Chapter 2: Units and Measurement

Fill in the blanks:

The volume of a cube of side $1\ cm$ is equal to..... m^3. The surface area of a solid cylinder of radius $2.0\ cm$ and height $10.0\ cm$ is equal to... $(mm)^2$. A vehicle moving with a speed of $18\ km\ h^{-1}$ covers .... m in $1\ s$. The relative density of lead is 11.3. Its density is .... $g\ cm^{-3}$ or .... $kg\ m^{-3}$.

Solution:

$1\ cm = \frac{1}{100} m$

Volume of the cube $= 1\ cm^3$

But, $1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$

$= \left(\frac{1}{100}\right) m \times \left(\frac{1}{100}\right) m \times \left(\frac{1}{100}\right) m$

$\therefore 1\ cm^3 = 10^{-6}\ m^3$

Hence, the volume of a cube of side $1\ cm$ is equal to $10^{-6}\ m^3$.

The total surface area of a cylinder of radius r and height h is

$S = 2\pi r\ (r + h)$.

Given that,

$r = 2\ cm$

$r = 2 \times 1\ cm$

$r = 2 \times 10\ mm$

$r = 20\ mm$

$h = 10\ cm$

$h = 10 \times 10\ mm$

$h = 100\ mm$

$h = 15072$

$h = 1.5 \times 10^4\ mm^2$

$\therefore S = 2 \times 3.14 \times 20 \times (20 + 100)$

Using the conversion,

$1 km/h = \frac{5}{18} m/s$

$18 km/h = 18 \times \frac{5}{18} = 5 m/s$

Therefore, distance can be obtained using the relation:

Distance = Speed × Time

$Distance = 5 \times 1$

$= 5\ m$

Hence, the vehicle covers $5m$ in $1s$.

Relative density of a substance is given by the relation,

$$Relative\ density = \frac{Density\ of\ substance}{Density\ of\ water}$$

Density of water $= 1\ g/cm^3$

Density of lead = Relative density of lead × Density of water

$= 11.3 \times 1$

$= 11.3\ g/cm^3$

Again, $1g = \frac{1}{1000} kg$

$1\ cm^3 = 10^{-6}\ m^3$

$1\ g/cm^3 = \frac{10^{-3}}{10^{-6}} kg/m^3$

$= 10^3\ kg/m^3$

$\therefore 11.3\ g/cm^3 = 11.3 \times 10^3\ kg/m^3$

2.2 Fill in the blanks by suitable conversion of units:

$1\ kg\ m^2\ s^{-2} = ...g\ cm^2\ s^{-2}$

$1\ m = ...\ ly$

$3.0\ m\ s^{-2} = ...\ km\ h^{-2}$

$G = 6.67 \times 10^{-11}\ N\ m^2\ (kg)^{-2} = ...\ (cm)^3\ s^{-2}\ g^{-1}$.

Solution:

$1\,kg = 10^3\,g$

$1\,m^2 = 10^4\,cm^2$

$1\,kg\,m^2\,s^{-2} = 1\,kg \times 1\,m^2 \times 1\,s^{-2}$

$= 10^3\,g \times 10^4\,cm^2 \times 1\,s^{-2} = 10^7\,g\,cm^2\,s^{-2}$

Light year is the total distance travelled by light in one year.

$1\,ly = Speed\ of\ light \times One\ year$

$= \left(3 \times 10^8\,m/s\right) \times \left(365 \times 24 \times 60 \times 60\,s\right)$

$= 9.46 \times 10^{15}\,m$

$\therefore 1m = \dfrac{1}{9.46 \times 10^{15}}$

$= 1.057 \times 10^{-16}$ ly

$1\,m = 10^{-3}\,km$

Again, $1\,s = \dfrac{1}{3600}\,h$

$1\,s^{-1} = 3600\,h^{-1}$

$1\,s^{-2} = \left(3600\right)^2\,h^{-2}$

$\therefore 3m\,s^{-2} = \left(3 \times 10^{-3}\,km\right) \times \left(\left(3600\right)^2\,h^{-2}\right)$

$= 3.88 \times 10^{-4}\,km\,h^{-2}$

$1\,N = 1\,kg\,m\,s^{-2}$

$1\,kg = 10^{-3}\,g^{-1}$

$1\,m^3 = 10^6\,cm^3$

$\therefore 6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$

$= 6.67 \times 10^{-11} \times \left(1\,kg\,m\,s^{-2}\right)\left(1\,m^2\right)\left(1\,s^{-2}\right)$

$= 6.67 \times 10^{-11} \times \left(1\,kg \times 1\,m^3 \times 1\,s^{-2}\right)$

$= 6.67 \times 10^{-11} \times \left(10^{-3}\,g^{-1}\right) \times \left(10^6\,cm^3\right) \times \left(1\,s^{-2}\right)$

$= 6.67 \times 10^{-8}\,cm^3\,s^{-2}\,g^{-1}$

2.3 A calorie is a unit of heat or energy and it equals about 4.2 J where 1J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 α^{-1} β^{-2} γ^{2} in terms of the new units.

Solution:

Given that,

$1\ calorie = 4.2\ (1\ kg)\ (1\ m^2)\ (1\ s^{-2})$

New unit of mass $= \alpha\ kg$

Hence, in terms of the new unit, $1\ kg = \frac{1}{\alpha} = \alpha^{-1}$

In terms of the new unit of length,

$1\ m = \frac{1}{\beta} = \beta^{-1}$ or $1\ m^2 = \beta^{-2}$

And, in terms of the new unit of time,

$1\ s = \frac{1}{\gamma} = \gamma^{-1}$

$1\ s^2 = \gamma^{-2}$

$1\ s^{-2} = \gamma^{2}$

$\therefore 1\ calorie = 4.2\ (1\ \alpha^{-1})\ (1\ \beta^{-2})\ (1\ \gamma^{2}) = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^{2}$

2.4 Explain this statement clearly:

"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison". In view of this, reframe the following statements wherever necessary:

Atoms are very small objects a jet plane moves with great speed the mass of Jupiter is very large the air inside this room contains a large number of molecules a proton is much more massive than an electron the speed of sound is much smaller than the speed of light.

Solution:

Because a dimensionless quantity can be large or tiny in comparison to some standard reference, the above assertion is correct. The coefficient of friction, for example, is dimensionless. Sliding friction has a higher coefficient than rolling friction, although it is lower than static friction. In comparison to a soccer ball, an atom is an extremely little item. A jet plane can go at a faster rate than a bicycle. Jupiter's mass is enormous when compared to that of a cricket ball. In comparison to the air in a

geometric box, the air in this room includes a huge number of molecules. A proton has a mass that is larger than that of an electron. Sound travels at a slower rate than light.

2.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Solution:

Distance between the Sun and the Earth:

$= Speed\ of\ light \times Time\ taken\ by\ light\ to\ cover\ the\ distance$

Given that in the new unit,

Speed of light $= 1\ unit$

Time taken,

$t = 8\ min\ 20\ s$

$t = 500\ s$

$\therefore$ Distance between the Sun and the Earth

$= 1 \times 500$

$= 500\ units$

2.6 Which of the following is the most precise device for measuring length: a Vernier calipers with 20 divisions on the sliding scale a screw gauge of pitch 1 mm and 100 divisions on the circular scale an optical instrument that can measure length to within a wavelength of light?

Solution:

A device with minimum count is the most suitable to measure length.

Least count of Vernier callipers

$= 1\ standard\ division\ (SD) - 1\ vernier\ division\ (VD)$

$= 1 - \frac{9}{10}$

$= \frac{1}{10}$

$= 0.01\ cm$

Least count of screw gauge =

$= \frac{Pitch}{Number\ of\ Divisions}$

$= \frac{1}{1000}$

$= 0.001\ cm$

Least count of an optical device $=$ $Wavelength\ of\ \ light \sim 10^{-5}\ cm$

$=\ 0.00001\ cm$

Hence, it can be inferred that an optical instrument is the most suitable device to measure length.

2.7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is $3.5\ mm$. What is the estimate on the thickness of hair?

Solution:

Magnification of the microscope $=\ 100$

Average width of the hair in the field of view of the microscope $=\ 3.5\ mm$

$\therefore$ Actual thickness of the hair is $\frac{3.5}{100} =\ 0.035\ mm.$

2.8 Answer the following:

You are given a thread and a metre scale. How will you estimate the diameter of the thread? A screw gauge has a pitch of $1.0\ mm$ and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?

The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Solution:

Wrap the thread around a smooth, homogeneous rod in such a way that the resulting coils are very close together. Using a metre scale, determine the thread's length. The thread's diameter is determined by the relationship,

$$Diameter = \frac{Length\ of\ \ thread}{Number\ of\ \ turns}$$

The number of divisions on a circular scale cannot be increased to improve the accuracy of a screw gauge. The precision of the circular scale can be improved by increasing the number of divisions, but only to a limited amount.

A collection of 100 measurements is more dependable than a set of 5 measurements because the former has many fewer random mistakes than the latter.

2.9 The photograph of a house occupies an area of $1.75\ cm^2$ on a $35\ mm$ slide. The slide is projected on to a screen, and the area of the house on the screen is $1.55\ m^2$. What is the linear magnification of the projector-screen arrangement?

Solution:

Area of the house on the slide $= 1.75\ cm^2$

Area of the image of the house formed on the screen $= 1.55\ m^2$

$= 1.55 \times 10^4\ cm^2$

Arial magnification, $m_a = \dfrac{Area\ of\ image}{Area\ of\ object}$

$= \dfrac{1.55}{1.75} \times 10^4$

$\therefore$ Linear magnifications, $m_l = \sqrt{m_a}$

$= \sqrt{\dfrac{1.55}{1.75} \times 10^4}$

$= 94.11$

2.10 State the number of significant figures in the following:

a) $0.007\ m^2$

b) $2.64 \times 10^{24}\ kg$

c) $0.2370\ g\ cm^{-3}$

d) $6.320\ J$

e) $6.032\ N\ m^{-2}$

f) $0.0006032\ m^2$

Solution:

a) The given quantity is $0.007\ m^2$.

If the number is less than one, then all zeros on the right of the decimal point (but left to the first non-zero) are insignificant. This means that here, two zeros after the decimal are not significant. Hence, only 7 is a significant figure in this quantity.

b) The given quantity is $2.64 \times 10^{24}\ kg$.

Here, the power of 10 is irrelevant for the determination of significant figures. Hence, all digits i.e., $2,\ 6\ and\ 4$ are significant figures.

c) The given quantity is $0.2370\ g\ cm^{-3}$.

For a number with decimals, the trailing zeroes are significant. Hence, besides digits $2,\ 3$ and $7,\ 0$ that appears after the decimal point is also a significant figure.

d) The given quantity is $6.320\ J$.

The trailing zeroes in a decimal number are crucial. As a result, all four digits in the supplied amount are meaningful.

e) The given quantity is $6.032\ Nm^{-2}$.

All zeroes between two non-zero digits are always significant.

f) The given quantity is $0.0006032\ m^2$.

The zeroes to the right of the decimal point (but left to the first non-zero) are meaningless if the number is less than one. As a result, all three zeroes preceding 6 are not significant figures. Between two non-zero digits, all zeros are always meaningful. As a result, the last four numbers are significant figures.

2.11 The length, breadth and thickness of a rectangular sheet of metal are $4.234\ m$, $1.005\ m$, and $2.01\ cm$ respectively. Give the area and volume of the sheet to correct significant figures.

Solution:

Length of sheet, $l = 4.234\ m$

Breadth of sheet, $b = 1.005\ m$

Thickness of sheet, $h = 2.01\ cm = 0.0201\ m$

The given table lists the respective significant figures:

Quantity	Number	Significant Figure
l	4.234	4
b	1.005	4
h	2.01	3

Hence, area and volume both must have least significant figures i.e. 3.

Surface area of the sheet $= 2\left(l \times b + b \times h + h \times l\right)$

$= 2\left(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234\right)$

$= 2\left(4.25517 + 0.02620 + 0.08510\right)$

$= 2 \times 4.360$

$= 8.72\ m^2$

Volume of the sheet $= l \times b \times h$

$= 4.234 \times 1.005 \times 0.0201$

$= 0.0855\ m^3$

This number has only 3 significant figures i.e., $8,\ 5,\ and\ 5$.

2.12 The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is

(a) The total mass of the box,

(b) The difference in the masses of the pieces to correct significant figures?

Solution:

Mass of grocer's box $= 2.300\ kg$

Mass of gold piece I $= 20.15g = 0.02015\ kg$

Mass of gold piece II $= 20.17\ g = 0.02017\ kg$

Total mass of the box

$= 2.3 + 0.02015 + 0.02017$

$= 2.34032\ kg$

In addition, the final result should retain as many decimal places as there are in the number with the least decimal places. Hence, the total mass of the box is $2.3\ kg$.

Difference in masses

$= 20.17 - 20.15$

$= 0.02\ g$

In subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.

2.13 A physical quantity P is related to four observables a, b, c and d as follows:

$$P = \frac{a^3 b^2}{\left(\sqrt{cd}\right)}$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Solution:

$$P = \frac{a^3 b^2}{\left(\sqrt{cd}\right)}$$

$$\frac{\triangle P}{P} = \frac{3\triangle a}{a} + \frac{2\triangle b}{b} + \frac{1}{2}\frac{\triangle c}{c} + \frac{\triangle d}{d}$$

$$\left(\frac{\triangle P}{P}\times 100\right)\% = \left(3\times\frac{\triangle a}{a}\times 100 + 2\times\frac{\triangle b}{b}\times 100 + \frac{1}{2}\frac{\triangle c}{c}\times 100 + \frac{\triangle d}{d}\times 100\right)\%$$

$$= 3\times 1 + 2\times 3 + \frac{1}{2}\times 4 + 2$$

$$= 3 + 6 + 2 + 2$$

$$= 13\%$$

Percentage error in $P = 13\ \%$

Value of P is given as 3.763.

By rounding off the given value to the first decimal place, we get $P = 3.8$.

2.14 A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion:

$$\mathbf{y = a \sin}\left(\frac{2\pi t}{T}\right)$$

$$\mathbf{y = a \sin vt}$$

$$\mathbf{y} = \left(\frac{\mathbf{a}}{T}\right)\mathbf{\sin}\left(\frac{t}{a}\right)$$

$$\mathbf{y} = \left(a\sqrt{2}\right)\left(\sin\frac{2\pi t}{T} + \cos\frac{2\pi t}{T}\right)$$

a = maximum displacement of the particle,

v = speed of the particle.

T = time-period of motion).

Rule out the wrong formulas on dimensional grounds.

Solution:

Correct:

$$y = a \sin\left(\frac{2\pi t}{T}\right)$$

Dimension of $y = M^0 L^1 T^0$

Dimension of $a = M^0 L^1 T^0$

Dimension of $\sin\frac{2\pi t}{T} = M^0 L^0 T^0$

$\therefore$ Dimension of L.H.S = Dimension of R.H.S

Hence, the given formula is dimensionally correct.

Incorrect

$y = a \sin vt$

Dimension of $y = M^0 L^1 T^0$

Dimension of $a = M^0 L^1 T^0$

Dimension of $vt = M^0 L^1 T^{-1} \times M^0 L^0 T^1 = M^0 L^1 T^0$

But the argument of the trigonometric function must be dimensionless, which is not so in the given case. Hence, the given formula is dimensionally incorrect.

Incorrect:

$$\mathbf{y} = \left(\frac{\mathbf{a}}{T}\right) \mathbf{sin}\left(\frac{t}{a}\right)$$

Dimension of $y = M^0 L^1 T^0$

Dimension of $\frac{a}{T} = M^0 L^1 T^{-1}$

Dimension of $\frac{t}{a} = M^0 L^{-1} T^1$

But the argument of the trigonometric function must be dimensionless, which is not so in the given case. Hence, the formula is dimensionally incorrect.

Correct :

$$\mathbf{y} = \left(a\sqrt{2}\right)\left(\sin\frac{2\pi t}{T} + \cos\frac{2\pi t}{T}\right)$$

Dimension of $y = M^0 L^1 T^0$

Dimension of $a = M^0 L^1 T^0$

Dimension of $\frac{t}{T} = M^0L^0T^0$

The dimensions of y and an are the same since the trigonometric function's argument must be dimensionless (which is true in this case). As a result, the formula given is dimensionally correct.

2.15 A famous relation in physics relates 'moving mass' m to the 'rest mass' m0 of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:

$$m = \frac{m_o}{(1-v^2)^{\frac{1}{2}}}$$

Solution:

Given the relation,

$$m = \frac{m_o}{(1-v^2)^{\frac{1}{2}}}$$

Dimension of $m = M^1L^0T^0$

Dimension of $m_0 = M^1L^0T^0$

Dimension of $v = M^0L^1T^{-1}$

Dimension of $v^2 = M^0L^2T^{-2}$

Dimension of $c = M^0L^1T^{-1}$

Only when the dimensions of L.H.S and R.H.S are the same will the supplied formula be dimensionally correct.

This is only possible when the factor, $(1-v^2)^{\frac{1}{2}}$ is dimensionless i.e., $(1-v^2)$ is dimensionless. Only if v_2 is split by c_2 is this possible. As a result, the right relationship is

$$m = \frac{m_o}{(1-\frac{v^2}{c^2})^{\frac{1}{2}}}$$

2.16 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by $\overset{o}{A}$: $1\overset{o}{A} = 10^{-10}m$ .The size of a hydrogen atom is about 0.5Å , what is the total atomic volume in m3 of a mole of hydrogen atoms?

Solution:

Radius of hydrogen atom, $r = 0.5\overset{o}{A} = 0.5 \times 10^{-10}m$

Volume of hydrogen atom $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3$

$= 0.524 \times 10^{-30}\ m^3$

1mole of hydrogen contains 6.023×10^{23} hydrogen atoms.

$\therefore$ Volume of 1 mole of hydrogen atoms

$= 6.023 \times 10^{23} \times 0.524 \times 10^{-30}$

$= 3.16 \times 10^{-7} m^3$

2.17 One mole of an ideal gas at standard temperature and pressure occupies $22.4\ L$ (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about $1\overset{o}{A}$). Why is this ratio so large?

Solution:

Radius of hydrogen atom, $r = 0.5\overset{o}{A} = 0.5 \times 10^{-10} m$

Volume of hydrogen atom $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3$

$= 0.524 \times 10^{-30}\ m^3$

Now, 1 mole of hydrogen contains 6.023×10^{23} hydrogen atoms.

Volume of 1 mole of hydrogen atoms, $V_a = 6.023 \times 10^{23} \times 0.524 \times 10^{-30}$

$= 3.16 \times 10^{-7} m^3$

Molar volume of 1 mole of hydrogen atoms at STP,

$V_m = 22.4\ L = 22.4 \times 10^{-3} m^3$

$\therefore \frac{V_m}{V_a} = \frac{22.4 \times 10^{-3}}{3.16 \times 10^{-7}}$

$= 7.08 \times 10^4$

As a result, the molar volume is 7.08×10^4 times the atomic volume. As a result, in hydrogen gas, the inter-atomic separation is substantially larger than the size of a hydrogen atom.

2.18 Explain this common observation clearly: If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

Solution:

A line of sight is an imaginary line that connects an object with the observer's eye. When we look at nearby stationary objects like as trees, houses, and other structures while riding in a moving train, they appear to move quickly in the other direction because the line of sight changes so quickly.

Distant things, such as trees, stars, and other celestial bodies, on the other hand, appear to be immobile due to their great distance. As a result, the line of sight does not abruptly change direction.

2.19 The principle of 'parallax' in section 2.3.1 is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit $\approx 3 \times 10^{11} m$. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of meters?

Solution:

Diameter of Earth's orbit $= 3 \times 10^{11} m$

Radius of Earth's orbit, $r = 1.5 \times 10^{11} m$

Let the distance parallax angle be $= 4.847 \times 10^{-6} rad$.

Let the distance of the star be D.

Parsec is defined as the distance at which the average radius of the Earth's orbit subtends an angle of **1"**.

We have $\theta = \frac{r}{D}$

$$D = \frac{r}{\theta} = \frac{1.5 \times 10^{11}}{4.847 \times 10^{-6}}$$

$$= 0.309 \times 10^{-6} \approx 3.09 \times 10^{16}\ m$$

Hence, $1\ parsec \approx 3.09 \times 10^{16} m$.

2.20 The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

Solution:

Distance of the star from the solar system $= 4.29\ ly$

1 light year is the distance travelled by light in one year.

1 light year = Speed of light × 1 year

$= 3 \times 10^{8} \times 365 \times 24 \times 60 \times 60$

$= 94608 \times 10^{11} m$

$\therefore 4.29\ ly = 405868.32 \times 10^{11} m$

$1\ parsec = 3.08 \times 10^{16} m$

$$\therefore 4.29\ ly\ = \frac{405868.32\times10^{11}}{3.08\times10^{16}} = \ 1.32\ parsec$$

Using the relation,

$$\theta = \frac{d}{D}$$

Where,

Diameter of Earth's orbit,

$$d = 3\times10^{11}\ m$$

Distance of the star from the Earth,

$$D = 405868.32\times10^{12}\ m$$

$$\therefore \theta = \frac{3\times10^{11}}{405868.32\times10^{11}}$$

$$= 7.39\times10^{-6}\ rad$$

But, $1\ sec\ =\ 4.85\ \times\ 10^{-6}\ rad$

$$7.39\times10^{-6}\ rad = \frac{7.39\times10^{-6}}{4.85\times10^{-6}}$$

$$= 1.52^{o}$$

2.21 Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Solution:

It is undeniably true that exact measurements of physical quantities are critical for scientific progress. Ultra-shot laser pulses, for example, are used to monitor time intervals in a variety of physical and chemical processes (time intervals of 10^{-15} s).

The inter-atomic separation or inter-planer spacing is determined using X-ray spectroscopy. The advancement of mass spectrometers has made it feasible to precisely measure the mass of atoms.

2.22 Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations.

Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):

The total mass of rain-bearing clouds over India during the Monsoon the mass of an elephant the wind speed during a storm the number of strands of hair on your head the number of air molecules in your classroom.

Solution:

During monsoons, a metrologist records about $215\ cm$ of rainfall in India i.e., the height of water column,

$h = 215\ cm = 2.15\ m$

Area of country, $A = 3.3 \times 10^{12}\ m^2$

Hence, volume of rain water,

$V = A \times h$

$= 7.09 \times 10^{12}\ m^3$

Density of water, $\rho = 1 \times 10^3\ kg\ m^{-3}$

Hence, mass of rainwater $= \rho \times V = 7.09 \times 10^{15}\ kg$

Hence, the total mass of rain-bearing clouds over India is approximately $7.09 \times 10^{15}\ kg$.

Consider a ship of known base area floating in the sea. Measure its depth in sea (say d_1).

Volume of water displaced by the ship, $V_b = A\ d_1$

Now, move an elephant on the ship and measure the depth of the ship (d_2) in this case.

Volume of water displaced by the ship with the elephant on board, $V_{be} = Ad_2$

Volume of water displaced by the elephant $= Ad_2 - Ad_1$

Density of water $= D$

Mass of elephant $= AD\left(d_2 - d_1\right)$

Wind speed during a storm can be measured by an anemometer. As wind blows, it rotates.

The value of wind speed is determined by the anemometer's revolution in one second.

Area of the head surface carrying hair $= A$

A screw gauge can be used to calculate the diameter and thus the radius of a hair.

Let it be r .

$\therefore$ Area of one hair $= \pi r^2$

Number of strands of hair

$= \dfrac{Total\ surface\ area}{Area\ of\ one\ hair}$

$= \frac{A}{r^2}$

Let the volume of the room be V.

One mole of air at NTP occupies 22.41

i.e., $22.4 \times 10^{-3}\ m^3\ volume$

Number of molecules in one mole $= 6.023 \times 10^{23}$

$\therefore$ Number of molecules in room of volume V

$= \frac{6.023 \times 10^{23}}{22.4 \times 10^{-3}} \times V$

$= 134.915 \times 10^{26}\ V$

$= 1.35 \times 10^{28}\ V$

2.23 The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding $107\ K$, and its outer surface at a temperature of about $6000\ K$. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun $= 2.0 \times 10^{30}$ kg, radius of the Sun $= 7.0 \times 10^{8}$ m.

Solution:

Mass of the Sun, $M = 2.0 \times 10^{30}\ kg$

Radius of the Sun, $R = 7.0 \times 10^{8}\ m$

Volume of the Sun,

$V = \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(7.0 \times 10^{8}\right)^3$

$= \frac{88}{21} \times 343 \times 10^{24}$

$= 1437.3 \times 10^{24} m^3$

Density of the Sun

$= \frac{Mass}{Volume}$

$$= \frac{2.0\times10^{30}}{1437.3\times10^{24}} \sim 1.4\times10^{3}\ kg/m^{5}$$

The Sun has a density that is similar to that of solids and liquids. The enormous gravitational attraction of the inner layers on the outer layer of the Sun is responsible for the high density.

2.24 When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be of arc. Calculate the diameter of Jupiter.

Solution:

Distance of Jupiter from the Earth,

$D = 824.7 \times 10^{6}\ km$

$= 824.7 \times 10^{9}\ m$

Angular diameter $= 35.72^{o} = 35.72\times4.874\times10^{-6}\ rad$

Diameter of Jupiter $= d$

Using the relation,

$$\theta = \frac{d}{D}$$

$$d = \theta D$$

$$= 824.7\times10^{9}\times35.72\times4.872\times10^{-6}$$

$$= 143520.76\times10^{3}$$

$$= 1.435\times10^{5}\ km$$

Additional Exercises

2.25 A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v: $\tan\theta = v$ and checks that the relation has a correct limit: as , θ $v \to 0, \theta \to 0$ as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Solution:

Answer is Incorrect:

On dimensional ground, the relation is $\tan\theta = v$

Dimension of R.H.S $= M^{0}\ L^{1}\ T^{-1}$

Dimension of L.H.S $= M^{0}\ L^{0}\ T^{0}$

($\therefore$ The trigonometric function is considered to be a dimensionless quantity)

Dimension of R.H.S is not equal to the dimension of L.H.S. Hence, the given relation is not correct dimensionally.

2.26 It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about $0.02\ s$. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of $1\ s$?

Solution:

Difference in time of caesium clocks $= 0.02\ s$

Time required for this difference $= 100\ years$

$= 100 \times 365 \times 24 \times 60 \times 60$

$= 3.15 \times 10^{9}\ s$

In $3.15 \times 10^{9}\ s$, the caesium clock shows a time difference of $0.02\ s$.

In $1\ s$, the clock will show a time difference of $\frac{0.02}{3.15\times10^{9}}\ s$.

Hence, the accuracy of a standard caesium clock in measuring a time interval of $1\ s$ is

$$\frac{3.15\times10^{9}}{0.02} = 157.5\times10^{9}\ s \approx 1.5\times10^{11}\ s$$

2.27 Estimate the average mass density of a sodium atom assuming its size to be about 2.5 (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: $970\ kg\ m^{-3}$. Are the two densities of the same order of magnitude? If so, why?

Solution:

Diameter of sodium atom = Size of sodium atom $= 2.5\ \mathring{A}$

Radius of sodium atom, $r = \frac{1}{2}\times 2.5\ \mathring{A} = 1.25\ \mathring{A}$

$= 1.25 \times 10^{-10}\ m$

According to the Avogadro hypothesis, one mole of sodium contains 6.023×10^{23} atoms has a mass of $23\ g$ or $23 \times 10^{-3}\ kg$.

$\therefore$ Mass of one atom $= \frac{23\times10^{-3}}{6.023\times10^{23}}\ kg$

Density of sodium atom,

$$\rho = \frac{\frac{23\times10^{-3}}{6.023\times10^{23}}}{\frac{4}{3}\times3.14\times(1.25\times10^{-10})^3}$$

$$= 4.67\times10^{-3}\ kg\ m^{-3}$$

It is given that the density of sodium in crystalline phase is $970\ kg\ m^{-3}$.

Hence, the density of sodium atom and the density of sodium in its crystalline phase are not in the same order. This is because in solid phase, atoms are closely packed. Thus, the inter-atomic separation is very small in the crystalline phase.

2.28 The unit of length convenient on the nuclear scale is a fermi: $1\ f = 10^{-15}\ m$. Nuclear sizes obey roughly the following empirical relation: $r = r_0 A^{1/3}$ where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise. 2.27

Solution:

Radius of nucleus r is given by the relation,

$r = r_0 A^{\frac{1}{3}}$ .... (i)

$r_0 = 1.2\ f$

$= 1.2 \times 10^{-15}\ m$

Volume of nucleus, $V = \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi\left(r_0 A^{\frac{1}{3}}\right)^3$$

$$= \frac{4}{3}\pi r_0^3 A$$

Now, the mass of a nuclei M is equal to its mass number i.e.

$M = A\ amu = A \times 1.66 \times 10^{-27}\ kg$

Density of nucleus,

$$\rho = \frac{Mass\ of\ nucleus}{Volume\ of\ nucleus}$$

$$= \frac{A\times1.66\times10^{-27}}{\frac{4}{3}\pi r_0^3 A}$$

$$= \frac{3 \times 1.66 \times 10^{-27}}{4\pi r_0^3} \ kg / m^3$$

This relation shows that nuclear mass depends only on constant r_0. As a result, all nuclei's nuclear mass densities are almost the same.
Density of sodium nucleus is given by,

$$\rho_{Sodium} = \frac{3 \times 1.66 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}$$

$$= \frac{4.98}{21.17} \times 10^{18}$$

$$= 2.29 \times 10^{17} \ kg \ m^{-3}$$

2.29 A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser asa source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Solution:

Time taken by the laser beam to return to Earth after reflection from the Moon $= \ 2.56 \ s$
Speed of light $= \ 3 \times 10^8 m / s$

Time taken by the laser beam to reach Moon $= \frac{1}{2} \times 2.56$

$= 1.28 \ s$
Radius of the lunar orbit
= Distance between the Earth and the Moon
$= \ 1.28 \times 3 \times 10^8$
$= 3.84 \times 10^8 m$

$= \ 3.84 \times 10^5 km$

2.30 A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water $= \ 1450 \ m \ s^{-1}$).

Solution:
Allow a space of 'S' between the ship and the enemy submarine.
Sound speed in water $= \ 1450 \ m / s$
Transmission and reception of sonar waves have a time lag $= \ 77 \ s$
Sound waves travel twice as far as the distance between the ship and the submarine during this time

lag.

The amount of time it takes for the sound to reach the submersible is measured in seconds.

$= \frac{1}{2} \times 77$

$= 38.5\ s$

$\therefore$ Distance between the ship and the submarine

$(S) = 1450 \times 38.5$

$= 55825\ m$

$= 55.8\ km$

2.31The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Solution:

Time taken by quasar light to reach Earth $= 3$ billion years

$= 3 \times 10^9$ years

$= 3 \times 10^9 \times 365 \times 24 \times 60 \times 60\ s$

Speed of light $= 3 \times 10^8\ m/s$

Distance between the Earth and quasar

$= (3 \times 10^8) \times (3 \times 10^9 \times 365 \times 24 \times 60 \times 60)$

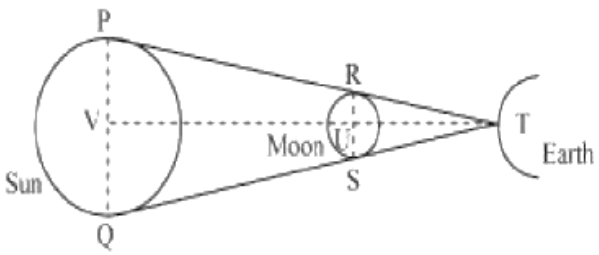
$= 283824 \times 10^{20}\ m$

$= 2.8 \times 10^{22}\ km$

2.32 It is a well-known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.

Solution:

The position of the Sun, Moon, and Earth during a lunar eclipse is shown in the given figure.



Distance of the Moon from the Earth $= 3.84 \times 10^8 m$

Distance of the Sun from the Earth $= 1.496 \times 10^{11} m$

Diameter of the Sun $= 1.39 \times 10^9 \ m$

It can be observed that ΔTRS and ΔTPQ are similar. Hence, it can be written as:

$$\frac{PQ}{RS} = \frac{VT}{UT}$$

$$\frac{1.39 \times 10^9}{RS} = \frac{1.496 \times 10^{11}}{3.84 \times 10^8}$$

$$RS = \frac{1.39 \times 3.84}{1.496} \times 10^6$$

$$= 3.57 \times 10^6 \ m$$

Hence, the diameter of the Moon is $3.57 \times 10^6 m$.

2.33 A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~ 15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?

Solution:

One relation consists of some fundamental constants that give the age of the Universe by:

$$t = \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \times \frac{1}{m_\rho m_\varepsilon^2 c^3 G}$$

Where,

$t =$ Age of Universe

$e =$ Charge of electrons

$e = 1.6 \times 10^{-19} C$

$\varepsilon_0 =$ Absolute permittivity

$m_p =$ Mass of protons

$= 1.67 \times 10^{-27} kg$

$m_e =$ Mass of electrons

$= 9.1 \times 10^{-31} kg$

$c =$ Speed of light

$= 3 \times 10^8 m/s$

G = Universal gravitational constant

$= 6.67 \times 10^{11} Nm^2 kg^{-2}$

Also, $\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \ Nm^2/C^2$

Substituting these values in the equation, we get

$$t = \frac{(1.6\times10^{-19})^4 \times (9\times10^9)^2}{(9.1\times10^{-31})^2 \times 1.67\times10^{-27} \times (3\times10^8)^3 \times 6.67\times10^{-11}}$$

$$= \frac{(1.6)^4 \times 81}{9.1\times1.67\times27\times6.67} \times 10^{-76+18+62+27-24+11}$$

$$= \frac{(1.6)^4 \times 81}{9.1\times1.67\times27\times6.67\times365\times24\times3600} \times 10^{-76+18+62+27-24+11}$$

$= 6\times10^{-9}\times10^{18} \ years$

$= 6$ billion years