{1}{a}\right)-a\left(\frac{1}{b}+\frac{1}{c}\right) = c\left(\frac{1}{a}+\frac{1}{b}\right)-b\left(\frac{1}{c}+\frac{1}{a}\right)$

$\frac{b(a+c)}{ac}-\frac{a(b+c)}{bc} = \frac{c(a+b)}{ab}-\frac{b(a+c)}{ac}$

$\frac{b^2a+b^2c-a^2b-a^2c}{abc} = \frac{c^2a+c^2b-b^2a-b^2c}{abc}$

$b^2a-a^2b+b^2c-a^2c = c^2a-b^2a+c^2b-b^2c$

$ab(b-a)+c\left(b^2-a^2\right) = a\left(c^2-b^2\right)+bc(c-b)$

$ab(b-a)+c(b-a)(b+a) = a(c-b)(c+b)+bc(c-b)$

$(b-a)(ab+cb+ca) = (c-b)(ac+ab+bc)$

$b-a = c-b$

Hence proved

17. If a, b, c, d are in GP, prove that $\left(a^n+b^n\right), \left(b^n+c^n\right), \left(c^n+d^n\right)$ are in GP

Answer: Given a, b, c, d are in GP

$b^2 = ac$ (i)

$c^2 = bd$ (ii)

$ad = bc$ (iii)

We have to prove that $\left(a^n+b^n\right), \left(b^n+c^n\right), \left(c^n+d^n\right)$ are in GP

$\left(b^n+c^n\right)^2 = \left(a^n+b^n\right)\left(c^n+d^n\right)$

From L.H.S.

$\left(b^n+c^n\right)^2 = b^{2n}+2b^nc^n+c^{2n}$

$= \left(b^2\right)^n+2b^nc^n+\left(c^2\right)^n$

$= (ac)^n+2b^nc^n+(bd)^n$ [by using (i) and (ii)]

$= a^nc^n+b^nc^n+b^nc^n+b^nd^n$

$= a^nc^n+b^nc^n+a^nd^n+b^nd^n$ [by using (iii)]

$= c^n\left(a^n+b^n\right)+d^n\left(a^n+b^n\right)$

$= \left(a^n+b^n\right)\left(c^n+d^n\right)$

= R.H.S.

Hence proved

18. If a and b are the roots of $x^2-3x+p = 0$ and c, d are roots of $x^2-12x+q = 0$, where a, b, c, d form a G.P. Prove that $(q+p):(q-p) = 17:15$

Answer: Given: a and b are the roots of $x^2-3x+p = 0$

$a+b = 3$ and $ab = p$ (i)

Also, c and d are the roots of $x^2-12x+q = 0$

$c+d = 12$ and $cd = q$ (ii)

As a, b, c, d are in G.P.

Let $a = x$, $b = xr$, $c = xr^2$, $d = xr^3$

From (i) and (ii)

we get $x+xr = 3$

$x(1+r) = 3$

$xr^2+xr^3 = 12$

$xr^2(1+r) = 12$

On dividing, we get

$$\frac{xr^2(1+r)}{x(1+r)} = \frac{12}{3}$$

$r^2 = 4$

$r = \pm 2$

When $r = 2$, $x = \frac{3}{1+2} = \frac{3}{3} = 1$

When $r = -2$, $x = \frac{3}{1-2} = \frac{3}{-1} = -3$

Case (1):

When $r = 2$ and $x = 1$, $ab = x^2r = 2$, $cd = x^2r^5 = 32$

$$\frac{q+p}{q-p} = \frac{32+2}{32-2} = \frac{34}{30} = \frac{17}{15}$$

i.e., $(q+p):(q-p) = 17:15$

Case (2):

When $r = -2$, $x = -3$, $ab = x^2r = -18$, $cd = x^2r^5 = -288$

$$\frac{q+p}{q-p} = \frac{-288-18}{-288+18} = \frac{-306}{-270} = \frac{17}{15}$$

i.e., $(q+p):(q-p) = 17:15$

19. The ratio of the AM and GM of two positive numbers a and b, is $m:n$ **. Show that**

$a:b = \left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$

Answer: Let a and b be two nos.

A.M. $= \frac{a+b}{2}$ and G.M. $= \sqrt{ab}$

According to question:-

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

$$\frac{(a+b)^2}{4(ab)} = \frac{m^2}{n^2}$$

$$(a+b)^2 = \frac{4abm^2}{n^2}$$

$$(a+b) = \frac{2\sqrt{ab}m}{n} \qquad \text{(i)}$$

By identity $(a-b)^2=(a+b)^2-4ab$, we get

$$(a-b)^2=\frac{4abm^2}{n^2}-4ab=\frac{4ab\left(m^2-n^2\right)}{n^2}$$

$$(a-b)=\frac{2\sqrt{ab}\sqrt{m^2-n^2}}{n} \qquad \text{(ii)}$$

Add (i) and (ii), we get

$$2a=\frac{2\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

$$a=\frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

Substitute value of a in (i), we get

$$b=\frac{2\sqrt{ab}}{n}m-\frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

$$=\frac{\sqrt{ab}}{n}m-\frac{\sqrt{ab}}{n}\sqrt{m^2-n^2}$$

$$=\frac{\sqrt{ab}}{n}\left(m-\sqrt{m^2-n^2}\right)$$

$$a:b=\frac{a}{b}=\frac{\frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)}{\frac{\sqrt{ab}}{n}\left(m-\sqrt{m^2-n^2}\right)}=\frac{\left(m+\sqrt{m^2-n^2}\right)}{\left(m-\sqrt{m^2-n^2}\right)}$$

20. If a, b, c are in AP; b, c, d are in GP and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in AP. Prove that a, c, e are in GP

Answer: Given that a, b, c are in AP

$$b-a=c-b \qquad \text{(i)}$$

And given that b, c, d are in GP

$$c^2=bd \qquad \text{(ii)}$$

And it is also given that $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in AP

$$\frac{1}{d}-\frac{1}{c}=\frac{1}{e}-\frac{1}{d}$$

$$\frac{2}{d}=\frac{1}{c}+\frac{1}{e} \qquad \text{(iii)}$$

From (i), we get

2b = a+c

$$b = \frac{a+c}{2}$$

From (ii), we get

$$d = \frac{c^2}{b}$$

Substitute values in (iii), we get

$$\frac{2b}{c^2} = \frac{1}{c}+\frac{1}{e}$$

$$\frac{2(a+c)}{2c^2} = \frac{1}{c}+\frac{1}{e}$$

$$\frac{a+c}{c^2} = \frac{e+c}{ce}$$

$$\frac{a+c}{c} = \frac{e+c}{e}$$

(a+c)e = (e+c)c

$ae+ce = ec+c^2$

$c^2 = ae$

Hence, a, c, e are in G.P.

21. Find the sum of the following series up to n terms

(i) 5+55+555+...

Answer: Let $S_n = 5+55+555+\ldots$ to n terms

$$= \frac{5}{9}[9+99+999+\ldots \text{ to n terms}]$$

$$= \frac{5}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\ldots \text{ to n terms}\right]$$

$$= \frac{5}{9}\left[\left(10+10^2+10^3+\ldots\text{n terms}\right)-(1+1+\ldots\text{n terms})\right]$$

$$= \frac{5}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$$

$$= \frac{5}{9}\left[\frac{10\left(10^n-1\right)}{9}-n\right]$$

$$= \frac{50}{81}\left(10^n-1\right)-\frac{5n}{9}$$

(ii) .6+.66+.666+...

Answer: Let $S_n = 0.6+0.66+0.666+\ldots$ to n terms

$$= 6[0.1+0.11+0.111+\ldots \text{ to n terms}]$$

$$= \frac{6}{9}[0.9+0.99+0.999+\ldots \text{ to n terms}]$$

$$= \frac{6}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+\ldots \text{ to n terms}\right]$$

$$= \frac{2}{3}\left[(1+1+\ldots\text{n terms})-\frac{1}{10}\left(1+\frac{1}{10}+\frac{1}{10^2}+\ldots\text{n terms}\right)\right]$$

$$= \frac{2}{3}\left[n-\frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right)\right]$$

$$= \frac{2}{3}n-\frac{2}{30}\times\frac{10}{9}\left(1-10^{-n}\right)$$

$$= \frac{2}{3}n-\frac{2}{27}\left(1-10^{-n}\right)$$

22. Find the 20^{th} term of the series $2\times4+4\times6+6\times8+\ldots+n$ terms

Answer: We have given that $2\times4+4\times6+6\times8+\ldots n$ terms

n^{th} term $= a_n = 2n\times(2n+2) = 4n^2+4n$

$a_{20} = 4(20)^2+4(20) = 4(400)+80 = 1600+80 = 1680$

23. Find the sum of the first n terms of the series $3+7+13+21+31+\ldots$

Answer: Given that $3+7+13+21+31+\ldots$

$S = 3+7+13+21+31+\ldots+a_{n-1}+a_n$

$S = 3+7+13+21+\ldots+a_{n-2}+a_{n-1}+a_n$

Subtract both the above equations, we get

$S-S = \left[3+\left(7+13+21+31+\ldots+a_{n-1}+a_n\right)\right]-[(3+7+13+21+31+\ldots+a_{n-1})+a_n]$

$S-S = 3+\left[(7-3)+(13-7)+(21-13)+\ldots+\left(a_n-a_{n-1}\right)\right]-a_n$

$0 = 3+[4+6+8+\ldots(n-1) \text{ terms }]-a_n$

$a_n = 3+[4+6+8+\ldots(n-1) \text{ terms}]$

$$a_n = 3+\left(\frac{n-1}{2}\right)[2\times4+(n-1-1)2]$$

$$= 3+\left(\frac{n-1}{2}\right)[8+(n-2)2]$$

$$= 3+\frac{(n-1)}{2}(2n+4) = 3+(n-1)(n+2)$$

$$= 3+\left(n^2+n-2\right) = n^2+n+1$$

$$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^2+\sum_{k=1}^{n} k+\sum_{k=1}^{n} 1$$

$$= \frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}+n$$

$$= n\left[\frac{(n+1)(2n+1)+3(n+1)+6}{6}\right]$$

$$= n\left[\frac{2n^2+3n+1+3n+3+6}{6}\right]$$

$$= n\left[\frac{2n^2+6n+10}{6}\right] = \frac{n}{3}\left(n^2+3n+5\right)$$

24. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively. Show that $9S_2^2 = S_3\left(1+8S_1\right)$

Answer: As it is given that S_1, S_2, S_3 are the sum of first n natural numbers. So,

$S_1 = \frac{n(n+1)}{2}$

$S_3 = \frac{n^2(n+1)^2}{4}$

By using $S_3(1+8S_1) = \frac{n^2(n+1)^2}{4}\left[1+\frac{8n(n+1)}{2}\right]$

$= \frac{n^2(n+1)^2}{4}\left[1+4n^2+4n\right]$

$= \frac{n^2(n+1)^2}{4}(2n+1)^2$

$= \frac{[n(n+1)(2n+1)]^2}{4}$ (i)

Also, $9S_2^2 = 9\frac{[n(n+1)(2n+1)]^2}{(6)^2} = \frac{9}{36}[n(n+1)(2n+1)]^2$

$= \frac{[n(n+1)(2n+1)]^2}{4}$ (ii)

Hence, by (i) and (ii), we get our desired result

25. Find the sum of the following series up to n terms: $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$

Answer: n^{th} term is $\frac{1^3+2^3+3^3+\ldots+n^3}{1+3+5+\ldots+(2n-1)} = \frac{\left[\frac{n(n+1)}{2}\right]^2}{1+3+5+\ldots+(2n-1)}$

Here, 1,3,5,…(2n-1) is an A.P.

where a = first term, last term = (2n-1), number of terms = n

$$1+3+5+....+(2n-1) = \frac{n}{2}[2\times1+(n-1)2] = n^2$$

$$a_n = \frac{n^2(n+1)^2}{4n^2} = \frac{(n+1)^2}{4} = \frac{1}{4}n^2+\frac{1}{2}n+\frac{1}{4}$$

$$S_n = \sum_{K=1}^{n} a_K = \sum_{K=1}^{n}\left(\frac{1}{4}K^2+\frac{1}{2}K+\frac{1}{4}\right)$$

$$= \frac{1}{4}\frac{n(n+1)(2n+1)}{6}+\frac{1}{2}\frac{n(n+1)}{2}+\frac{1}{4}n$$

$$= \frac{n[(n+1)(2n+1)+6(n+1)+6]}{24}$$

$$= \frac{n\left[2n^2+3n+1+6n+6+6\right]}{24}$$

$$= \frac{n\left(2n^2+9n+13\right)}{24}$$

26. Show that $\frac{1\times2^2+2\times3^2+\ldots+n\times(n+1)^2}{1^2\times2+2^2\times3+\ldots+n^2\times(n+1)} = \frac{3n+5}{3n+1}$

Answer: n^{th} term of numerator $= n(n+1)^2 = n^3+2n^2+n$

n^{th} term of denominator $= n^2(n+1) = n^3+n^2$

$$\frac{1\times2^2+2\times3^2+\ldots+n\times(n+1)^2}{I^2\times2+2^2\times3+\ldots+n^2\times(n+1)} = \frac{\sum_{K-1}^{a} a_K}{\sum_{k=1} a_K} = \frac{\sum_{K-1}^{n}\left(K^3+2K^2+K\right)}{\sum_{K=1}^{n}\left(K^3+K^2\right)} \quad \text{(i)}$$

$$\sum_{K=1}^{n}\left(K^3+2K^2+K\right) = \frac{n^2(n+1)^2}{4}+\frac{2n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{2}{3}(2n+1)+1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+8n+4+6}{6}\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+11n+10\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+6n+5n+10\right]$$

$$= \frac{n(n+1)}{12}[3n(n+2)+5(n+2)]$$

$$= \frac{n(n+1)(n+2)(3n+5)}{12} \qquad \text{(ii)}$$

Also, $\sum_{K=1}^{n}\left(K^3+K^2\right) = \frac{n^2(n+1)^2}{4}+\frac{n(n+1)(2n+1)}{6}$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+4n+2}{6}\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+7n+2\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+6n+n+2\right]$$

$$= \frac{n(n+1)}{12}[3n(n+2)+1(n+2)]$$

$$= \frac{n(n+1)(n+2)(3n+1)}{12} \qquad \text{(iii)}$$

From (i), (ii), (iii), we get

$$\frac{1\times2^2+2\times3^2+\ldots+n\times(n+1)^2}{1^2\times2+2^2\times3+\ldots+n^2\times(n+1)} = \frac{n(n+1)(n+2)(3n+5)}{\frac{n(n+1)(n+2)(3n+1)}{12}} = \frac{n(n+1)(n+2)(3n+5)}{n(n+1)(n+2)(3n+1)} = \frac{3n+5}{3n+1}$$

27. A farmer buys a used tractor for Rs. 12000 . He pays Rs. 6000 cash and agrees to pay the balance in annual installments of Rs. 500 plus 12% interest on the unpaid amount. How much will be the tractor cost him

Answer: Given that farmer buys a used tractor for Rs. 12000 cash

So, amount which is unpaid = Rs. 12000-Rs. 6000 = Rs. 6000

Now, according to question, interest paid annually i.e.

12% of 6000, 12% of 5500, 12% of 5000…12% of 500

Hence, total interest paid

= 12% of 6000+12% of 5500+12% of 5000+…+12% of 500

= 12% of (6000+5500+5000+…+500)

= 12% of (500+1000+1500+…+6000)

So, series in AP i.e. $500, 1000, 1500 \ldots 6000$ where

$a(=\text{first term}) = d(=\text{common difference}) = 500$

Let number of terms of the A.P. = n

6000 = 500+(n-1)500

1+(n-1) = 12

n = 12

So, sum = $= \frac{12}{2}[2(500)+(12-1)(500)] = 6[1000+5500] = 6(6500) = 39000$

Now, total interest which have to be paid

= 12% of (500+1000+1500+…+6000)

= 12% of 39000 = Rs 4680

Hence, cost of the tractor = (Rs 12000+ Rs 4680) = Rs 16680

28. Shamshad ali buys a scooter for Rs. 22000 . He pays Rs. 4000 cash and agrees to pay the balance in annual installment of Rs. 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him

Answer: Given Shamshad ali buys a scooter for Rs. 22000 and He pays Rs. 4000 cash

So, remaining unpaid amount = Rs. 22000-Rs. 4000 = Rs. 18000

Interest which is paid annually

10% of 18000, 10% of 17000, 10% of 16000…10% of 1000

Hence, total interest which is paid

= 10% of 18000+10% of 17000+10% of 16000+…+10% of 1000

= 10% of (18000+17000+16000+…+1000)

= 10% of (1000+2000+3000+…+18000)

So, series in AP i.e. $1000, 2000, \ldots, 18000$ where

$a(=\text{first term}) = d(=\text{common difference}) = 1000$

Let number of terms of the A.P. = n

$18000 = 1000+(n-1)(1000)$

$n = 18$

So, $1000+2000+\ldots+18000 = \frac{18}{2}[2(1000)+(18-1)(1000)]$

$= 9[2000+17000] = 171000$

Total interest which have to be paid = 10% of (18000+17000+16000+…+1000)

= 10% of Rs. 171000

= Rs. 17100

Hence, Cost of the scooter = Rs. 22000+Rs. 17100 = Rs. 39100

29. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8^{th} set of letter is mailed.

Answer: Nos. of letters mailed to form a G.P. is $4, 4^2, \ldots 4^8$

Here, a = 4, r = 4, n = 8

As $S_n = \frac{a(r^n-1)}{r-1}$

$S_8 = \frac{4(4^8-1)}{4-1} = \frac{4(65536-1)}{3} = \frac{4(65535)}{3} = 4(21845) = 87380$

Now, given the cost to mail 1 letter = 50 paise

So, Cost of mailing 87380 letters = $\text{Rs. } 87380 \times \frac{50}{100} = \text{Rs. } 43690$

30. A man deposited Rs. 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

Answer: As it is given that man deposited Rs. 10000 in a bank at the rate of 5% simple interest annually = $\frac{5}{100} \times$ Rs. 10000 = Rs. 500

So, interest in I^{st} year = $10000+\underbrace{500+500+\ldots+500}_{14\text{ times}}$

Now, Amount in fifteenth year = Rs. 10000+14× Rs. 500

= Rs. 10000+ Rs. 7000

= Rs. 17000

Amount after twenty years = Rs. 10000+20× Rs. 500

= Rs. 10000+ Rs. 10000 = Rs. 20000

31.A manufacturer reckons that the value of a machine, which costs him Rs. 15625 will depreciate each year by 20% Find the estimated value at the end of 5 years.

Answer: Here, it is given that cost of machine = Rs. 15625

And it will depreciate each year by 20%

So, after every year its value is 80% of original cost

Now, value at the end of five years = $15625\times\underbrace{\frac{4}{5}\times\frac{4}{5}\times\ldots\times\frac{4}{5}}_{5\text{ times}} = 5\times1024 = 5120$

32. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed

Answer: Let no. of days in which 150 workers finish work = x

According to question:

$150x = 150+146+142+\ldots(x+8)$ terms

This series is in AP where $a = 146, d = -4, n = (x+8)$

$150x = \frac{(x+8)}{2}[2(150)+(x+8-1)(-4)]$

$150x = (x+8)[150+(x+7)(-2)]$

$150x = (x+8)(150-2x-14)$

$150x = (x+8)(136-2x)$

$75x = (x+8)(68-x)$

$75x = 68x-x^2+544-8x$

$x^2+75x-60x-544 = 0$

$x^2+15x-544 = 0$

$x^2+32x-17x-544 = 0$

$x(x+32)-17(x+32) = 0$

$(x-17)(x+32) = 0$

$x = 17$ or $x = -32$

Thus, x can't be negative. So, $x = 17$

Hence, no. of days in which work was completed = 17

Thus, no. of days = (17+8) = 25