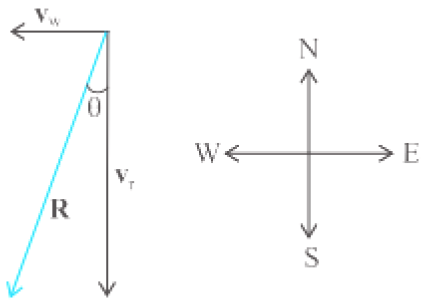
Chapter 4: MOTION IN A PLANE

EXAMPLES

1. Rain is falling vertically with a speed of 35 m/ s. Winds starts blowing after sometime with a speed of 12 m/ s in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?



Solution: The vectors v_r and v_w represent the rain and wind velocity, respectively .in Figure 4.7 and are pointing in the direction of the issue. We can use the rule of vector addition to Solutionve this problem. have a look at the outcome of v_r also v_w is R, as demonstrated in the diagram R has a magnitude of:

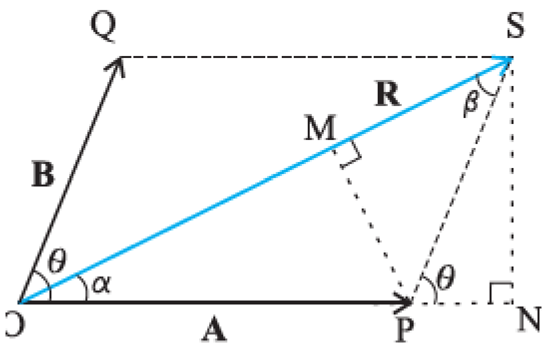
$$R=\sqrt{v^2{}_r+v^2{}_w}=\sqrt{35^2+12^2}\,m/s^{-1}=37m/s^{-1}$$

$$\tan\theta=\frac{v_w}{v_r}=\frac{12}{35}=0.343$$

$$\theta=\tan^{-1}(0.343)=19^0$$

As a result, the youngster should hold his umbrella in the vertical plane at a 19^0 angle with the vertical in the direction of the east.

2.Find the magnitude and direction of the resultant of two vectors A and B in terms of their magnitudes and angle θ between them.



Solution: Let OP and OQ stand for the two vectors A and B that form a q-angle .Then, using the vector parallelogram approach, OS also stands for the resultant vector R.

$R = A + B$

$OS^2 = ON^2 + SN^2$

$SN = B \sin\theta$

$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

In Δ OSN, SN = OS $\sin\alpha$ = R $\sin\alpha$, and

in Δ PSN, SN = PS $\sin\theta$ = B $\sin\theta$

Therefore:

$R\sin\alpha = B\sin\theta$

or, $\dfrac{R}{\sin\theta} = \dfrac{B}{\sin\alpha}$

Similarly:

$PM = A\sin\alpha = B\sin\beta$

or, $\dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha}$

Combining equations:

$$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$$

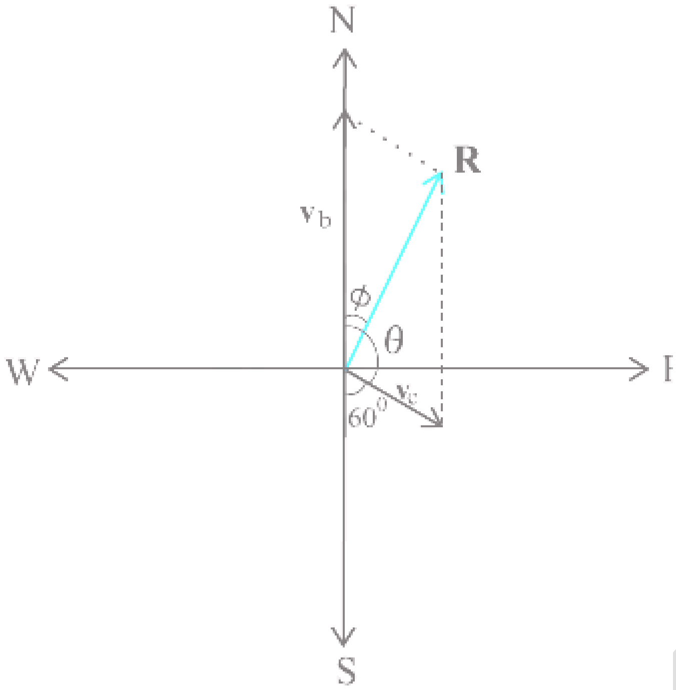
$$\sin\alpha = \frac{B}{R}\sin\theta$$

Where R is given as:

$$\tan\alpha = \frac{SN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta}$$

3. A motorboat is racing towards north at $25\ km/h$ and the water current in that region is $10\ km/h$ in the direction of 60° east of south. Find the resultant velocity of the boat

Solution: The vector v_b represents the motorboat's velocity, while the vector v_c represents the water current. The problem specifies the directions. Making use of the outcome of the parallelogram method of addition R is obtained in the direction indicated in the diagram.



We can obtain the magnitude of R using the Law of cosine:

$$R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120^0}$$
$$= \sqrt{25^2 + 10^2 + 2\times 25\times 10\times(-\frac{1}{2})} = 22 km/hr$$

To obtain the direction, we apply the Law of sines:

$$\frac{R}{\sin\theta} = \frac{v_c}{\sin\phi} = \frac{10\sin 120^0}{21.8} = 0.397 = \phi = 23.4^0$$

4. The position of a particle is given by $r = 3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$ where t is in seconds and the coefficient have the proper units for r to be in meters.

(a) Find v(t) and a(t) of the particle.

(b) Find the magnitude and direction of v(t) at t=1.0 s.

Solution:

$$v(t) = \frac{dr}{dt} = \frac{d}{dt}(3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k})$$
$$= 3.0\hat{i} + 4.0\hat{j}$$
$$a(t) = \frac{dv}{dt} = +4.0\hat{j}$$
$$Magnitude\ is\ v = \sqrt{3^2 + 4^2} = 5.0 ms$$

$$v_0t+\frac{1}{2}at^2=r=r_0+v_0t+\frac{1}{2}at^2$$

direction is: $x=x_0+v_{ax}t+\frac{1}{2}a_xt^2$

$$y=y_0+v_{ay}t+\frac{1}{2}a_yt^2$$

The motions in the x- and y-directions can be considered independently, according to one interpretation. To put it another way, motion in a plane (in two dimensions) can be divided into two parts. Distinct one-dimensional one-dimensional motions along two axes with constant acceleration Directions that are perpendicular to each other. This is a significant point as a result, and it is valuable for analysing object motion. Two-dimensionally the same is true for the other three dimensions.

5. **A particle starts from origin at t = 0 with a velocity** $5.0\hat{i}m/s$ **and move sin x-y plane under action of a force which produces a constant acceleration of** $(3.0\hat{i}+2.0\hat{j})m/s^2$**. (a) What is the y-coordinate of the particle at the instant its x-coordinate is 84 m ? (b) What is the speed of the particle at this time?**

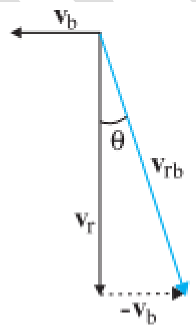
Solution: From Eq For $r_0 = 0$, the position of particle given by:

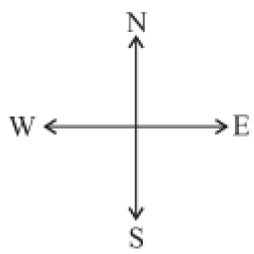
$$r(t)=v_0t+\frac{1}{2}at^2 = 5.0\,\hat{i}t+(1/2)\,(3.0\hat{i}+2.0\hat{j})t^2 = (5.0\,t+1.5t^2)\hat{i}+1.0\,t^2\,\hat{j}$$

Therefore: $x(t)=5.0t+1.5^2$

6. Rain is falling vertically with a speed of 35 m/ s. A woman rides a bicycle with a speed of $12m/s$ **in east to west direction. What is the direction in which she should hold her umbrella?**

Solution: v_r represents the speed of the rain, and v_b indicates the speed of the woman's bicycle. Both of these speeds are in relation to the ground. Because the woman is riding a bicycle, the rain is falling at a faster rate.





her is the velocity of rain relative to the velocity of the bicycle she is riding. That is $v_{rb} = v_r - v_b$ .It is given by:

$\tan\theta = \frac{v_b}{v_r} = \frac{12}{35} = 0.34 = \theta = 19^0$

As a result, the woman should hold her umbrella at a 19- degree angle to the vertical, facing west.

7. Galileo, in his book Two new sciences, stated that "for elevations which exceed or fall short of 45° by equal amounts, the ranges are equal". Prove this n statement.

Solution: For a projectile launched with velocity v_0 at an angle θ_0, the range is given by:

$$R = \frac{v_0 \sin 2\theta_0}{g}$$

Now, for angles, $(45^0 + \alpha)$ and $(45^0 - \alpha)$, 2θo is $(90^0 + 2\alpha)$ and $(90^0 - 2\alpha)$, respectively. The values of sin $(90^0 + 2\alpha)$ and sin $(90^0 - 2\alpha)$ are the same, equal to that of cos 2α .As a result, ranges are equivalent at altitudes that are equal amounts α above or below 45°.

8. A hiker stands on the edge of a cliff $490m$ above the ground and throws a stone horizontally with an initial speed of $15m/s$. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take g = $9.8ms^{-2}$)

Solution: At the edge of the cliff, we choose the origins of the x- and $y\,axis$, as well as $t = 0$s for the moment the stone is flung. Choose the positive x-axis direction to be parallel to the initial velocity requires the y-axis to be in the positive direction upward in a vertical direction The motion's x- and y-components can be considered separately. The motion equations are:

$$x(t) = x_o + v_{ox}\, t$$
$$y(t) = y_o + v_{oy}\, t + (1/2)\, a_y\, t^2$$

Here: $x_o = y_o = 0,\ v_{oy} = 0,\ a_y = -g = -9.8\ m\ s^{-2}$,
$v_{ox} = 15\ m\ s^{-1}$

The stone hits the ground where: $y(t) = -\ 490\ m.$
$-\ 490\ m = -(1/2)(9.8)\ t^2$.

This gives $t = 10\ s.$

The velocity components are: $v_x = v_{ox}$ *and*
$v_y = v_{oy} - g\ t$

So that when stone hits the ground: $v_{ox} = 15\ m\ s^{-1}$
$v_{oy} = 0 - 9.8 \times 10 = -\ 98\ m\ s^{-1}$

Therefore speed of stone is: $\sqrt{v_x{}^2 + v_y{}^2} = \sqrt{15^2 + 98^2} = 99m/s$

9. **A cricket ball is thrown at a speed of $28\ m\ s^{-1}$ in a direction 30^0 above the horizontal. Calculate (a) the maximum height**

Solution: The maximum height is given by

$$h_m = \frac{(v_0 \sin\theta_0)^2}{2g} = \frac{(28\sin 30^0)^2}{2(9.8)}m = \frac{14\times14}{2\times9.8} = 10m$$

(b) The time taken by the ball to return to the same level,

Solution: The time taken to return to the same level is

$$T_f = \frac{(2v_0 \sin\theta_0)}{g} = \frac{(2\times28\sin 30^0)^2}{(9.8)} = 2.9s$$

(c) The distance from the thrower to the point where the ball returns to the same level.

Solution: The distance from the thrower to the point where the ball returns to the same level is:

$$R = \frac{(v_0 \sin 2\theta_0)}{g} = \frac{(28\times28\sin 60^0)}{(9.8)} = 69m$$

10. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s.
(a) What is the angular speed, and the linear speed of the motion?
Solution: This is an example of uniform circular motion. Here $R = 12cm$. The angular speed ω is given by:

$$\omega = \frac{2\pi}{T} = 2\pi\times\frac{7}{100} = 0.44rad/s$$

$$v = \omega R = 0.44\times12 = 5.3cm/s$$

(b) Is the acceleration vector a constant vector? What is its magnitude?
Solution: At every point, the direction of velocity v is along the tangent to the circle. The speed is increasing in the direction of the circle's centre. Since this is a constantly changing direction. The acceleration is not a constant vector in this case. The magnitude of acceleration, on the other hand, is significant. Constant:

$$a = \omega^2 R = (0.44)^2\times12 = 2.3cm/s^2$$

(EXERCISES)

1. State, for each of the following physical quantities, if it is a scalar or a vector: Volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.
Solution: Scalar: Volume, mass, speed, density, number of moles, angular frequency

Vector: Acceleration, velocity, displacement, angular velocity

A scalar quantity is defined Solutionely by its magnitude. It doesn't have a directional component to it. Scalar physical quantities include volume, mass, speed, density, number of moles, and angular frequency.

The magnitude of a vector quantity, as well as the direction with which it is connected, are both given. This category includes acceleration, velocity, displacement, and angular velocity.

2. Pick out the two scalar quantities in the following list: Force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Solution: Work and current are scalar quantities. The dot product of force and displacement equals work done. Work is a scalar physical quantity since the dot product of two quantities is always a scalar.

Only the magnitude of a current may be described. Its direction isn't taken into consideration. As a result, it's a scalar quantity.

3. Pick out the only vector quantity in the following list: Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.
Solution: Impulse

The product of force and time determines the impulse. Because force is a vector quantity, it produces a vector amount when it is multiplied by time (a scalar quantity).

4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) Adding any two scalars,

Solution: Meaningful. Only when two scalar variables represent the same physical quantity can their addition make sense.

(b) Adding a scalar to a vector of the same dimensions,

Solution: Not meaningful, It is meaningless to add a vector quantity to a scalar quantity.

(c) Multiplying any vector by any scalar

Solution: Meaningful. A vector can be multiplied by a scalar. Force, for example, is multiplied by time to produce impulse.

(d) Multiplying any two scalars

Solution: Meaningful, A scalar can be multiplied with another scalar of the same or different dimensions, regardless of the physical quantity it represents.

(e) Adding any two vectors

Solution: Meaningful. Only when two vector variables represent the same physical quantity does their addition make sense.

(f) Adding a component of a vector to the same vector.

Solution: Meaningful. Because both vectors have the same dimensions, a component of one can be added to the other.

5. Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar,

Solution: True. A vector's magnitude is a number. As a result, it is a scalar.

(b) Each component of a vector is always a scalar

Solution: False. A vector's components are also vectors.

(c) The total path length is always equal to the magnitude of the displacement vector of a particle

Solution: False. Displacement is a vector quantity, whereas total path length is a scalar variable. As a result, the total path length is always greater than the displacement magnitude. Only when a particle moves in a straight line does it become equal to the amount of displacement.

(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time,

Solution: True. It's because the entire route length is always higher than or equal to the amount of a particle's displacement.

(**e) Three vectors not lying in a plane can never add up to give a null vector.**

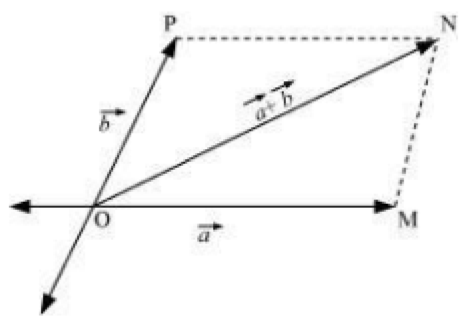
Solution: True. Three vectors that do not reside on a plane cannot be represented in the same order by the sides of a triangle.

*

6. Establish the following vector inequalities geometrically or otherwise:

a) $|a + b| \leq |a| + |b|$

Solution: Let two vectors $\vec{a}\, and\, \vec{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we can write:

$\left|\overrightarrow{OM}\right| = \left|\vec{a}\right|$ ….. (i)

$\left|\overrightarrow{MN}\right| = \left|\overrightarrow{OP}\right| = \left|\vec{b}\right|$ …… (ii)

$\left|\overrightarrow{ON}\right| = \left|\overrightarrow{a+b}\right|$ …… (iii)

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in $\triangle OMN$, we have : $ON < (OM + MN)$

$\left|\overrightarrow{a+b}\right| < \left|\vec{a}\right| + \left|\vec{b}\right|$ …… (iv)

If the two vectors $\vec{a}\ and\ \vec{b}$ act along a straight line in the same direction, then we can write:

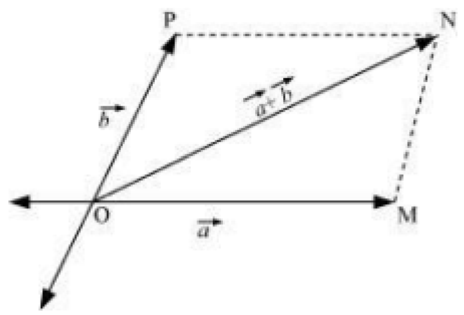
$\left|\overrightarrow{a+b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$ ….. (v)

Combining equations (iv) and (v), we get:

$\left|\overrightarrow{a+b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|$

b. $\left|\overrightarrow{a+b}\right| \geq \left|\vec{a}\right| + \left|\vec{b}\right|$

Solution: Let two vectors $\vec{a}\ and\ \vec{b}$ be represented by the adjacent sides of a parallelogram OMNP, as shown in the given figure.



Here, we have:

$\left|\overrightarrow{OM}\right| = \left|\vec{a}\right|$

$\left|\overrightarrow{MN}\right| = \left|\overrightarrow{OP}\right| = \left|\vec{b}\right|$

$\left|\overrightarrow{ON}\right| = \left|\overrightarrow{a+b}\right|$

In a triangle, each side is smaller than the sum of the other two sides. Therefore, in ΔOMN , we have:

$ON + MN > OM$

$ON + OM > MN \quad (OP = MN)$

$|\overrightarrow{ON}| > |\overrightarrow{OM} - \overrightarrow{OP}|$

$|\overrightarrow{a+b}| > ||a| - (b)|$ ……. (iv)

If the two vectors $\vec{a}\,and\,\vec{b}$ act along a straight line in the same direction, then we can write:

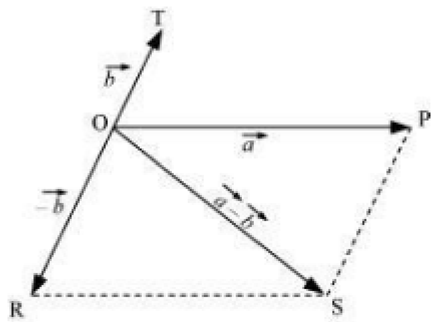
$|\overrightarrow{a+b}| = ||a| - (b)|$ …. (v)

Combining equations (iv) and (v), we get:

$|\overrightarrow{a+b}| \geq ||a| - (b)|$

c. $|\overrightarrow{a-b}| \leq ||a| + |b||$

Solution: Let two vectors $\vec{a}\,and\,\vec{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



Here we have:

$|\overrightarrow{OR}| = |\overrightarrow{PS}| = |\vec{b}|$

$|\overrightarrow{OP}| = |\vec{a}|$

In a triangle, each side is smaller than the sum of the other two sides. Therefore, in ΔOPS , we have:

$OP + PS > OS$

$|\overrightarrow{a-b}| < ||\vec{a}| + |\overrightarrow{-b}||$

$|\overrightarrow{a-b}| < ||\vec{a}| + |\vec{b}||$ …. (III)

If the two vectors act in a straight line but in opposite directions, then we can write:

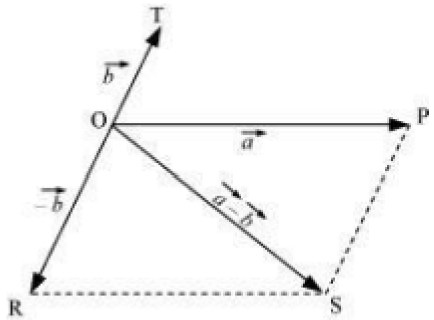
$\left|\overrightarrow{a-b}\right| = \left|\left|\vec{a}\right| + \left|\vec{b}\right|\right|$ …… (iv)

Combining equations (iii) and (iv), we get:

$\left|\overrightarrow{a-b}\right| \le \left|\left|\vec{a}\right| + \left|\vec{b}\right|\right|$

d. $\left|\overrightarrow{a-b}\right| \ge \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$

Solution: Let two vectors $\vec{a}\ and\ \vec{b}$ be represented by the adjacent sides of a parallelogram PORS, as shown in the given figure.



For the given parallelogram, the following relations may be written:

$$\begin{aligned} OS + PS &> OP \\ OS &> OP - PS \end{aligned} \quad \text{……. (i)}$$

$\left|\overrightarrow{a-b}\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$ ….. (iii)

On the LHS, the quantity is always positive, but on the RHS, it can be either positive or negative. We use modulus on both sides to make both values positive:

$\left|\left|\overrightarrow{a-b}\right|\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$

$\left|\overrightarrow{a-b}\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$ …….. (iv)

We can write if the two vectors act in a straight line but in opposing directions:

$\left|\overrightarrow{a-b}\right| = \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$ ……. (v)

Combining equations (iv) and (v), we get:

$\left|\overrightarrow{a-b}\right| \ge \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$

7. Given $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \mathbf{0}$, which of the following statements are correct:

a) a, b, c, and d must each be a null vector,

Solution: Incorrect. It is not necessary for all four supplied vectors to be null vectors in order to make $a+b+c+d=0$. There are other different combinations that can result in a zero sum.

b) The magnitude of ($\mathbf{a} + \mathbf{c}$) equals the magnitude of ($b+ \mathbf{d}$),

Solution: Correct. A + b + c + d = 0 a + c = – (b + d) a + c = – (b + d) Taking both sides' modulus, we get: | a + c | = | – (b + d)| = | b + d | As a result, the magnitudes of (a + c) and (b + d) are the same.

c) **The magnitude of a can never be greater than the sum of the magnitudes of b, c, and d.**

Solution: Correct. a + b + c + d = 0 a = (b + c + d) Taking modulus both sides, we get: | a | = | b + c + d |

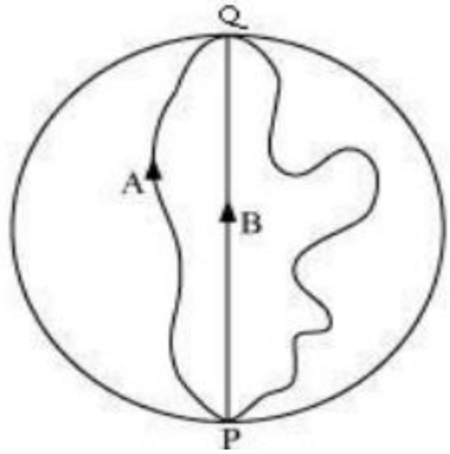
$|a| \le |a| + |b| + |c|$ …….. (i)

The magnitude of an is equal to or less than the total of the magnitudes of b, c, and d, as shown in equation I As a result, the magnitude of vector a can never exceed the sum of the magnitudes of vectors b, c, and d.

d) $\mathrm{c} + b$ Must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear?

Solution: Correct. For $\mathbf{a} + \mathbf{c} + b + \mathrm{d} = \mathbf{0}$. Only if ($\mathbf{c} + b$) lie in a plane including a and d can the resulting sum of the three vectors a, (b + c), and d be zero, assuming that these three vectors are represented by the three sides of a triangle. If a and d are collinear, the vector (b + c) must be in the same line as a and d. Only when this inference is true will the vector sum of all vectors be zero.

8. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.20. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of the path skated?



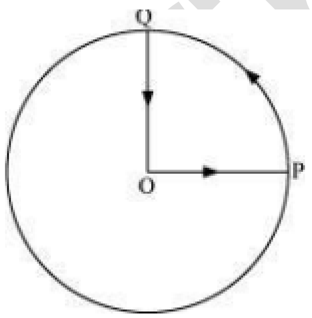
Solution: The smallest distance between a particle's initial and final coordinates determines displacement. In the example, all of the girls begin at point P and work their way to point Q. Their displacements will have the same magnitude as the diameter of the ground.

Radius of the ground = $200m$

Diameter of the ground = $2\times200=400m$

Displacement is determined by the shortest distance between a particle's initial and final coordinates. All the girls in the example start at point P and work their way to point Q. The magnitude of these displacements will be the same as the diameter of the ground.

9. A cyclist starts from the centre O of a circular park of radius $1km$, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along OQ as shown in Fig. 4.21. If the round trip takes $10\min$, what is the



(a) Net displacement,

Solution: The minimum distance between a body's initial and final positions determines displacement. After 10 minutes of cycling, the biker returns to the starting position in this scenario. As a result, he has no net displacement.

(b) Average velocity,

Solution: Average velocity is given by the relation:

Average velocity= $\dfrac{Net\ displacement}{Total\ time}$

speed= $\dfrac{Total\ path\,length}{Total\,time}$

Total path Because the cyclist's net displacement is zero, his average velocity is also zero.

(b) Average speed of the cyclist?

Solution: Average speed of the cyclist is given by the relation:

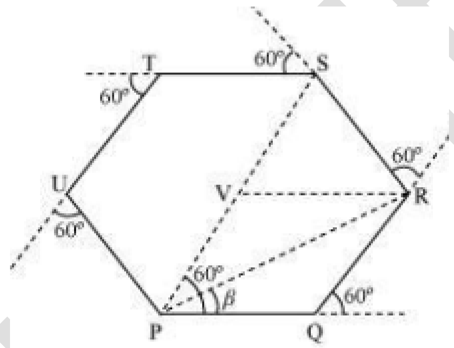
Average length=
$$OP+PQ+QO=1+\frac{1}{4}(2\pi\times 1)+1$$
$$=2+\frac{1}{2}\pi=3.570km$$

Time taken = $10\min=\dfrac{10}{60}=\dfrac{1}{6}h$

Average speed $=\dfrac{3.570}{\frac{1}{6}}=21.42km/hr$

10. On an open ground, a motorist follows a track that turns to his left by an angle of 60^0 after every $500\,\text{m}$. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Solution: The motorist's path is a regular hexagon with a 500 -meter side, as indicated in the diagram.



Let the motorist start from point P.

The motorist takes the third turn at S.

Magnitude of displacement $=PS=PV+VS=500+500=1000m$

Total path length $=PQ+QR+RS=500+500+500=1500m$

The motorist takes the sixth turn at point P, which is the starting point.

Magnitude of displacement = 0

Total path length
$$=PQ+QR+RS+ST+TU+UP=500+500+500+500+500+500=3000m$$

The motorist takes the eight turn at point R .Magnitude of displacement = PR

$$=\sqrt{PQ^2+QR^2+2.PQ.QR\cos 60^\circ}$$

$$=\sqrt{250000+250000+(500000\times\frac{1}{2})}$$

$$=866.03m$$

$$\beta=\tan^{-1}(\frac{500\sin 60^\circ}{500+500\cos 60^\circ})=30^0$$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30^0 with PR.

Total path length = Circumference of the hexagon $+ PQ + QR$

$$= 6\times 500+500+500=4000\ m$$

The magnitude of displacement and the total path length corresponding to the required

Turns is shown in the given table:

Turn	**Magnitude of displacement (m)**	**Total path length (m)**
Third	1000	1500
Sixth	0	3000
Eighth	$866.03; 30^\circ$	4000

11. A passenger arriving in a new town wishes to go from the station to a hotel located $10km$ away on a straight road from the station. A dishonest cabman takes him along a circuitous path $23km$ long and reaches the hotel in $28\min$. What is

(a) The average speed of the taxi,

Solution: Total distance travelled = $23km$

Total time taken = $28\min=\frac{28}{60}h$

Average speed of the taxi = $\frac{Total\ distance\ travelled}{Total\ time} = \frac{23}{\frac{28}{60}} = 49.29 km/h$

(a) The magnitude of average velocity? Are the two equal?

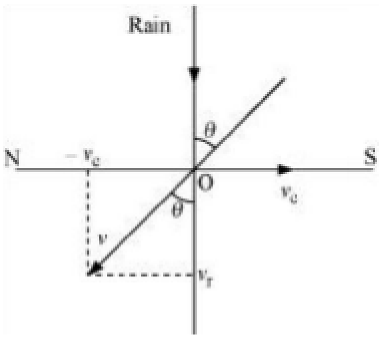
Solution: Distance between the hotel and the station = $10km$ = Displacement of the car

Average velocity y $= \frac{10}{\frac{28}{60}} = 21.43 km/h$

As a result, the two physical values (average velocity and average speed) are not equal.

12. Rain is falling vertically with a speed of $30\, m/s^{-1}$. A woman rides a bicycle with a speed of $10\, m/s^{-1}$ in the north to south direction. What is the direction in which she should hold her umbrella?

Solution: The described situation is shown in the given figure:



$$Here,\ v_c = Velocity\ of\ the\ cyclist$$
$$v_r = Velocity\ of\ falling\ rain$$

The woman must hold her umbrella in the direction of the rain's relative velocity (v) with respect to her in order to protect herself from the rain.

$$v = v_r + (-v_c)$$
$$= 30 + (-10) = 20 m/s$$
$$\tan\theta = \frac{v_c}{v_r} = \frac{10}{30}$$
$$\theta = \tan^{-1}(\frac{1}{3})$$
$$= \tan^{-1}(0.333) = 18^0$$

As a result, the woman must hold the umbrella roughly 18^0 to the south of the vertical.

13. A man can swim with a speed of $4.0\ \mathrm{km/h}$ in still water. How long does he take to cross a river $1.0 km$ wide if the river flows steadily at $3.0\ \mathrm{km/h}$ and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Solution: Speed of the man, $v_m = 4km/hr$

Width of the river = $1km$

Time taken to cross the river $= \dfrac{Width\ of\ the\ river}{Speed\ of\ the\ river} = \dfrac{1}{4}h = \dfrac{1}{4} \times 60 = 15\min$

Speed of the river, $v_r = 3km/hr$

Distance covered with flow of the river = $v_r \times t$

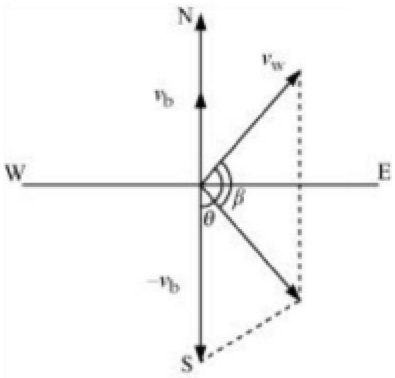
$$= 3 \times \frac{1}{4} = \frac{3}{4}km$$
$$= 750m$$

14. In a harbour, wind is blowing at the speed of $72\ \mathrm{km/h}$ and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of $51\mathrm{km/h}$ to the north, what is the direction of the flag on the mast of the boat?

Solution: Velocity of boat, $v_b = 51\ km/h$

Velocity of wind, $v_w = 72\ km/h$

The flag is flapping in the direction of the north-east. It indicates that the wind is blowing from the north-east. When the ship starts sailing north, the flag will follow the direction of the wind's relative velocity (v_{wb}) in relation to the boat.



The angle between v_w and $(-v_b) = 90° + 45°$

$$\tan\beta = \frac{51\sin(90+45)}{72+51\cos(90+45)}$$

$$= \frac{51\sin(90+45)}{72+51(-\cos 45)} = \frac{51\times\frac{1}{\sqrt{2}}}{72-51\times\frac{1}{\sqrt{2}}}$$

$$= \frac{51}{72\times1.414-51}$$

$$\beta = \tan^{-1}(1.0038) = 45.11^0$$

Angle in relation to the east = $45.11° - 45° = 0.11°$ As a result, the flag will float nearly due east.

15. The ceiling of a long hall is $25m$ high. What is the maximum horizontal distance that a ball thrown with a speed of $40\,\text{m}/s^{-1}$ can go without hitting the ceiling of the hall?

Solution: Speed of the ball, $u = 40\ m/s$

Maximum height, $h = 25\ m$

The highest height achieved by a body propelled at an angle in projectile motion is defined by the relation:

$$h = \frac{u^2\sin^2\theta}{2g}$$

$$25 = \frac{(40)^2\sin^2\theta}{2\times9.8}$$

$$sin^2\ \theta = 0.30625$$

$$sin\ \theta = 0.5534$$

$$\theta = sin^{-1}\left(0.5534\right) = 33.60°$$

$$R = \frac{u^2\sin 2\theta}{g}$$

Horizontal range, $= \dfrac{(40)^2\times\sin 2\times33.60}{9.8}$

$$= \frac{1600\times0.922}{9.8} = 150.53m$$

16. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Solution: Maximum horizontal distance, $R = 100\ m$

The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is $45°$, *i.e.*, $\theta = 45°$.

The horizontal range for a projection velocity v, is given by the relation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$100 = \frac{u^2}{g} \sin 90^\circ \; \ldots\ldots \text{(i)}$$

$$\frac{u^2}{g} = 100$$

When the ball is hurled vertically upward, it will reach its maximum height. At the maximum height H, the final velocity v is 0 for such motion.

Acceleration, $a = -g$

$$v^2 - u^2 = -2gH$$

$$H = \frac{1}{2} \times \frac{u^2}{g} = \frac{1}{2} \times 100 = 50m$$

*

17. A stone tied to the end of a string $80cm$ long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in $25s$, what is the magnitude and direction of acceleration of the stone?

Solution; Length of the string, $l = 0.8m$

Number of revolutions $= 14$

Time taken = $25s$

Frequency, $v = \frac{Number\ of\ revolutions}{time\ taken} = \frac{14}{25} hz$

Angular frequency, $\omega = 2\pi v = 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} rad\ s^{-1}$

Centripetal acceleration,

$$a_c = \omega^2 r = (\frac{88}{25})^2 \times 0.8 = 9.91 m/s^2$$

At all times, the direction of centripetal acceleration is always along the string, toward the centre.

18. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of $900 \mathbf{km/h}$ Compare its centripetal acceleration with the acceleration due to gravity.

Solution: Radius of the loop: $r = 1\,km = 1000\,m$

Speed of aircraft , $v = 900\ km/h$

$$= 900 \times \frac{5}{18} = 250 m/s$$

Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{(250)^2}{1000} = 62.5 m/s^2$

Acceleration due to gravity, $g = 9.8\ m/s2$

$$\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38$$

$$a_c = 6.38g$$

19. Read each statement below carefully and state, with reasons, if it is true or false:

a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

Solution: False, in circular motion, a particle's net acceleration is not always directed toward the centre along the radius of the circle. Only in the situation of uniform circular motion does this happen.

b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

Solution: True, A particle appears to move tangentially to the circular path at a location on the path. As a result, the particle's velocity vector is always parallel to the tangent at a given position.

c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Solution: True, The direction of the acceleration vector in uniform circular motion (UCM) is toward the circle's centre. It does, however, vary over time. A null vector is the average of these vectors over one cycle.

20. The position of a particle is given by $r = 3.0t\,\hat{i} - 2.0t\,\hat{j} + 4.0\,\hat{k}$

Where t is in seconds and the coefficients have the proper units for r to be in metres.

a)**Find the v and a of the particle?**
Solution: The position of the particle is given by:

$$r = 3.0t\,\hat{i} - 2.0t\,\hat{j} + 4.0\,\hat{k}$$

Velocity $\vec{v}$, of the particle is given as:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0t\,\hat{i} - 2.0t\,\hat{j} + 4.0\,\hat{k})$$
$$\vec{v} = 3.0t\,\hat{i} - 4.0t\,\hat{j}$$

Acceleration $\vec{a}$, of the particle is given as:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.0t\,\hat{i} - 4.0t\,\hat{j})$$
$$\vec{a} = -4.0t\,\hat{j}$$

$8.54\ m/s,\ 69.45^\circ$ below the x-axis

b) What is the magnitude and direction of velocity of the particle at $t = 2.0s$?

Solution: The magnitude of velocity is given by:

$$|\vec{v}| = \sqrt{3^2 + (-8)^2} = \sqrt{73} = 8.54m/s$$

$$Direction, \theta = \tan^{-1}(\frac{v_y}{v_x})$$

$$= \tan^{-1}(\frac{-8}{3}) = \tan^{-1}(2.667) = -69.45^0$$

The negative sign indicates that the direction of velocity is below the x-axis.

21. A particle starts from the origin at $t = 0s$ with a velocity of $10\hat{j}$ m/ s and moves in the x-y plane with a constant acceleration of $(\mathbf{8.0}\hat{i} + \mathbf{2.0}\hat{j})ms^{-2}$

a) At what time is the x-coordinate of the particle $16m$? What is the y-coordinate of the particle at that time?

Solution: Velocity of the particle = $10\vec{j}m/s$

Acceleration of the particle = $(8.0\vec{i} + 2.0\vec{j})ms^{-2}$

Also,

$$\vec{a} = \frac{d\hat{v}}{dt} = 8.0\hat{i} + 2.0\,\hat{j}$$
$$d\vec{v} = (8.0\hat{i} + 2.0\hat{j})dt$$

Integrating both sides:

$$\vec{v}(t) = 8.0t\,\hat{i} + 2.0t\,\hat{j} + \vec{u}$$

Where,

$\vec{u}$ = Velocity vector of the particle at t = 0

$\vec{v}$ = Velocity vector of the particle at time t

But, $v = \frac{dr}{dt} \quad dr = vdt = (8.0t\,\hat{i} + 2.0t\,\hat{j} + \vec{u})dt$

Integrating the equations with the conditions: $at\ t = 0;\ r = 0\ and\ at\ t = t;\ r = r$

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\hat{i} + \frac{1}{2}\times 2.0t^2\,\hat{j}$$
$$= \vec{u}t + 4.0t^2\hat{i} + t^2\,\hat{j}$$
$$= (10.0\,\hat{j})t + 4.0t^2\hat{i} + t^2\,\hat{j}$$
$$x\hat{i} + y\hat{j} = 4.0t^2\hat{i} + (10t + t^2)\,\hat{j}$$

b) What is the speed of the particle at the time?

Solution: Velocity of the particle is given by:

$$\vec{v}(t) = 8.0\times 2\hat{i} + 2.0\times 2\,\hat{j} + 10\,\hat{j}$$
$$= 16\hat{i} + 14\,\hat{j}$$
$$Speed\ \ of\ \ the\ \ particle:$$
$$|\vec{v}| = \sqrt{(16)^2 + (14)^2} = \sqrt{452} = 21.26m/s$$

23. For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = (\frac{1}{2})(v(t_1) + v(t_2))$

Solution: False: It is assumed that the particle's motion is random. As a result, this equation cannot calculate the particle's average velocity.

(b) $v_{average} = \frac{[r(t_1) - r(t_2)]}{(t_2 - t_1)}$

Solution: True: This equation can be used to depict the particle's arbitrary motion.

(c) $v(t) = v(0) + at$

Solution: False: The particle's motion is completely random. The particle's acceleration may also be non-uniform. As a result, this equation cannot reflect the particle's velocity in space.

(d) $r(t) = r(0) + v(0)t + (\frac{1}{2})at^2$

Solution: False: The particle's velocity is arbitrary, and its acceleration may be non-uniform. As a result, this equation cannot capture particle motion in space.

(e) $a_{average} = \frac{[v(t_2) - v(t_1)]}{(t_2 - t_1)}$

Solution: True: The arbitrary motion of the particle can be represented by this equation.

24. Read each statement below carefully and state, with reasons and examples, if it is true or false:

A scalar quantity is one that

a) is conserved in a process

Solution: False: Energy is not preserved in inelastic collisions, despite being a scalar quantity.

b) can never take negative values

Solution: False. Temperature, although being a scalar measure, can have negative values.

c) must be dimensionless

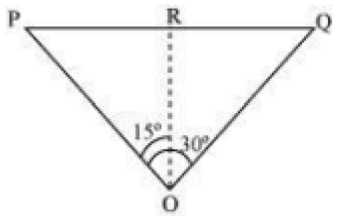
Solution: False: The whole length of a path is a scalar quantity. It does, however, have a length dimension.

d) has the same value for observers with different orientations of axes

Solution: True: For observers with varied axes orientations, the value of a scalar does not change.

25. An aircraft is flying at a height of $3400m$ above the ground. If the angle subtended at a ground observation point by the aircraft positions $10.0s$ apart is 30^0, what is the speed of the aircraft?

Solution: The positions of the observer and the aircraft are shown in the given figure.



Height of the aircraft from ground: $OR = 3400M$

Angle subtended between the positions, $\angle POQ = 30^0$

Time = $10s$

$In \triangle PRO$:

$\tan 15^\circ = \dfrac{PR}{OR}$,

$PR = OR \tan 15^0 = 3400 \times \tan 15^0$

$\triangle PRO$

Is similar to $\triangle RQO.$

$PR = RQ$

$PQ = PR + RQ$

$= 2PR = 2 \times 3400 \tan 15^\circ$

Speed of aircraft is: $\dfrac{1822.4}{10} = 12.24 m/s$

26. A vector has magnitude and direction. Does it have a location in space? Can it vary wit time? Will two equal vectors a and b at different locations in space necessarily havem identical physical effects? Give examples in support of your answer.

Solution: No; Yes; No

A vector, in general, has no distinct spatial places. This is because when a vector is relocated in such a way that its magnitude and direction remain the same, it remains invariant. A position vector, on the other hand, has a specific place in space. A vector's value can change over time. The displacement vector of a moving particle, for example. at a specific rate that changes over time Two equal vectors at separate points in space do not have to generate the same result. effect on the body Two equal forces acting on an object at different places, for example, can force the body to rotate, yet they cannot provide an equal turning effect when used together.

27. A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector?

Solution: NO

A physical quantity with both magnitude and direction does not necessarily have to be regarded a vector. Current, for example, is a scalar quantity despite its magnitude and direction. The law of vector addition is the most important prerequisite for a physical quantity to be termed a vector. In general, a body's rotation around an axis is not a vector quantity since it does not obey the law of vector addition. A rotation of a tiny angle, on the other hand, obeys the law of vector addition and is thus called a vector.

28. Can you associate vectors with

(a) The length of a wire bent into a loop,

Solution: No, A vector cannot be linked to the length of a wire bent into a loop.

(b) a plane area,

Solution: Yes, A plane area can be linked to an area vector. This vector's direction is normal to the plane area, either inward or outward.

(c) A sphere

Solution: A vector cannot be linked to the volume of a sphere. A sphere's area, on the other hand, can be related with an area vector.

29. A bullet fired at an angle of 30^0 with the horizontal hits the ground $3.0km$ away. By adjusting its angle of projection, can one hope to hit a target $5.0km$ m away? Assume the muzzle speed to the fixed, and neglect air resistance.

Solution: No

Range $R = 3km$

Angle of projection, $\theta = 30^\circ$

Acceleration due to gravity, $g = 9.8m/s^2$

Horizontal range for the projection velocity u_o is given by the relation:

$$R = \frac{u_0{}^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0{}^2}{g}\sin 60^\circ \text{ ....(i)}$$

$$\frac{u_0{}^2}{g} = 2\sqrt{3}$$

The maximum range ($R_{\max}$) is achieved by the bullet when it is fired at an angle of 45° with the horizontal, that is,

$$R_{\max} = \frac{u_0{}^2}{g} \text{ .... (ii)}$$

On comparing equations (i) and (ii), we get:

$$R_{\max} = 3\sqrt{3} = 2\times 1.732 = 3.46km$$

Hence, the bullet will not hit a target 5 km away.

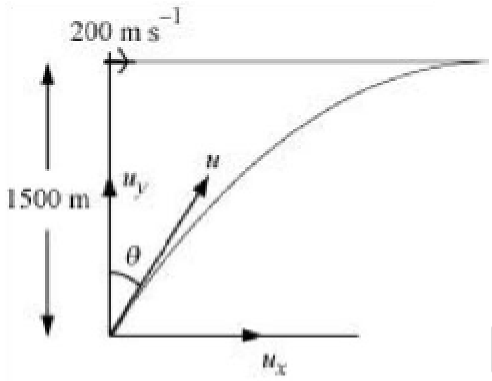
30. A fighter plane flying horizontally at an altitude of $1.5km$ with speed $720km/hr$ passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed $600\,m/s^{-1}$ to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 m)

Solution: Height of the fighter plane $=1.5km=1500m$

Speed of the fighter plane, $v=720km/hr=200m/s$

Let θ be the angle with the vertical so that the shell hits the plane. The situation is shown

in the given figure.



Muzzle velocity of the gun, $u=600m/s$

Time taken by the shell to hit the plane = t

Horizontal distance travelled by the shell = $u_x t$

Distance travelled by the plane = vt

The shell hits the plane. Hence, these two distances must be equal.

$u_x t \;= vt$

$u\sin\theta = v \quad \sin\theta \frac{v}{u} = \frac{200}{600} = 0.33$

$\theta = \sin^{-1}(0,33) = 19.5^0$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (H) higher than the maximum height achieved by the shell.

$$H = \frac{u^2\sin^2(90-\theta)}{2g} = \frac{(600)^2\cos^2\theta}{2g} = 18000\times(0.943)^2 = 160006.482km$$

31. A cyclist is riding with a speed of $27 km/hr$. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of $0.50 \mathrm{m}/s$ every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

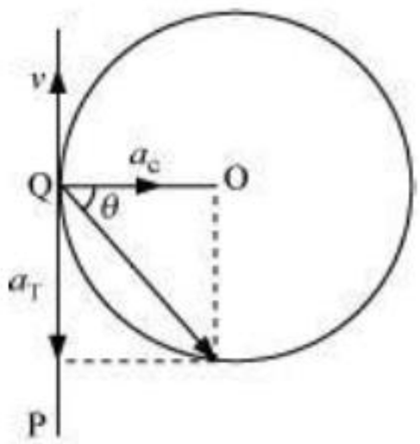
Solution: Speed of the cyclist, $v = 27 km/hr = 7.5 m/s$

Radius of the circular turn, $r = 80m$

Centripetal acceleration is given as:

$$a_c = \frac{v^2}{r} = \frac{(7.5)^2}{80} = 0.7 m/s^2$$

The situation is shown in the given figure:



Assume the cyclist starts from point P and rides toward point Q. At point Q, he uses the brakes and reduces the bicycle's speed by $0.5 m/s^2$. This acceleration is parallel to the tangent at Q and in the opposite direction of the object's motion.

Since the angle between a_c and a_T is 90°, the resultant acceleration a is given by:

$$a = \sqrt{a_c^{\ 2} + a_T^{\ 2}} = \sqrt{(0.7)^2 + (0.5)^2} = 0.86 m/s^2$$

$$\tan\theta = \frac{a_c}{a_T}$$

Where θ is the angle of the resultant with direction of velocity

$$\tan\theta = \frac{0.7}{0.5}$$

$$\theta = \tan^{-1}(1.4) = 54.46^0$$

32. Show that for a projectile the angle between the velocity and the *x*-axis as a function of

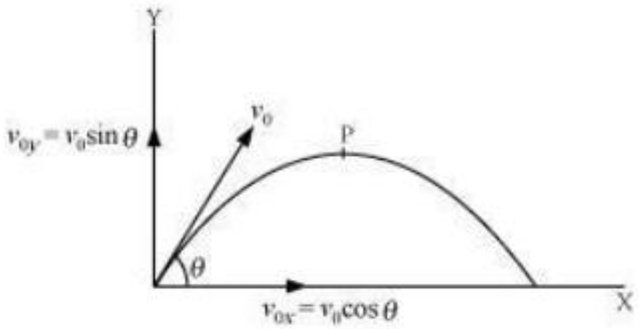
time is given by $\theta(t) = \tan^{-1}(\frac{v_{0_y} - gt}{v_{0x}})$

Show that the projection angle θ_0 **for a projectile launched from the origin is given by**

$$\theta_0 = \tan^{-1}(\frac{4h_m}{R})$$

Where the symbols have their usual meaning.

Solution: Let v_{0x} and v_{0_y} respectively be the initial components of the velocity of the projectile along horizontal (x) and vertical (y) directions. Let v_x *and* v_y respectively be the horizontal and vertical components of velocity at a point P.



Time taken by the projectile to reach point P = t

Applying the first equation of motion along the vertical and horizontal directions, we get:

$$v_x \ and \ v_y = v_{0_y} = gt$$

$$And \ v_x = v_{0x}$$

Maximum vertical height, $h_m = \frac{u_0{}^2 \sin^2\theta}{2g}$ …..(i)

Horizontal range, $R = \frac{u_0{}^2 \sin^2\theta}{g}$ …… (ii)

Solutionving equations (i) and (ii), we get:

$$-\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin^2\theta} = \frac{\sin\theta \times \sin\theta}{2 \times 2\sin\theta\cos\theta} = \frac{1\sin\theta}{4\sin\theta} = \frac{1}{4}\tan\theta$$

$$\tan\theta = (\frac{4h_m}{R}) \quad \theta = \tan^{-1}(\frac{4h_m}{R})$$