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Chapter 5: Laws of Motion

Examples

Example 5.1 An astronaut accidentally gets separated out of his small spaceship accelerating in inter stellar space at a. constant rate of $100\,\mathrm{m\,s^{-2}}$. What is the acceleration of the astronaut the instant after he is outside the spaceship? (Assume that there are no nearby stars to exert gravitational force on him.)

Solution:
Because there are no nearby stars exerting gravitational pull on him and the little spaceship exerts min or gravitational attraction on him, the net force acting on the astronaut once he exits the vessel is zero. The astronaut's acceleration is zero, i.e. Newton's first rule of motion.

Example 5.2 A bullet of mass $0.04\,\mathrm{kg}$ moving with a speed of $90\,\mathrm{m\,s^{-1}}$ enters a heavy wooden block and is stopped after a distance of $60\,\mathrm{cm}$. What is the average resistive force exerted by the block on the bullet?

Solution: The retardation $'a'$ of the bullet (assumed constant) is given by

$$a=\frac{-u^2}{2s}=\frac{-90\times 90}{2\times 0.6}\,\mathrm{m\,s^{-2}}=-6750\,\mathrm{m\,s^{-2}}$$

The retarding force, by the second law of motion, is

$$=0.04\,\mathrm{kg}\times 6750\,\mathrm{m\,s^{-2}}=270\,\mathrm{N}$$

The bullet's real resistive force, and hence its retardation, may not be uniform. As a result, the respons e simply reveals the average resistive force.

Example 5.3 The motion of a particle of mass m is described by $y=ut+\frac{1}{2}gt^2$. Find the force acting on the particle.

Solution: It is known that,

$$y=ut+\frac{1}{2}gt^2$$

Now,

$$v=\frac{\mathrm{d}y}{\mathrm{d}t}=u+gt$$

acceleration, $a=\frac{\mathrm{d}v}{\mathrm{d}t}=g$

Then the force is given by Eq. (5.5)

$F = ma = mg$

As a result, the above equation explains the motion of a particle subjected to gravitational acceleration ; with y representing the position coordinate in the direction of g.

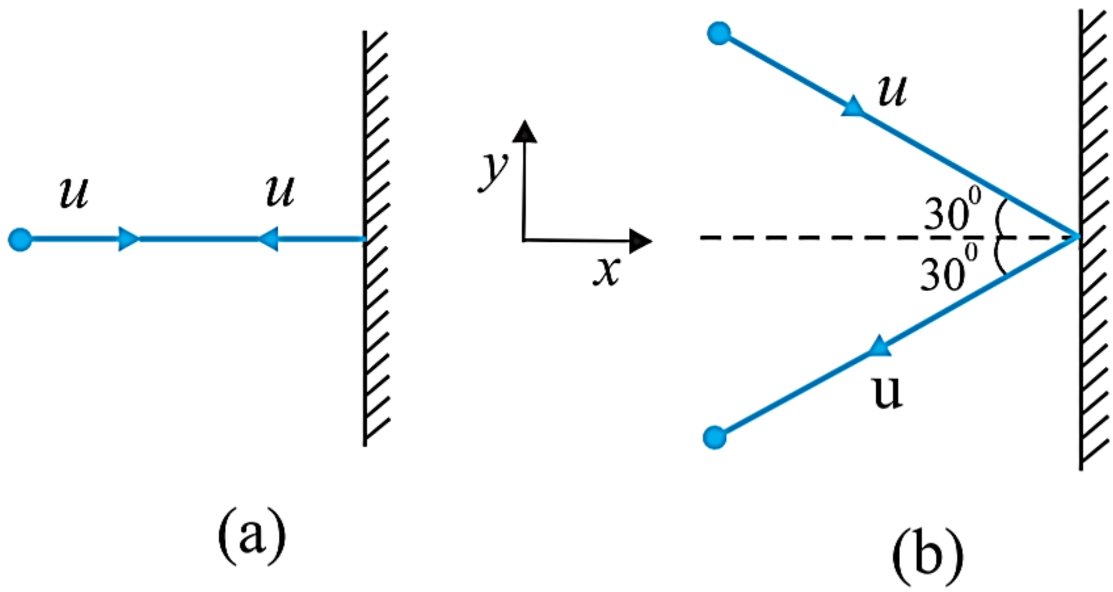
Example 5.4 A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of $12\,\mathrm{m\,s^{-1}}$. If the mass of the ball is $0.15\,\mathrm{kg}$, determine the impulse imparted to the ball. (Assume linear motion of the ball)

Solution: Change in momentum

$$= 0.15 \times 12 - (-0.15 \times 12)$$
$$= 3.6\,\mathrm{N\,s}$$

Impulse $= 3.6\,\mathrm{N\,s}$ when the ball is in linear motion.

Example 5.5 Two identical billiard balls strike a rigid wall with the same speed but at different angles, and get reflected without any change in speed, as shown in figure.



What is

(i) The direction of the force on the wall due to each ball?

Solution: It might be that the force on the wall in case (a) is normal to the wall, while that in case (b) is inclined at 30° to the normal. This Solution is wrong. The force n the wall is normal to the wall in both cases.

By using the second law, and then use the third law

Let u be the speed f each ball before and after collision with the wall, and m the mass of each ball.

Choose the x and y axes as shown in the figure, and consider he change in momentum of the ball in each case:

case (a),

$$\left(p_x\right)_{\text{initial}}=mu\ \left(p_y\right)_{\text{initial}}=0\left(p_x\right)_{\text{final}}=-mu\ \left(p_y\right)_{\text{final}}=0$$

Impulse is the change in momentum vector.

Therefore, $x-$ component of impulse $=-2\mathrm{mu}$

SO, the force on the ball due to the wall is normal to the wall, along the negative -direction. Using Newton's third law of motion, the force on the wall due to the ball is normal to the wall along the positive $\mathrm{x}-$direction.

Because the little time required for the collision has not been indicated in the problem, the quantity of force cannot be determined.

Case (b)

$$\left(p_x\right)_{\text{intul}}=mu\cos 30^{\circ},\left(p_y\right)_{\text{mitital}}=-mu\sin 30^{-}\left(p_x\right)_{\text{find}}=-mu\cos 30^{\circ},\left(p_y\right)_{\text{frud}}=-mu\sin 30^{\prime}$$

Note, while p_x changes sign after collision, p_y does not. Therefore,

$x-$component of impulse $=-2\,\mathrm{m}u\cos 30^{\circ}$ $y-$component of impulse $=0$

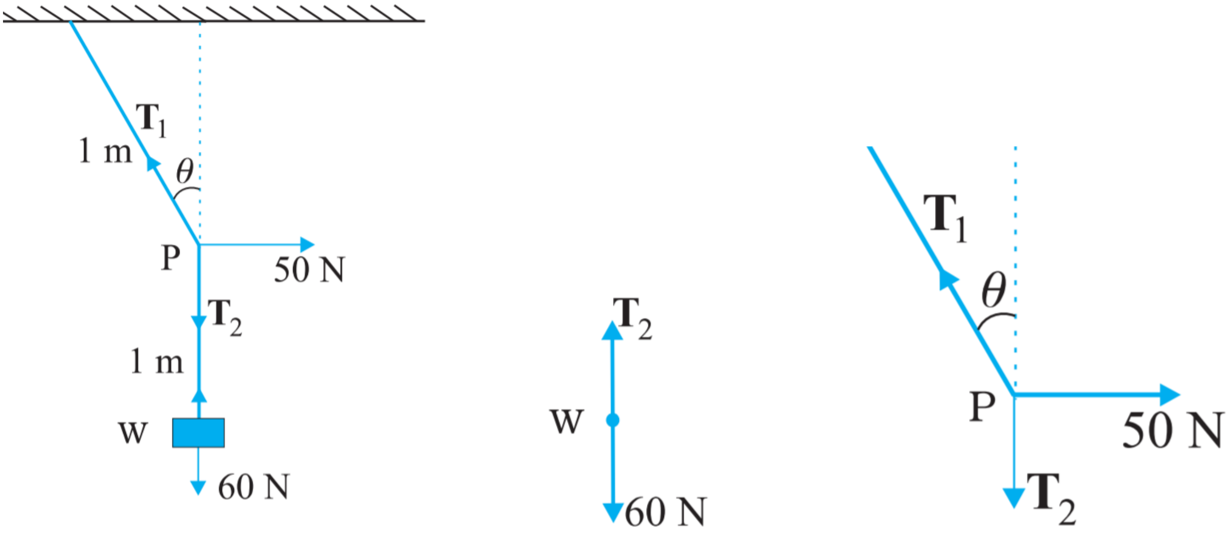
The direction of impulse (and force) is the same as in (a) and is normal to the wall along the negative $x-$direction. As before, using Newton's third law, the force on the wall due to the ball is normal to the wall along the positive $x-$direction.

(ii) The ratio of the magnitudes of impulses imparted to the balls by the wall ?

Solution: The ratio of the magnitudes of the impulses imparted to the balls in (a) and (b) is

$$2mu/\left(2mu\cos 30^{\circ}\right)=\frac{2}{\sqrt{3}}=1.2$$

Example 5.6 A mass of $6\,\mathrm{kg}$ is suspended by a rope of length $2\,\mathrm{m}$ from the ceiling. A force of $50\,\mathrm{N}$ in the horizontal direction is applied at the midpoint P of the rope, as shown. What is the angle the rope makes with the vertical in equilibrium? (Take $g=10\,\mathrm{m\,s^{-2}}$). Neglect the mass of the rope.



(a) (b) (c)

Solution: Figures b and c are known as free-body diagrams. Figure b is the free-body diagram of W and Fig. c is the free-body diagram of point P .

Consider the equilibrium of the weight W.

$$Clearly, T_2 = 6 \times 10 = 60\,\text{N}$$

Consider the equilibrium of the point P under the action of three forces - the tensions T_1 and T_2 , and the horizontal force $50\,\text{N}$.
The resulting force's horizontal and vertical components must each disappear separately:

$$T_1 \cos\theta = T_2 = 60\,\text{N}$$
$$T_1 \sin\theta = 50\,\text{N}$$

which gives that

$$\tan\theta = \frac{5}{6} \text{ or } \theta = \tan^{-1}\left(\frac{5}{6}\right) = 40^\circ$$

It's worth noting that the solution is independent of the rope's length (which is assumed to be massless) or the point at which the horizontal force is applied.

Example 5.7 Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary. given that the co-efficient of static friction between the box and the train's floor is 0.15.

Solution: Because the box's acceleration is caused by static friction,

$ma = f_s \le \mu_s$

$N = \mu_s mg$

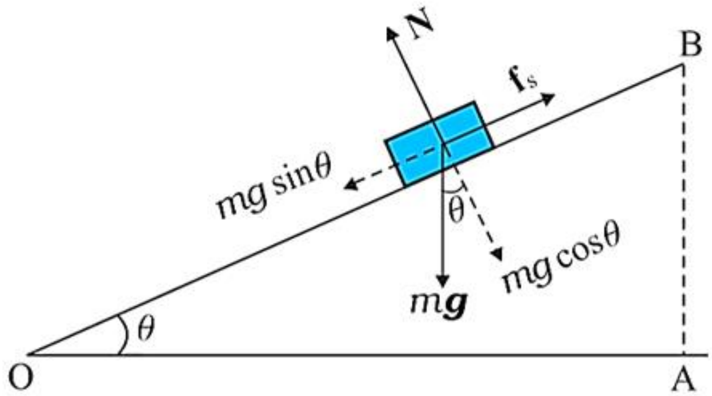
Let, $a \le \mu_s g$

$\therefore a_{\max} = \mu_s g$

$= 0.15 \times 10\, \text{m s}^{-2}$

$= 1.5\, \text{m s}^{-2}$

Example 5.8 A mass of $4\,\text{kg}$ rests on a horizontal plane. The plane is gradually inclined until at an angle $\theta = 15^\circ$ with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface?



Solution: The forces acting on a block of mass m at rest on an inclined plane are:

(i) the weight mg acting vertically downwards

(ii) the normal force N of the plane on the block, and

(iii) the static frictional force f_s opposing the impending motion.

The resultant of these forces must be zero in equilibrium. We can find the weight mg by resolving it i n the two ways illustrated.

$mg \sin\theta = f_s,$

$mg \cos\theta = N$

As θ increases, the self-adjusting frictional force, f_s increases until at $\theta = \theta_{\max}$ f_s achieves its maximum value, $(f_s)_{\max} = \mu_s N.$

Therefore,

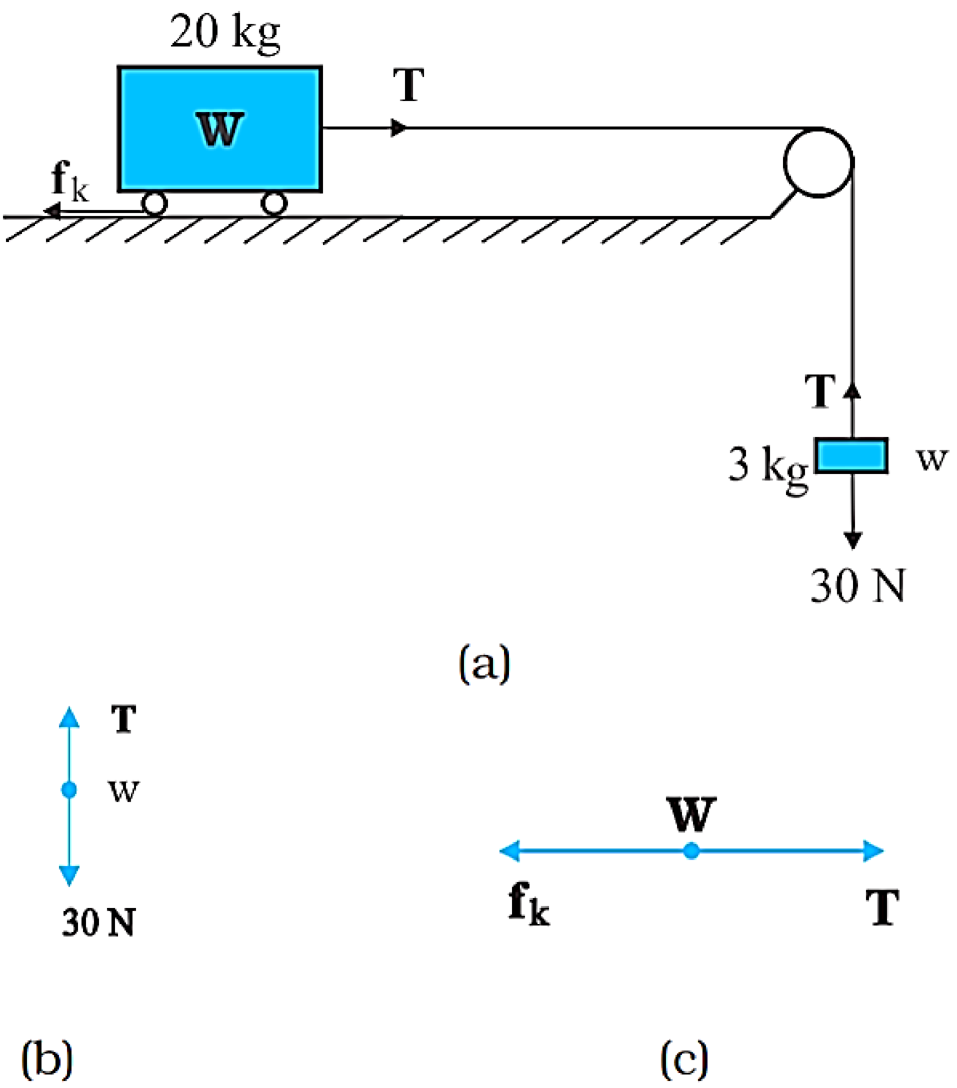
$\tan\theta_{\max} = \mu_s$ or $\theta_{\max} = \tan^{-1} \mu_s$

When θ becomes just a little more than $\theta_{\max}$.

There is a small net force on the block and it begins to slide. Note that $\theta_{\max}$ depends only on μ_s and is independent of the mass of the block.

$$For\theta_{\max} = 15^{\circ},$$
$$\mu_s = \tan 15^{\circ}$$
$$= 0.27$$

Example 5.9 What is the acceleration of the block and trolley system shown in a fig. a, if the coefficient of kinetic friction between the trolley and the surface is 0.04 ? What is the tension in the string? (Take $g = 10\,\mathrm{m\,s}^2$). Neglect the mass of the string.



Solution: As the string is inextensible, and the pully is smooth, the $3\,\mathrm{kg}$ block and the $20\,\mathrm{kg}$ trolley both have same magnitude of acceleration. According to the second law to motion of the block,

$$30 - T = 3a$$

According to the second law to motion

Now,

$f_k = \mu_k N,$
Here , $\mu_k = 0.04,$
$N = 20 \times 10N = 200\,\text{N}.$

Thus the equation for the motion of the trolley is $T - 0.04 \times 200 = 20aOrT - 8 = 20a$

These equations give,

$$a = \frac{22}{23}\text{ms}^{-2} = 0.96\,\text{ms}^{-2} and$$
$$T = 27.1\text{N}.$$

Example 5.10 A cyclist speeding at $18\,\text{km/h}$ on a level road takes a sharp circular turn of radius 3m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

Solution:
Frictional force alone can supply the centripetal force required to keep a bike going in a circular circle without skidding on an unbanked road. The frictional force is insufficient to deliver the requisite cent ripetal force if the cyclist's speed is too high, or if the turn is too acute (i.e. of too short a radius), or bo th.

The condition for the cyclist not to slip is given by

$$v^2 \le \mu_s Rg$$

Now,

$R = 3\text{m}, g = 9.8\,\text{ms}^2, \mu_s = 0.1.$
$\mu_s Rg = 2.94\,\text{m}^2\,\text{s}^{-2}$
$v = 18\,\text{km/h} = 5\,\text{ms}^{-1};$
$v^2 = 25\,\text{m}^2\,\text{s}^2.$

The condition is disregarded. During the circular turn, the rider will slip.

Example 5.11 Acircular racetrack of radius $300\,\text{m}$ is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the

(a) optimum speed of the racecar to avoid wear and tear on its tyres, and

Solution:
On a banked road, the frictional force and the horizontal component of the normal force combine to g enerate centrifugal force, which keeps the automobile going in a circular turn without skidding. The n ormal response component is sufficient to give the required centripetal force at the optimal speed, and the frictional force is not required.

The optimum speed v_o is given as

$$v_o = (Rg\tan\theta)^{1/2}$$

Here $R = 300\,\mathrm{m}, \theta = 15^{\circ}, g = 9.8\,\mathrm{m\,s^{-2}}$; we have

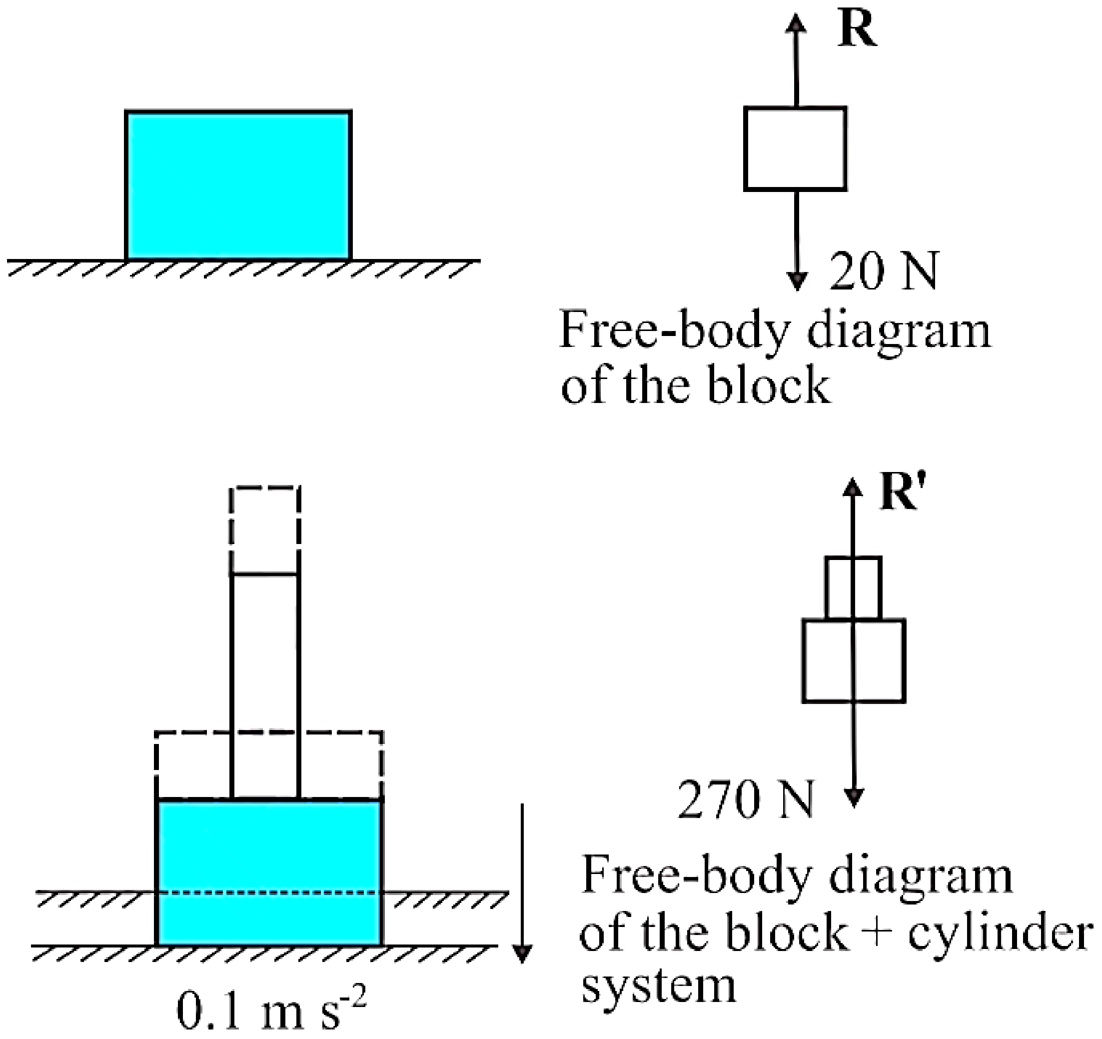
$$v_o = 28.1\,\mathrm{m\,s^{-1}}$$

(b) maximum permissible speed to avoid slipping?

Solution: The maximum permissible speed $v_{\max}$ is given by

$$v_{\max} = \left(Rg\frac{\mu_s + \tan\theta}{1-\mu_s\tan\theta}\right)^{1/2} = 38.1\,\mathrm{m\,s^{-1}}$$

Example 5.12 See A wooden block of mass $2\,\mathrm{kg}$ rests on a soft horizontal floor. When an iron cylinder of mass $25\,\mathrm{kg}$ is placed on top of the block, the floor yields steadily and the block and the cylinder together go down with an acceleration of $0.1\,\mathrm{m s^{2}}$.



What is the action of the block on the floor (a) before and (b) after the floor yields? Take $g = 10\,\text{m}\,\text{s}^{-2}$. Identify the action-reaction pairs in the problem.

Solution:

(a) The block has come to a halt on the floor. Its free-body diagram shows two forces on the block, the force of gravitational attraction by the earth equal to $2 \times 10 = 20\,\text{N}$; and the normal force R of the floor on the block. According to the First Law of motion,the net force on the block must be zero i.e., $R = 20\,\text{N}$.By applying third law of the motion on the the action of the block (i.e. the force exerted on the floor by the block) is equal to $20\,\text{N}$ and directed vertically downwards.

(b) The system (block + cylinder) accelerates downwards with 0.1ms^2. The free-body diagram of the system shows two forces on the system: the force of gravity due to the earth $(270\,\text{N})$; and the normal force R' by the floor.
The internal forces between the block and the cylinder are not shown in the system's free-body diagram.Applying the second law to the system,

$$270 - R' = 27 \times 0.1\,\text{N}$$

$$ie. R' = 267.3\,\text{N}$$

By the third law, the action of the system on the floor is equal to $267.3\,\text{N}$ vertically downward.
Action-reaction pairs:

For (a): (i) the force of gravity $(20\,\text{N})$ on the block by the earth (say, action); the force of gravity on the earth by the block (reaction) equal to $20\,\text{N}$ directed upwards (not shown in the figure).

(ii) the force on the floor by the block (action); the force on the block by the floor (reaction).

For (b): (i) the force of gravity $(270\ N)$ on the system by the earth (say, action); the force of gravity on the earth by the system (reaction), equal to $(270\ N)$,

directed upwards (not shown in the figure).

(ii) the system's force on the floor (activity); the system's force on the floor (reaction).

In addition, for (b), the cylinder's force on the block and the block's force on the cylinder form an acti on-reaction pair:
The crucial thing to remember is that an action-
reaction pair is made up of reciprocal forces between two bodies that are always equal and opposing.
An action-
reaction pair can never be formed by two equal and opposite forces acting on the same body.
The normal force on the mass by the floor and the force of gravity on the mass in (a) or (b) are not acti on-reaction pairings.These forces happen to be equal and opposite for (a) since the mass is at rest.
They are not so for case (b), as seen already. The weight of the system is $270\,\text{N}$, while the normal force R' is $267.3\,\text{N}$.

Exercises

5.1 Give the magnitude and direction of the net force acting on

a) A drop of rain falling down with a constant speed,

Solution:
Net force is zero.The raindrops are falling at a steady rate.As a result, the acceleration is zero.

The net force exerted on the rain drop is zero, according to Newton's second law of motion.

b) a cork of mass $10\,\text{g}$ floating on water,

Solution: Net force is zero. The cork's weight is acting downward.

It is counter-balanced by the water's buoyant force in the upward direction, 44.

As a result, there is no net force acting on the floating cork.

c) A kite skilfully held stationary in the sky,

Solution: Net force is zero.In the sky, the kite is stationary, i.e. it is not moving at all.

As a result, according to Newton's first rule of motion, there is no net force acting on the kite.

d) A car moving with a constant velocity of $30 \mathrm{~km} / \mathrm{h}$ on a rough road,

Solution: Net force is zero.The automobile is going at a consistent speed down a bumpy route.

As a result, it has no acceleration. There is no net force operating on the automobile, according to Newton's second law of motion.

e) A high-speed electron in space far from all material objects, and free of electric and magnetic fields.

Solution: Net force is zero.All fields have no affect on the high-speed electron.

As a result, there is no net force acting on the electron.

5.2 A pebble of mass $0.05 \mathrm{kg}$ is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble,

a) During its upward motion,

b) During its downward motion,

c) At the highest point where it is momentarily at rest. Do your Solutions change if the pebble was thrown at an angle of 45° with the horizontal direction?

Ignore air resistance.

Solution: $0.5 \mathrm{N}$, in vertically downward direction, in all cases.

Gravitational acceleration always operates downward, regardless of the direction of motion of an item.

In all three scenarios, the gravitational force is the only force acting on the stone.
Newton's second law of motion gives its magnitude as:

$F = m \times a$

Where, $F =$ Net force

$m =$ Mass of the pebble $= 0.05 \mathrm{kg}$

$a = \mathrm{g} = 10 \mathrm{m/s}^2$

$\therefore F = 0.05 \times 10 = 0.5 \mathrm{N}$

The net force on the pebble in all three cases is $0.5 \mathrm{N}$ and this force acts in the downward direction.

The horizontal and vertical components of velocity will be present if the pebble is hurled at an angle of 45° with the horizontal. Only the vertical component of velocity reaches zero at the highest point.
Throughout its travel, however, the stone will have a horizontal component of velocity.
The net force applied on the stone is unaffected by this component of velocity.

5.3 Give the magnitude and direction of the net force acting on a stone of mass 0.1kg,

a) Just after it is dropped from the window of a stationary train,

Solution: 1N; vertically downward

Mass of the stone, $m = 0.1\text{kg}$

Acceleration of the stone, $a = \text{g} = 10\,\text{m/s}^2$

The net force exerted on the stone, according to Newton's second law of motion, is:

$F = ma = m\text{g}$

$= 0.1 \times 10 = 1\text{N}$

Gravitational acceleration always works in the downward direction.

b) just after it is dropped from the window of a train running at a constant velocity of $36\,\text{km/h}$,

Solution: 1N; vertically downward

The train is travelling at a steady speed.
As a result, its acceleration in the horizontal direction, where it is moving, is zero.
As a result, there is no horizontal force acting on the stone.
The net force exerted on the stone is due to gravity's acceleration, and it is always vertically downwar d.

This force has a magnitude of $1N$.

c) Just after it is dropped from the window of a train accelerating with $1\,\text{m}\,\text{s}^{-2}$,

Solution: 1N; vertically downward

It is given that the train is accelerating at the rate of $1\,\text{m/s}^2$.

Therefore, the net force acting on the stone, $F = ma = 0.1 \times 1 = 0.1\text{N}$.

This force has a horizontal component to it.
When the stone is dropped, the horizontal force F no longer affects it.
This is due to the fact that the force exerted on a body at any one time is determined by the current cir cumstance rather than previous ones.
As a result, the net force acting on the stone is determined only by gravity's acceleration.
$F = mg = 1\text{N}$

This force works in a vertical downward direction.

d) Lying on the floor of a train which is accelerating with $1\,\text{m}\,\text{s}^{-2}$**, the stone being at rest relative to the train. Neglect air resistance throughout.**

Solution: 0.1N; in the direction of motion of the train.

The typical response of the floor balances the weight of the stone. The train's horizontal motion is the only source of acceleration.

Acceleration of the train, $a = 0.1\,\mathrm{m/s^2}$

The net force acting on the stone will be directed in the train's direction of travel.
Its size is determined by:

$F = ma$

$= 0.1 \times 1$

$= 0.1\,\mathrm{N}$

5.4 One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v the net force on the particle (directed towards the centre) is:

$(i) T,$

$(ii) T - \dfrac{mv^2}{l}$

$(iii) T + \dfrac{mv^2}{l}$

$(iv) 0$

T is the tension in the string. [Choose the correct alternative].

Solution: $(i)\mathrm{T}$

The tension created in the string provides the centripetal force when a particle linked to a string circles in a circular motion around a centre. As a result, in the given example, the particle's net force is the tension T, i.e.

$F = T = \dfrac{mv^2}{l}$

Where F is the net force acting on the particle.

5.5 A constant retarding force of $50\,\mathrm{N}$ is applied to a body of mass $20\,\mathrm{kg}$ moving initially with a speed of $15\,\mathrm{ms^{-1}}$. How long does the body take to stop?

Solution: Retarding force, $F = -50\,\mathrm{N}$

Mass of the body, $m = 20\,\mathrm{kg}$

Initial velocity of the body, $u = 15\,\mathrm{m/s}$

Final velocity of the body, $v = 0$

The acceleration (a) produced in the body may be estimated using Newton's second rule of motion:

$F = ma$
$-50 = 20 \times a$
$\therefore a = \frac{-50}{20}$
$= -2.5\,\text{m/s}^2$

The time (t) it takes for the body to come to rest may be computed using the first equation of motion:

$v = u + at$
$\therefore t = \frac{-u}{a}$
$= \frac{-15}{-2.5}$
$= 6\,\text{s}$

5.6 A constant force acting on a body of mass $3.0\,\text{kg}$ changes its speed from $2.0\,\text{m s}^{-1}$ to $3.5\,\text{ms}^{-1}$ in $25\,\text{s}$. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force?

Solution: $0.18\,\text{N}$; in the direction of motion of the body

Mass of the body, $m = 3\,\text{kg}$

Initial speed of the body, $u = 2\,\text{m/s}$

Final speed of the body, $v = 3.5\,\text{m/s}$

Time, $t = 25\,\text{s}$

The acceleration (a) produced in the body may be estimated using the first equation of motion:

$v = u + at$

$\therefore a = \frac{v-u}{t}$
$= \frac{3.5-2}{25}$
$= \frac{1.5}{25}$
$= 0.06\,\text{m/s}^2$

Newton's second law of motion states that force is equal to:

$F = ma$
$= 3 \times 0.06$
$= 0.18\,\text{N}$

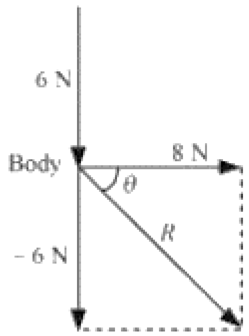
The net force applied on the body is in the direction of its motion since the application of force does not affect the direction of the body.

5.7 A body of mass $5\,\text{kg}$ is acted upon by two perpendicular forces $8\,\text{N}$ and $6\,\text{N}$. Give the magnitude and direction of the acceleration of the body.

Solution: $2\,\text{m/s}^2$, at an angle of 37° with a force of $8\,\text{N}$.

Mass of the body, $m = 5\,\text{kg}$

The following is a representation of the situation:



The sum of two forces is expressed as:

$$R = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = 10\,\text{N}$$

θ is the angle made by R with the force of $8\,\text{N}$

$$\therefore \theta = \tan^{-1}\left(\frac{-6}{8}\right) = -36.87^\circ$$

The negative sign indicates that θ is in the clockwise direction with respect to the force of magnitude $8\,\text{N}$.

As per Newton's second law of motion, the acceleration (a) of the body is given as:

$$F = ma$$

$$\therefore a = \frac{F}{m} = \frac{10}{5} = 2\,\text{m/s}^2$$

5.8 The driver of a three-wheeler moving with a speed of $36\,\text{km/h}$ sees a child standing in the middle of the road and brings his vehicle to rest in $4.0\,\text{s}$ just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is $400\,\text{kg}$ and the mass of the driver is $65\,\text{kg}$.

Solution: Initial speed of the three-wheeler, $u = 36\,\text{km/h}$

Final speed of the three-wheeler, $v = 10\,\text{m/s}$

Time, $t = 4\,\text{s}$

Mass of the three-wheeler, $m = 400\,\text{kg}$

Mass of the driver, $m' = 65\,\text{kg}$

Total mass of the system, $M = 400 + 65 = 465\,\text{kg}$

By applying the first law of motion, the acceleration (a) of the three-wheeler will be: $v = u + at$

$$\therefore a = \frac{v-u}{t} = \frac{0-10}{4} = -2.5\,\text{m/s}^2$$

The negative indication implies that the three-velocity wheeler's is decreasing with time.
The net force operating on the three-wheeler may be estimated using Newton's second law of motion:

$$F = Ma$$
$$= 465 \times (-2.5)$$
$$= -1162.5\,\text{N}$$

The minus symbol shows that the force is acting in the opposite direction of the three-motion wheeler's.

5.9 A rocket with a lift-off mass $20,000\,\text{kg}$ is blasted upwards with an initial acceleration of $5.0\,\text{ms}^2$. Calculate the initial thrust (force) of the blast.

Solution: Mass of the rocket, $m = 20,000\,\text{kg}$

Initial acceleration, $a = 5\,\text{m/s}^2$

Acceleration due to gravity, $g = 10\,\text{m/s}^2$

The net force (thrust) acting on the rocket may be calculated using Newton's second rule of motion:

$$F - mg = maF$$
$$= m(g + a)$$
$$= 20000 \times (10 + 5)$$
$$= 20000 \times 15$$
$$= 3 \times 10^5\,\text{N}$$

5.10 A body of mass $0.40\,\text{kg}$ moving initially with a constant speed of $10\,\text{ms}^{-1}$ to the north is subject to a constant force of $8.0\,\text{N}$ directed towards the south for $30\,\text{s}$. Take the instant the force is applied to be $t = 0$, the position of the body at that time to be $x = 0$, and predict its position at $t = -5\text{s}, 25\text{s}, 100\text{s}$.

Solution: Mass of the body, $m = 0.40\,\text{kg}$

Initial speed of the body, $u = 10\,\text{m/s}$ due north

Force acting on the body, $F = -8.0\,\text{N}$

Acceleration produced in the body, $a = \frac{F}{m} = \frac{-8.0}{0.40} = -20\,\text{m/s}^2$

At $t = -5\,\text{s}$

Acceleration,

$a' = 0 and$
$u = 10\,\text{m/s.}$

$s = ut + \frac{1}{2}a't^2$
$= 10 \times (-5) = -50\,\text{m}$

At $t = 25\,\text{s}$

Acceleration,

$a'' = -20\,\text{m/s}^2 and$
$u = 10\,\text{m/s}$
$s' = ut' + \frac{1}{2}a''t^2$
$= 10 \times 25 + \frac{1}{2} \times (-20) \times (25)^2$
$= 250 + 6250$
$= -6000\,\text{m}$

At $t = 100\,\text{s}$

For $0 \le t \le 30\,\text{s}$

$a = -20\,\text{m/s}^2$
$u = 10\,\text{m/s}$
$s_1 = ut + \frac{1}{2}a''t^2$
$= 10 \times 30 + \frac{1}{2} \times (-20) \times (30)^2$
$= 300 - 9000$
$= -8700\,\text{m}$

For $30' < t \le 100\,\text{s}$

As per the first equation of motion, for $t = 30\,\text{s}$, final velocity is given as:

$v = u + at$
$= 10 + (-20) \times 30$
$= -590\,\text{m/s}$

Velocity of the body after $30\text{s} = -590\,\text{m/s}$

For motion between $30\,s to 100\,s, i.e., in 70\,s$:

$$s_2 = v + \frac{1}{2}a''t^2$$
$= -590 \times 70$
$= -41300\,\text{m}$

So, total distance, $s'' = s_1 + s_2$

$= -8700 - 41300$
$= -50000\,\text{m}$

5.11 A truck starts from rest and accelerates uniformly at $2.0\,\text{ms}^{-2}$, At $t = 10\text{s}$, a stone is dropped by a person standing on the top of the truck ($6\,\text{m}$ high from the ground). What are the

(a) Velocity, and

(b) Acceleration of the stone at $t = 11\text{s}$? (Neglect air resistance.)

Solution:

Initial velocity of the truck, $u = 0$

Acceleration, $a = 2\,\text{m/s}^2$

Time, $r = 10\text{s}$

The ultimate velocity is provided by the first equation of motion:

$v - u + at$
$= 0 + 2 \times 10$
$= 20\,\text{m/s}$

The final velocity of the truck and hence, of the stone is $20\,\text{m/s}$.

At $t = 11\text{s}$, the horizontal component (v_x) of velocity, in the absence of air resistance, remains unchanged, i.e., $v_x = 20\,\text{m/s}$

The vertical component (v_y) of velocity of the stone is given by the first equation of motion as:

$v_y - u + a_y \delta t$

Where, $\delta t = 11 - 10 = 1\text{s}\; and a_y = \text{g} = 10\,\text{m/s}^2$

$\therefore v_y = 0 + 10 \times 1 = 10\,\text{m/s}$

The resultant velocity (v) of the stone is given as:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 10^2}$$
$$= \sqrt{400 + 100}$$
$$= \sqrt{500}$$
$$= 22.36\,\text{m/s}$$

Let θ be the angle made by the resultant velocity with the horizontal component of velocity, v_x

$$\therefore \tan\theta = \left(\frac{v_z}{v_x}\right)$$
$$\theta = \tan^{-1}\left(\frac{10}{20}\right)$$
$$= \tan^{-1}(0.5)$$
$$= 26.57^\circ$$

The horizontal force exerted on the stone becomes zero when it is dropped from the truck, yet the ston e continues to travel due to gravity.Hence, the acceleration of the stone is $10\,\text{m/s}^2$ and it acts vertically downward.

5.12 A bob of mass $0.1\,\text{kg}$ hung from the ceiling of a room by a string $2\,\text{m}$ long is set into oscillation. The speed of the bob at its mean position is $1\,\text{ms}^{-1}$. What is the trajectory of the bob if the string is cut when the bob is

(a) At one of its extreme positions,

Solution: Parabolic route, vertically downward. The bob's velocity is 0 at its most extreme position.

The bob will fall vertically to the ground if the string is severed at this point.

(b) At its mean position.

Solution:
The bob's velocity is $I\,m/s$ at its average location. The oscillating bob's motion is tangential to the di rection of this velocity. If the bob is severed at its average location, it will trace a projectile route with only the horizontal component of velocity, resulting in a parabolic path.

5.13 A man of mass $70\,\text{kg}$ stands on a weighing scale in a lift which is moving

a) Upwards with a uniform speed of $10\,\text{ms}^{-1}$,

Solution: Mass of the man, $m = 70\,\text{kg}$ Acceleration, a=0

Using Newton's second law of motion, we can write the equation of motion as:

$R - mg = ma$

Where, ma is the net force acting on the man.

As the lift is moving at a uniform speed, acceleration $a = 0$

$\therefore R = m\text{g} = 70 \times 10 = 700\,\text{N}$

$\therefore$ Reading on the weighing scale $= \frac{700}{\text{g}} = \frac{700}{10} = 70\,\text{kg}$

b) Downwards with a uniform acceleration of $5\,\text{ms}^{-2}$,

Solution: Mass of the man, $m = 70\,\text{kg}$

Acceleration, $a = 5\,\text{m/s}^2$ downwards

We can write the equation of motion as: Using Newton's second law of motion, we can write the equa tion of motion as:

$R = m(\text{g} - a)$
$= 70(10 - 5)$
$= 70 \times 5 = 350\,\text{N}$
$R + mg = ma$

So, Reading on the weighing scale $= \frac{350}{\text{g}} = \frac{350}{10} = 35\,\text{kg}$

c) Upwards with a uniform acceleration of $5\,\text{ms}^{-2}$ .What would be the readings on the scale in each case?

Solution: Mass of the man, $m = 70\,\text{kg}$

Acceleration, $a = 5\,\text{m/s}^2$ upwards

Using Newton's second law of motion. we can write the equation of motion as:

$R - m\text{g} = mR$
$= m(\text{g} + a)$
$= 70(10 + 5)$
$= 70 \times 15$
$= 1050\,\text{N}$

$\therefore$ Reading on the weighing scale $= \frac{1050}{\text{g}} = \frac{1050}{10} = 105\,\text{kg}$

d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity?

Solution: When the lift moves freely under gravity, acceleration $a = \mathrm{g}$

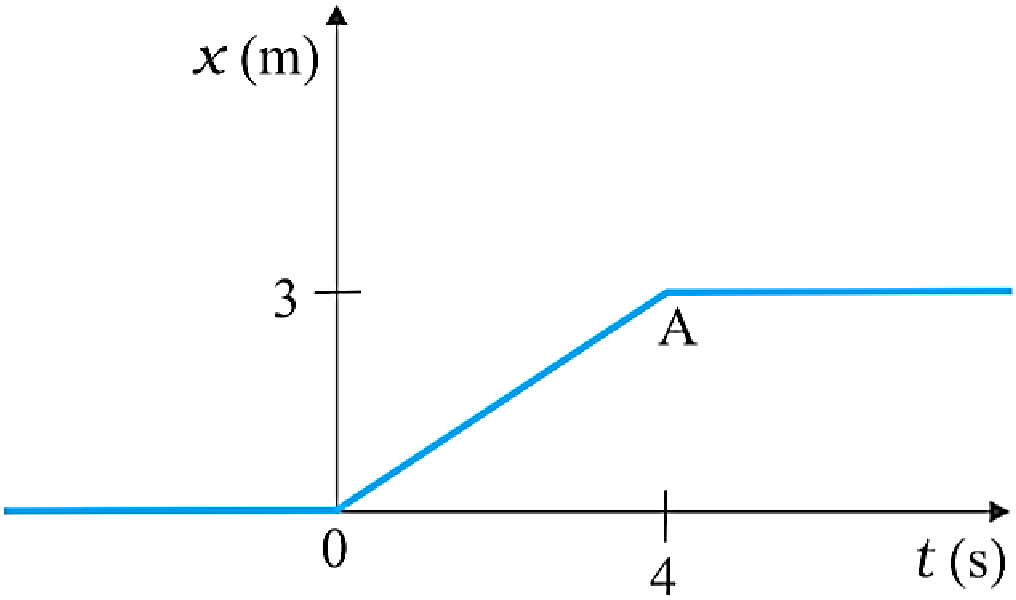
Using Newton's second law of motion, we can write the equation of motion as:

$$R + mg = ma$$
$$R = m(g - a)$$
$$= m(g - g)$$
$$= 0$$

So, Reading on the weighing scale $= \frac{0}{\mathrm{g}} = 0\,\mathrm{kg}$

The man will experience weightlessness.

5.14 Figure shows the position-time graph of a particle of mass $4\,\mathrm{kg}$**. What is the**



(a) Force on the particle for $t < 0, t > 4\,\mathrm{s}$**,** $0 < t < 4\,\mathrm{s}$ **?**

Solution: For $t < 0$

The location of the particle is coincident with the time axis, as can be seen in the graph.
It means that the particle's movement in this time interval is zero.
As a result, there is no force acting on the particle.
For $t > 4\mathrm{s}$.

The location of the particle in the depicted graph is parallel to the time axis, as can be seen.
It denotes that the particle is at rest at a $3m$ distance from the origin. As a result, there is no force acting on the particle.
For $0 < t < 4$

The provided position-time graph exhibits a constant slope, as can be seen.
As a result, the particle's acceleration is zero. As a result, there is no force acting on the particle.

(b) Impulse at $t=0$ and $t=4\text{s}$? (Consider one-dimensional motion only).

Solution: At $t=0$

Impulse $=$ Change in momentum

$mv-mu$

Mass of the particle, $m=4\text{kg}$

Initial velocity of the particle, $u=0$

Final velocity of the particle, $v=\frac{3}{4}\text{m/s}$

Ampulse $=4\left(\frac{3}{4}-0\right)=3\text{kg m/s}$

At $t=4\text{s}$

Initial velocity of the particle, $u=\frac{3}{4}\text{m/s}$

Final velocity of the particle, $v=0$

$\therefore$ Impulse $=4\left(0-\frac{3}{4}\right)=-3\text{kg m/s}$

5.15 Two bodies of masses 10kg and 20kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force $F=600\text{N}$ is applied to (i) A, (ii) B along the direction of string. What is the tension in the string in each case?

Solution: Horizontal force, $F=600\text{N}$

Mass of body $\text{A}, m_1=10\text{kg}$

Mass of body $B, m_2=20\text{kg}$

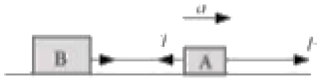
Total mass of the system, $m=m_1+m_2=30\text{kg}$

The acceleration (a) produced in the system may be estimated using Newton's second rule of motion:

$F=ma$

$\therefore a=\frac{F}{m}=\frac{600}{30}=20\text{m/s}^2$

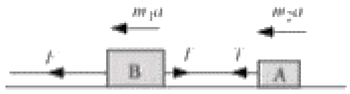
When force F is applied on body A :



The equation of motion can be written as:

$$F - T = m_1 a$$
$$\therefore T = F - m_1 a$$
$$= 600 - 10 \times 20$$
$$= 400\,\text{N}$$

When force F is applied on body B:

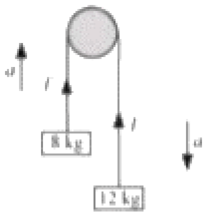


The equation of motion can be written as:

$$\text{F} - \text{T} = \text{m}_2\text{a}$$
$$T = F - m_2 a$$
$$\therefore T = 600 - 20 \times 20 = 200\,\text{N}$$

5.16 Two masses $8\,\text{kg}$ and $12\,\text{kg}$ are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses, and the tension in the string when the masses are released.

Solution:
The following diagram shows how the given system with two masses and a pulley may be represented :



Smaller mass, $m_1 = 8\,\text{kg}$

Larger mass, $m_2 = 12\,\text{kg}$

Tension in the string $= T$

Mass m_2, owing to its weight, moves downward with acceleration a, and mass m_1 moves upward.

Applying Newton's second law of motion to the system of each mass:

For mass m_1 The equation of motion can be written as:

$T - m_1 g = ma ...(1)$

For mass m_2 : The equation of motion can be written as:

$m_2 g - T = m_2 a ...(2)$

Adding equations (1) and (2), we get:

$$(m_2 - m_1) g = (m_1 + m_2) a$$

$$\therefore a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

$$= \left(\frac{12 - 8}{12 + 8}\right) \times 10 = \frac{4}{20} \times 10 = 2\,\text{m}/\text{s}^2$$

Therefore, the acceleration of the masses is $2\,\text{m}/\text{s}^2$.

Substituting the value of a in equation (2), we get:

$$m_2 g - T = m_2 \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

$$T = \left(m_2 - \frac{m_2^2 - m_1 m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{2 \times 12 \times 8}{12 + 8}\right) \times 10$$

$$= \frac{2 \times 12 \times 8}{20} \times 10$$

$$= 96\,\text{N}$$

Therefore, the tension in the string is $96\,\text{N}$.

5.17 A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei the products must move in opposite directions.

Solution: Let m_1 *and* m_2. be the respective masses of the parent nucleus and the two daughter nuclei. The parent nucleus is at rest.

Initial momentum of the system (parent nucleus) $= 0$

Let $v_1 and v_2$ be the respective velocities of the daughter nuclei having masses m_1 *and* m_2.

Total linear momentum of the system after disintegration $= m_1 v_1 + m_2 v_2$

According to the law of conservation of momentum:

Total initial momentum = Total final momentum

$$0 = m_1 v_1 + m_2 + v_2 v_1 = \frac{-m_2 v_2}{m_1}$$

The negative indication here suggests that the parent nucleus pieces are moving in opposing directions .

5.18 Two billiard balls each of mass $0.05\,\text{kg}$ moving in opposite directions with speed $6\,\text{ms}^{-1}$ collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?

Solution: Mass of each ball $= 0.05\,\text{kg}$

Initial velocity of each ball $= 6\,\text{m/s}$

Magnitude of the initial momentum of each ball, $p_i = 0.3\,\text{kg m/s}$

The balls alter their directions of motion after colliding, but their velocity magnitudes do not change.

Final momentum of each ball, $p_f = -0.3\,\text{kg m/s}$

Impulse imparted to each ball = Change in the momentum of the system

$$= p_\gamma - p_i$$
$$= -0.3 - 0.3 = -0.6\,\text{kg m/s}$$

The negative indication implies that the balls are receiving opposite-direction shocks.

5.19 A shell of mass $0.020\,\text{kg}$ is fired by a gun of mass $100\,\text{kg}$. If the muzzle speed of the shell is $80\,\text{ms}^{-1}$, what is the recoil speed of the gun?

Solution: Mass of the gun, $M = 100\,\text{kg}$.

Mass of the shell, $m = 0.020\,\text{kg}$

Muzzle speed of the shell, $v = 80\,\text{m/s}$

Recoil speed of the gun $= V$

Both the rifle and the shell are initially at rest.

Initial momentum of the system $= 0$

Final momentum of the system $= m\,v - M\;V$.

Because the shell and the gun are pointing in opposing directions, the negative sign emerges.According to the law of conservation of momentum:

Final momentum = Initial momentum

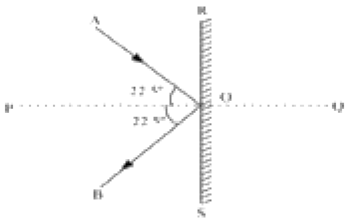
$mv - MV = 0$

$\therefore V = \dfrac{mv}{M}$

$= \dfrac{0.020 \times 80}{100 \times 1000} = 0.016\,\mathrm{m/s}$

5.20 A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to $54\,\mathrm{km/h}$. What is the impulse imparted to the ball? (Mass of the ball is $0.15\,\mathrm{kg}$.)

Solution: As indicated in the diagram below, the given scenario can be represented.



Where,

$\mathrm{AO} =$ Incident path of the ball

$\mathrm{OB} =$ Path followed by the ball after deflection

$\angle\mathrm{AOB} =$ Angle between the incident and deflected paths of the ball $= 45^\circ$

$\angle\mathrm{AOP} = \angle\mathrm{BOP} = 22.5^\circ = \theta$

Initial and final velocities of the ball $= v$

Horizontal component of the initial velocity $= v\cos\theta\, along\, \mathrm{RO}$

Vertical component of the initial velocity $= v\sin\theta\, along\, \mathrm{PO}$

Horizontal component of the final velocity $= v\cos\theta\, along\, \mathrm{OS}$

Vertical component of the final velocity $= v\sin\theta\, along\, OP$

There is no change in the horizontal components of velocities. The vertical components of velocities are in a clockwise orientation.

Impulse imparted to the ball $=$ Change in the linear momentum of the ball

$= mv\cos\theta - (-mv\cos\theta)$

$= 2mv\cos\theta$

Mass of the ball, $m = 0.15\,\mathrm{kg}$

Velocity of the ball,

$v = 54\,\text{km/h} = 15\,\text{m/s}$

$\therefore$ Impulse $= 2 \times 0.15 \times 15 \cos 22.5^{\circ} = 4.16\,\text{kg m/s}$

5.21 A stone of mass $0.25\,\text{kg}$ tied to the end of a string is whirled round in a circle of radius $1.5\,\text{m}$ with a speed of $40\,\text{rev/min}$ in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of $200\,\text{N}$?

Solution: Mass of the stone, $m = 0.25\,\text{kg}$

Radius of the circle, $r = 1.5\,\text{m}$

Number of revolution per second, $n = \frac{40}{60} = \frac{2}{3}\,\text{rps}$

Angular velocity, $\omega = \frac{v}{r} = 2\pi n \ldots\ldots\ldots\ldots\ldots\ldots(i)$

The centripetal force for the stone is provided by the tension T, in the string, i.e.,

$T = F_{\text{Centripetal}}$

$= \frac{mv^2}{r} = mre^2 = mr(2\pi n)^2$

$= 0.25 \times 1.5 \times \left(2 \times 3.14 \times \frac{2}{3}\right)^2$

$= 6.57N$

Maximum tension in the string, $T_{\max} = 200\,\text{N}$

$T_{\max} = \frac{m{v_{\max}}^2}{r}$

$\therefore v_{\max} = \sqrt{\frac{T_{\min} \times r}{m}}$

$= \sqrt{\frac{200 \times 1.5}{0.25}}$

$= \sqrt{1200} = 34.64\,\text{m/s}$

Therefore, the maximum speed of the stone is $34.64\,\text{m/s}$.

5.22 If, in 5.21, the speed of the stone is increased beyond the maximum permissible, value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks:

a) The stone moves radially outwards,

b) The stone flies off tangentially from the instant the string breaks,

c) The stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle?

Solution: Option (b) is correct.

The stone will go in the direction of the velocity at the time the string breaks.
The direction of the velocity vector is tangential to the path of the stone at that time, according to Newton's first rule of motion. As a result, as soon as the string snaps, the stone will fly off in a tangential direction.

5.23 Explain why

a) A horse cannot pull a cart and run in empty space,

Solution: A horse must push the earth backwards with some force in order to drive a waggon.

The horse's feet, in turn, are subjected to an equal and opposite reaction force from the ground.
The horse moves forward as a result of this response force.There is no response force in an empty place. As a result, a horse can't pull a cart and run in the open.

b) Passengers are thrown forward from their seats when a speeding bus stops suddenly,

Solution:
When a fast bus comes to a rapid stop, the lower section of a passenger's body in touch with the seat comes to an abrupt stop.
The top half, on the other hand, prefers to stay in motion (as per the first law of motion).
As a result, the upper body of the passenger is propelled forward in the direction of the bus's movement.

c) It is easier to pull a lawn mower than to push it,

Solution: While pulling a lawn mower, a force at an angle θ is applied on it, as shown in the following figure.

This applied force's vertical component acts upward. This decreases the mower's effective weight.
On the other hand, while pushing a lawn mower, a force at an angle θ is applied on it.

The vertical component of the applied force works in this situation in the direction of the mower's weight. This raises the mower's effective weight.Because the lawn mower's effective weight is lower in the first example, pulling the lawn mower is easier than pushing it.

d) A cricketer moves his hands backwards while holding a catch.

Solution: According to Newton's second law of motion, we have the equation of motion:

$$F = ma = m\frac{\Delta v}{\Delta t}$$

Where,

$F =$ Stopping force experienced by the cricketer as he catches the ball.

$m =$ Mass of the ball

$\Delta t =$ Time of impact of the ball with the hand

It is the impact force is inversely proportional to the impact time, i.e.,

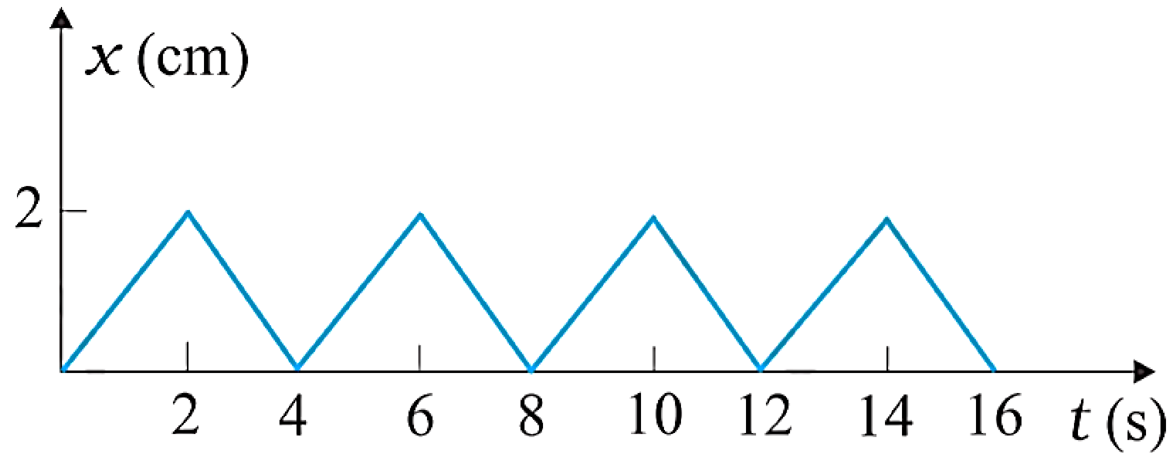
$$F \propto \frac{1}{\mathrm{M}}$$

It demonstrates that as the time of collision grows, the force received by the cricketer reduces, and vic e versa.

A cricketer slides his hand backwards while taking a catch to lengthen the time of impact Δt. As a result, the stopping force decreases, saving the cricketer's hands from being injured.

Additional Exercises

5.24 Figure shows the position-time graph of a body of mass $0.04\,\mathrm{kg}$. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the body? What is the magnitude of each impulse?



Solution: A ball rebounding between two walls located between at $x = 0$ and $x = 2\,\mathrm{cm}$; after every $2\,\mathrm{s}$, the ball receives an impulse of magnitude $0.08 \times 10^{-2}\,\mathrm{kg\,m/s}$ from the walls.

The given graph shows that a body changes its direction of motion after every $2\,\mathrm{s}$. Physically, this situation can be visualized as a ball rebounding to and fro between two stationary walls situated between positions $x = 0$ and $x = 2\,\mathrm{cm}$; Since the slope of the $x - t$ graph reverses after every $2\,\mathrm{s}$, the ball collides with a wall after every $2\,\mathrm{s}$. Therefore, ball receives an impulse after every $2\,\mathrm{s}$.

Mass of the ball, $m = 0.04\,\mathrm{kg}$

The ball's velocity is determined by the graph's slope.Using the graph; we can calculate initial velocity (u) as:

$$u = \frac{(2-0)\times10^{-2}}{(2-0)} = 10^{-2}\ \text{m/s}$$

Velocity of the ball before collision, $u = 10^{-2}\ \text{m/s}$

Velocity of the ball after collision, $v = -10^{-2}\ \text{m/s}$

(As the ball reverses its direction of motion, the negative sign appears.)

Magnitude of impulse $=$ Change in momentum

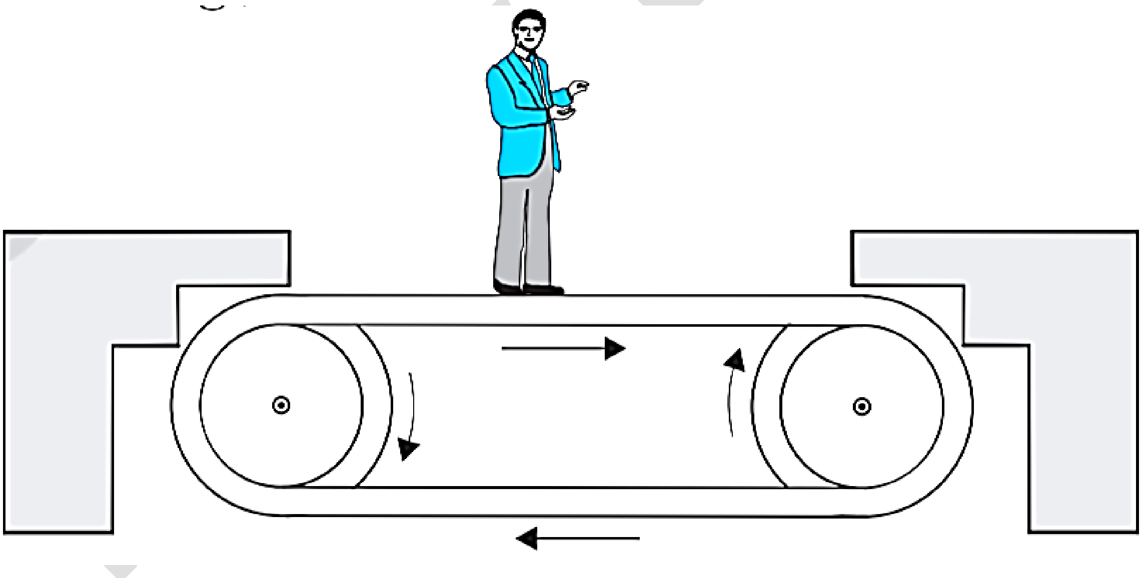
$= |mv - mu|$

$= |0.04(v-u)|$

$= \left|0.04\left(-10^{-2} - 10^{-2}\right)\right|$

$= 0.08\times10^{-2}\ \text{kg m/s}$

5.25 Figure shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with $1\,\text{m s}^{-2}$. What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt? (Mass of the man $= 65\,\text{kg}$.)



Solution: Mass of the man, $m = 65\,\text{kg}$

Acceleration of the belt, $a = 1\,\text{m/s}^2$

Coefficient of static friction, $\mu = 0.2$

The net force F, acting on the man is given by Newton's second law of motion as:

$F_{\text{is}} = ma_{=65\times1=65\ \text{N}}$

The man will continue to be stationary with respect to the conveyor belt until the net force on the man is less than or equal to the frictional force f_S, exerted by the belt, i.e.,

$$F'_{\text{mel}} = f_x$$

$$ma' = \mu mg$$

$$\therefore a' = 0.2 \times 10 = 2\,\text{m}/\text{s}^2$$

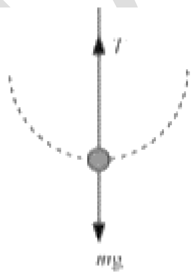
Therefore, the maximum acceleration of the belt up to which the man can stand stationary is $2\,\text{m}/\text{s}^2$.

5.26 A stone of mass m tied to the end of a string revolves in a vertical circle of radius R. The net forces at the lowest and highest points of the circle directed vertically downwards are: [Choose the correct alternative]

	LOWER POINT	HIGHEST POINT
(A)	$mg - T_1$	$mg + T_2$
(B)	$mg + T_1$	$mg - T_2$
(C)	$mg + T_1 - \frac{mv_1^2}{R}$	$mg - T_2 + \frac{mv_1^2}{R}$
(D)	$mg - T_1 - \frac{mv_1^2}{R}$	$mg + T_2 + \frac{mv_1^2}{R}$

$T_1 and v_1$ denote the tension and speed at the lowest point. $T_2 and v_2$ denote corresponding values at the highest point.

Solution: The following graphic depicts the free body diagram of the stone at its lowest position.

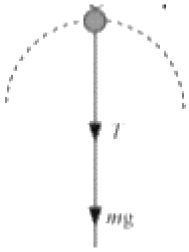


The net force exerted on the stone at this moment, according to Newton's second law of motion, is equ al to the centripetal force, i.e.

$$F_{i=1} = T - mg = \frac{mv_1^2}{R} \ldots(1)$$

Where, v_1 = Velocity at the lowest point

The following graphic depicts the stone's free body diagram at its highest position.



Using Newton's second law of motion, we have:

$$T + mg = \frac{mv_2^2}{R} \quad ...(2)$$

Where, v_2 = Velocity at the highest poine

It is clear from equations (1) and (2) that the net force acting at the lowest and the highest points are respectively $(T - m\,g)$ *and* $(T + m\,g)$.

5.27 A helicopter of mass $1000\,\text{kg}$ rises with a vertical acceleration of $15\,\text{ms}^{-2}$. The crew and the passengers weigh $300\,\text{kg}$. Give the magnitude and direction of the

a) Force on the floor by the crew and passengers,

Solution: Mass of the helicopter, $m_h = 1000\,\text{kg}$

Mass of the crew and passengers, $m_p = 300\,\text{kg}$

Total mass of the system, $m = 1300\,\text{kg}$

Acceleration of the helicopter, $a = 15\,\text{m/s}^2$

So, by applying Newton's second law of motion, the reaction force R, on the system by the floor can be calculated as:

$$R - m_p g = ma$$
$$= m_p(g + a)$$
$$= 300(10 + 15)$$
$$= 300 \times 25$$
$$= 7500\,\text{N}$$

The response force will likewise be directed upward because the chopper is accelerating vertically up ward. As a result of Newton's third law of motion, the force exerted on the floor by the crew and passe ngers is $7500N$ in a downward direction.
Using Newton's second law of motion, the reaction force R', experienced by the helicopter can be calculated as:

$R' - mg = ma$

$= m(g + a)$

$= 1300(10 + 15)$

$= 1300 \times 25$

$= 32500\,\text{N}$

b) Action of the rotor of the helicopter on the surrounding air,

Solution: The helicopter is being pushed higher by the reaction force of the surrounding air.

As a result, the rotor's action on the surrounding air will be $32500N$
, directed downward, according to Newton's third law of motion.

c) Force on the helicopter due to the surrounding air.

Solution: The force exerted by the surrounding air on the helicopter is $32500N$ pointing upward.

5.28 A stream of water flowing horizontally with a speed of $15\,\text{m}\,\text{s}^{-1}$ gushes out of a tube of cross-sectional area $10^{-2}\,\text{m}^2$, and hits a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming it does not rebound?

Solution: Speed of the water stream, $v = 15\,\text{m/s}$

Cross-sectional area of the tube, $A = 10^{-2}\,\text{m}^2$

Volume of water coming out from the pipe per second,

$V = Av = 15 \times 10^{-2}\,\text{m}^3/\text{s}$

Density of water, $\rho = 10^3\,\text{kg/m}^3$

Mass of water flowing out through the pipe per second $= \rho \times V = 150\,\text{kg/s}$

When the water hits the wall, it does not bounce back.
As a result, Newton's second rule of motion gives the force produced by the water on the wall as:

$F = \text{Rate of change of momentum} = \dfrac{mv}{t}$

5.29 Ten one-rupee coins are put on top of each other on a table. Each coin has a mass m. Give the magnitude and direction of

a) The force on the 7^{th} coin (counted from the bottom) due to all the coins on its top,

Solution: The weight of the three coins on top of the seventh coin exerts force on it.

Weight of one coin $= mg$

Weight of three coins $= 3mg$

Hence, the force exerted on the 7^{th} coin by the three coins on its top is $3mg$. This force acts vertically downward.

b) The force on the 7^{th} coin by the eighth coin,

Solution:
Because of the weight of the eighth coin and the other two coins (ninth and tenth) on its top, the eight h coin exerts force on the seventh coin.

Weight of the eighth coin $= mg$

Weight of the ninth coin $= mg$

Weight of the tenth coin $= mg$

Total weight of these three coins $= 3m\text{g}$

Hence, the force exerted on the 7^{th} coin by the eighth coin is $3mg$. This force acts vertically downward.

c) The reaction of the 6^{th} coin on the 7^{th} coin.

Solution: The 6^{th} coin experiences a downward force because of the weight of the four coins $(7^{th}, 8^{\text{th}}, 9^{\text{th}}, and 10^{\text{th}})$ on its top.

Therefore, the total downward force experienced by the 6^{th} coin is $4mg$.

As per Newton's third law of motion, the 6^{th} coin will produce an equal reaction force on the 7^{th} coin, but in the opposite direction. Hence, the reaction force of the 6^{th} coin on the 7^{th} coin is of magnitude 4mg. This force is vertical in nature.

5.30 An aircraft executes a horizontal loop at a speed of $720\,\text{km/h}$ with its wings banked at 15°. what is the radius of the loop?

Solution: Speed of the aircraft, $v = 720\,\text{km/h} = 720 \times \frac{5}{18} = 200\,\text{m/s}$

Acceleration due to gravity, $\text{g} = 10\,\text{m/s}^2$

Angle of banking, $\theta = 15^\circ$

For radius r, of the loop, we have the relation:

$$\tan\theta = \frac{v^2}{rg} r$$

$$= \frac{v^2}{g\tan\theta}$$

$$= \frac{200\times 200}{10\times \tan 15}$$

$$= \frac{4000}{0.268}$$

$$= 14925.37\,\text{m}$$

$$= 14.92\,\text{km}$$

5.31 A train runs along an unbanked circular track of radius $30\,\text{m}$ at a speed of $54\,\text{km/h}$. The mass of the train is $10^6\,\text{kg}$. What provides the centripetal force required for this purpose - The engine or the rails? What is the angle of banking required to prevent wearing out of the rail?

Solution: Radius of the circular track, $r = 30\,\text{m}$

Speed of the train, $v = 54\,\text{km/h} = 15\,\text{m/s}$

Mass of the train, $m = 10^6\,\text{kg}$

The lateral thrust of the rail on the wheel provides the centripetal force.
The wheel exerts an equal and opposite force on the rail, according to Newton's third law of motion.
The wear and damage of the rail are caused by this reaction force.

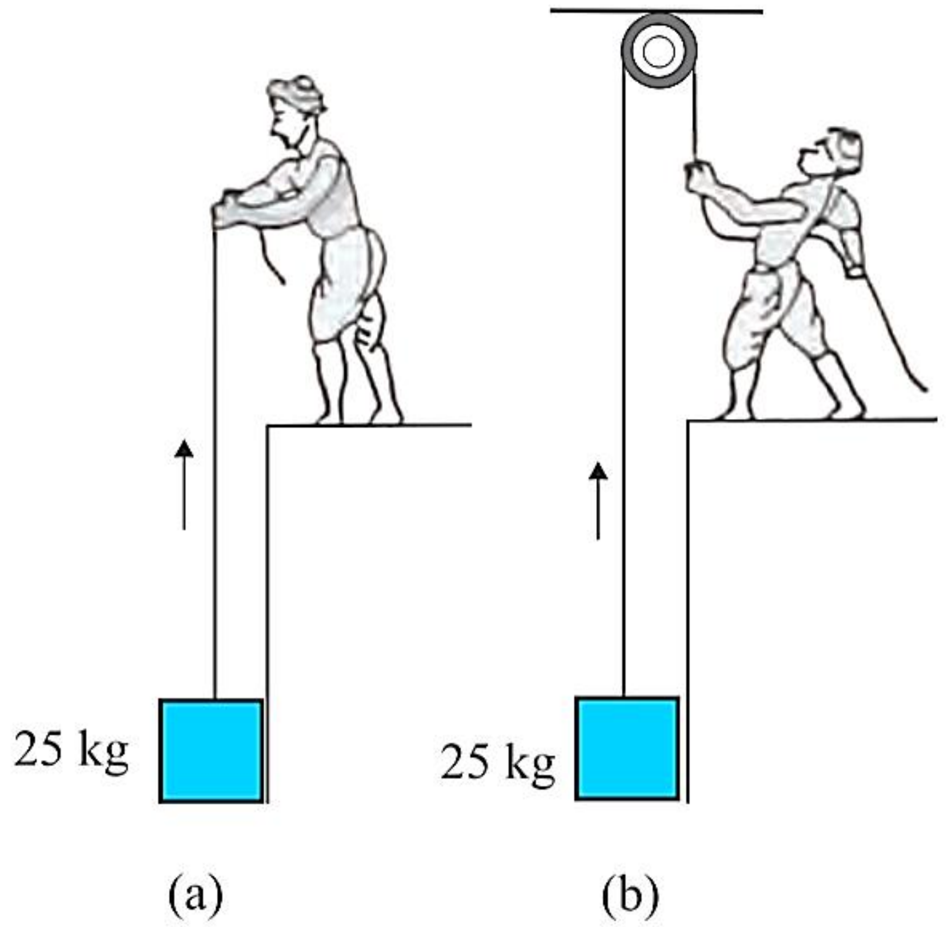
The angle of banking θ, is related to the radius r and speed v by the relation: $\tan\theta = \frac{y^2}{rg}$

$$= \frac{(15)^2}{30\times 10} = \frac{225}{300}$$

$$\theta = \tan^{-1}(0.75) = 36.87^\circ$$

Therefore, the angle of banking is about 36.87°.

5.32 A block of mass $25\,\text{kg}$ is raised by a $50\,\text{kg}$ man in two different ways as shown in figure. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of $700\,\text{N}$, which mode should the man adopt to lift the block without the floor yielding?



Solution: $750\,\text{N}\,and\,250\,\text{N}$ in the respective cases; Method (b)

Mass of the block, $m = 25\,\text{kg}$

Mass of the man, $M = 50\,\text{kg}$.

Acceleration due to gravity, $g = 10\,\text{m}/\text{s}^2$

Force applied on the block, $F = 25 \times 10 = 250\,\text{N}$

Weight of the man, $W = 50 \times 10 = 500\,\text{N}$

Case (a): When the guy directly raises the block .In this situation, the guy exerts an upward push.

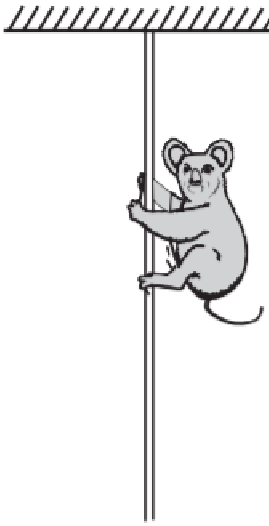
This makes him appear heavier.
Action on the floor by the man $= 250 + 500 = 750\,\text{N}$

Case (b):
When the guy uses a pulley to lift the block, In this situation, the guy exerts a downward force.

His perceived weight is reduced as a result of this.
Action on the floor by the man $= 500 - 250 = 250\,\text{N}$

If the floor can yield to a normal force of $700\,\text{N}$, then the man should adopt the second method to easily lift the block by applying lesser force.

5.33 A monkey of mass $40\,\text{kg}$ climbs on a rope which can stand a maximum tension of $600\,\text{N}$. In which of the following cases will the rope break: the monkey



a) Climbs up with an acceleration of $6\,\text{ms}^{-2}$

Solution: Mass of the monkey, $m = 40\,\text{kg}$

Acceleration due to gravity, $g = 10\,\text{m/s}$

Maximum tension that the rope can bear, $T_{\max} = 600\,\text{N}$

Acceleration of the monkey, $a = 6\,\text{m/s}^2$ upward

The equation of motion may be written as follows, using Newton's second law of motion:

$$T - mg = ma$$
$$\therefore T = m(g + a)$$
$$= 40(10 + 6)$$
$$= 640\,\text{N}$$

Since $T > T_{\max}$, the rope will break in this case.

b) Climbs down with an acceleration of $4\,\text{ms}^{-2}$

Solution: Acceleration of the monkey, $a = 4\,\text{m/s}^2$ downward

The equation of motion may be written as follows, using Newton's second law of motion:

$$mg - T = ma$$
$$\therefore T = m(\text{g} - a)$$
$$= 40(10 - 4)$$
$$= 240\,\text{N}$$

Since $T < T_{\max}$, the rope will not break in this case.

c) Climbs up with a uniform speed of $5\,ms^{-1}$

Solution: The monkey is climbing with a uniform speed of $5\,m/s$. Therefore, its acceleration is zero, i.e., $a = 0$,

The equation of motion may be written as follows, using Newton's second law of motion:

$$T - mg = 0$$
$$\therefore T = mg = 40 \times 10$$
$$= 400\,\mathrm{N}$$

Since $T < T_{max}$, the rope will not break in this case.

d) Falls down the rope nearly freely under gravity?

(Ignore the mass of the rope).

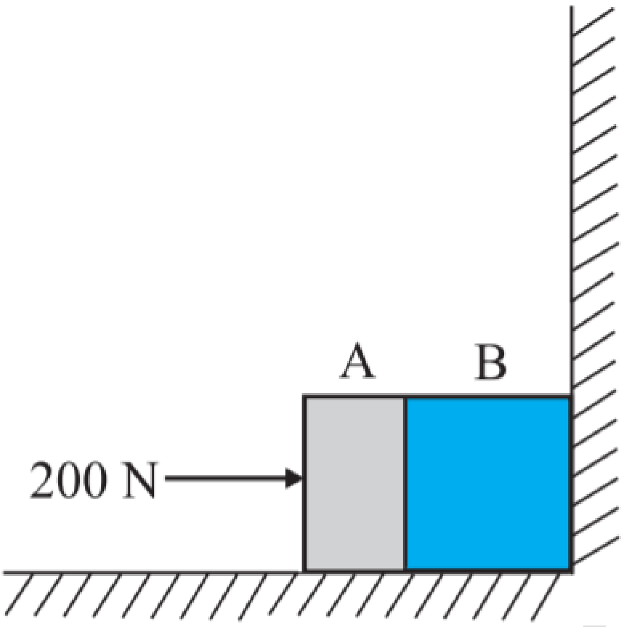
Solution: When the monkey falls freely under gravity, its will acceleration become equal to the acceleration due to gravity, i.e., $a = g$.

The equation of motion may be written as follows, using Newton's second law of motion:

$$mg - T = mg$$
$$\therefore T = m(g - g) = 0$$

Since $T < T_{max}$, the rope will not break in this case.

5.34 Two bodies A and B of masses $5\,kg$ and $10\,kg$ in contact with each other rest on a table against a rigid wall. The coefficient of friction between the bodies and the table is 0.15. A force of $200\,\mathrm{N}$ is applied horizontally to A. What are (a) the reaction of the partition (b) the action-reaction forces between A and B ? What happens when the wall is removed? Does the Solution to (b) change, when the bodies are in motion? Ignore the difference between μ_s and μ_k.



Solution: Mass of body $A, m_A = 5\,\text{kg}$ Mass of body $B, m_B = 10\,\text{kg}$

Applied force, $F = 200\,\text{N}$

Coefficient of friction, $\mu_s = 0.15$

The force of friction is given by the relation:

$$f_s = \mu(m_A + m_B)g$$
$$= 0.15(5+10)\times 10$$
$$= 1.5\times 15 = 22.5\,\text{N leftward}$$

Net force acting on the partition $= 200 - 22.5 = 177.5\,\text{N}$ rightward

As per Newton's third law of motion, the reaction force of the partition will be in the direction opposite to the net applied force.

Hence, the reaction of the partition will be $177.5\,\text{N}$, in the leftward direction.

Force of friction on mass A:

$$f_A = \mu m_A g$$
$$= 0.15\times 5\times 10$$
$$= 7.5\,\text{N leftward}$$

Net force exerted by mass A on $massB = 200 - 7.5 = 192.5N$ rightward

As per Newton's third law of motion, an equal amount of reaction force will be exerted by mass B on mass A, i.e, $192.5\,\text{N}$ acting leftward.

When the wall is removed, the two bodies will move in the direction of the applied force.

Net force acting on the moving system $= 177.5\,\text{N}$.

The equation of motion for the system of acceleration a, can be written as:

Net force $= \left(m_n + m_n\right)a$

$$\therefore a = \frac{\text{Net force}}{m_i + m_m}$$

$$= \frac{177.5}{5+10} = \frac{177.5}{15} = 11.83\,\text{m/s}^2$$

Net force causing mass A to move:

$$F_A = m_A a$$
$$= 5 \times 11.83$$
$$= 59.15\,\text{N}$$

Net force exerted by mass A on mass B $= 192.5 - 59.15 = 133.35\,\text{N}$

The direction of motion will be affected by this force. According to the Newton's third law of motion, an equal amount of force will be exerted by mass B on mass A, i.e., $133.3\,\text{N}$, acting opposite to the direction of motion.

5.35 A block of mass $15\,\text{kg}$ is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with $0.5\,\text{ms}^{-2}$ for $20\,\text{s}$ and then moves with uniform velocity. Discuss the motion of the block as viewed by (a), a stationary observer on the ground, (b) an observer moving with the trolley.

Solution: Mass of the block, $m = 15\,\text{kg}$

Coefficient of static friction, $\mu = 0.18$

Acceleration of the trolley, $a = 0.5\,\text{m/s}^2$

According to Newton's second law of motion, the force F exerted on the block by the trolley's motio n is given by the equation:

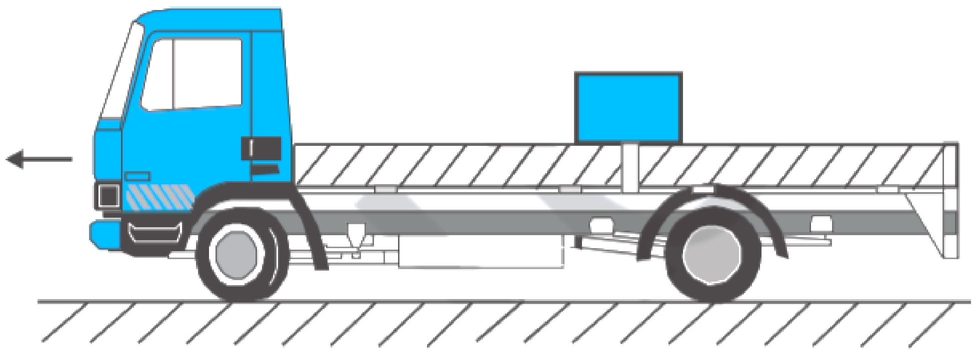
$$F = ma = 15 \times 0.5 = 7.5\,\text{N}$$

This force is applied in the trolley's forward motion.
The block and the trolley have a static friction force of:

$$f = \mu mg$$
$$= 0.18 \times 15 \times 10 = 27\,\text{N}$$

The applied external force is larger than the static friction force between the block and the trolley.
As a result, the block will appear to be at rest to a ground observer.
There will be no applied external force while the trolley moves at a constant speed.
In this case, the only force acting on the block is friction.
When travelling with the trolley, a spectator experiences some acceleration.
This is a noninertial frame of reference situation. A pseudo force of equal size opposes the frictional f orce pushing on the trolley rearward. This force, on the other hand, works in the opposite direction.
For the spectator travelling with the trolley, the trolley will appear to be at rest.

5.36 The rear side of a truck is open and a box of $40\,\mathrm{kg}$ mass is placed $5\,\mathrm{m}$ away from the open end as shown in Fig. 5.22. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with $2\,\mathrm{ms}^{-2}$. At what distance from the starting point does the box fall off the truck? (Ignore the size) of the box).



Solution: Mass of the box, $m = 40\,\mathrm{kg}$

Coefficient of friction, $\mu = 0.15$

Initial velocity, $u = 0$

Acceleration, $a = 2\,\mathrm{m/s^2}$

Distance of the box from the end of the truck, $s' = 5\,\mathrm{m}$

The force on the box induced by the truck's accelerated velocity is provided by Newton's second law o f motion:

$F = ma$
$= 40 \times 2 = 80\,\mathrm{N}$

A response force of
$80N$ is operating on the box in the rearward direction, according to Newton's third law of motion. The force of friction f operating between the box and the truck's floor opposes the bo x'sbackward motion. The following factors contribute to this force:
$f = \mu mg$
$= 0.15 \times 40 \times 10 = 60\,\mathrm{N}$

Net force acting on the block:

$F_{\text{net}} = 80 - 60 = 20\,\mathrm{N}$ backward

The backward acceleration produced in the box is given by:

$$a = \frac{F_{\text{net}}}{m} = \frac{20}{40} = 0.5\,\mathrm{m/s^2}$$

Using the second equation of motion, time t can be calculated as:

$$s' = ut + \frac{1}{2}a_{h+k}t^2$$

$$5 = 0 + \frac{1}{2} \times 0.5 \times t^2$$

$$\therefore t = \sqrt{20}\,\text{s}$$

Hence, the box will fall from the truck after $\sqrt{20}\,\text{s}$ from start.

The distance s, travelled by the truck in $\sqrt{20}\,\text{s}$ is given by the relation:

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 2 \times (\sqrt{20})^2$$

$$= 20\,\text{m}_2$$

5.37 A disc revolves with a speed of $33\frac{1}{3}\,\text{rev}/\text{min}$, and has a radius of $15\,\text{cm}$. Two coins are placed at $4\,\text{cm}$ and $14\,\text{cm}$ away from the centre of the record. If the co-efficient of friction between the coins and the record is 0.15, which of the coins will revolve with the record?

Solution: Coin placed at $4\,\text{cm}$ from the centre,

Mass of each $\text{coin} = m$

Radius of the disc, $r = 15\,\text{cm} = 0.15\,\text{m}$

Frequency of revolution,

$$v = 33\frac{1}{3} rev / min$$

$$= \frac{100}{3 \times 60} = \frac{5}{9}\,\text{rev}/\text{s}$$

Coefficient of friction, $\mu = 0.15$

The coin with a friction force higher than or equal to the centripetal force generated by the rotation of the disc will rotate with the disc in the present circumstance. If this isn't the case, the coin will fall out of the dispenser.
Coin placed at $4\,\text{cm}$:

Radius of revolution, $r^2 = 4\,\text{cm} = 0.04\,\text{m}$

Angular frequency, $\omega = 2\pi v = 2 \times \frac{22}{7} \times \frac{5}{9} = 3.49\,\text{s}^{-1}$

Frictional force, $f = \mu \text{mg} = 0.15 \times \text{m} \times 10 = 1.5\,\text{m}\,\text{N}$

Centripetal force on the coin:

$$F_{cent} = mr'e^2$$
$$= m \times 0.04 \times (3.49)^2$$
$$= 0.49m\,\text{N}$$

Since $f > F_{cent}$ the coin will revolve along with the record.

Coin placed at $14\,\text{cm}$:

Radius, $r'' = 14\,\text{cm} = 0$

Angular frequency, $\omega = 3.49\,\text{s}^{-1}$

Frictional force, $f = 1.5\,\text{m N}$

Centripetal force is given as:

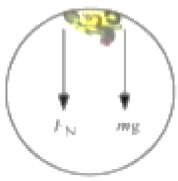
$$F_{cent} = mr^n e^2$$
$$= m \times 0.14 \times (3.49)^2$$
$$= 1.7m\,\text{N}$$

Since $f < F_{cent,}$, the coin will slip from the surface of the record.

5.38 You may have seen in a circus a motorcyclist driving in vertical loops inside a 'deathwell' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is $25\,\text{m}$?

Solution:

Because both the force of normal response and the weight of the biker act downward and are balanced by the centripetal force, a motorcyclist does not fall at the top point of a vertical loop in a death-well. The scenario is depicted in the diagram below.



The net force acting on the motorcyclist is the sum of the normal force (F_N) and the force due to gravity $(F_z = mg)$.

The equation of motion for the centripetal acceleration a_ε , can be written as:

$$F_{net} = ma_c$$
$$F_N + F_s = ma_e$$
$$F_N + mg = \frac{mv^2}{r}$$

The motorcyclist's speed provides a normal reaction.

At the minimum speed,

$$(v_{min}), F_N = 0mg = \frac{mv_{min}}{r}$$
$$\therefore v_{min} = \sqrt{rg}$$
$$= \sqrt{25 \times 10}$$
$$= 15.8\,\text{m/s}$$

5.39 A $70\,\text{kg}$ man stands in contact against the inner wall of a hollow cylindrical drum of radius 3m rotating about its vertical axis with 200rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?

Solution: Mass of the man, $m = 70\,\text{kg}$

Radius of the drum, $r = 3\text{m}$

Coefficient of friction, $\mu = 0.15$

Frequency of rotation,

$$v = 200 rev/min$$
$$= 200/60$$
$$= 10/3\ rev/s$$

The necessary centripetal force required for the rotation of the man is provided by the normal force $(F\text{N})$.

The guy adheres to the drum's wall while the floor circles.Hence, the weight of the man (mg) acting downward is balanced by the frictional force $\left(f = \mu F_N\right)$ acting upward.

Hence, the man will not fall until:

$$mg < fmg < \mu F_N$$
$$= \mu mr\omega^2 g < \mu r\omega^2$$
$$\omega > \sqrt{\frac{g}{\mu r}}$$

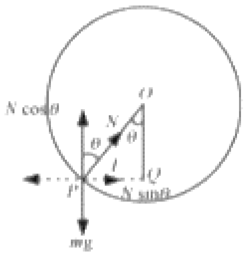
The minimum angular speed is given as:

$$\omega_{\min} = \sqrt{\frac{g}{\mu r}}$$

$$= \sqrt{\frac{10}{0.15 \times 3}} = 4.71 \text{rads}^{-1}$$

5.40 A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{g/R}$? Neglect friction.

Solution: Make an angle θ
between the radius vector between the bead and the centre and the vertical downward direction.



$\text{OP} = R = \text{Radius of the circle}$

$N = \text{Normal reaction}$

The vertical and horizontal force equations can be stated as follows:

$$Mg = N\cos\theta \ldots\ldots\ldots\ldots(i)$$
$$ml\omega^2 = N\sin\theta \ldots\ldots\ldots\ldots(ii)$$

In $\triangle OPQ$, we have:

$$\sin\theta = \frac{l}{R}$$
$$l = R\sin\theta \ldots\ldots\ldots(iii)$$

Substituting equation (iii) in equation (ii), we get:

$$m(R\sin\theta)\omega^2 = N\sin\theta$$
$$mR\omega^2 = N \ldots\ldots(iv)$$

Substituting equation (iv) in equation (i), we get:

$$mg = mR\omega^2\cos\theta$$

$$\cos\theta = \frac{g}{R\omega^2} \ldots.(v)$$

Since $\cos\theta \le 1$, the bead will remain at its lowermost point for $\frac{g}{R\omega^2} \le 1$, i.e.,

$$\omega \le \sqrt{\frac{g}{R}}$$

$$v = \sqrt{\frac{2g}{R}} or$$

$$\omega^2 = \frac{2g}{R} ....(iv)$$

On equating equations (v) and (iv), we get:

$$\frac{2g}{R} = \frac{g}{R\cos\theta}$$

$$\cos\theta = \frac{1}{2}$$

$$\therefore \theta = \cos^{-1}(0.5) = 60^{\circ}$$