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Chapter 6: WORK, ENERGY AND POWER

Examples

6.1 Find the angle between force $\mathbf{F}=(3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\mathbf{k})$ unit and displacement $\mathbf{d}=(5\hat{\mathbf{i}}+4\hat{\mathbf{j}}+3\mathbf{k})$ unit. Also find the projection of $\mathbf{F}$ on $\mathbf{d}$.

Solution:

$$\mathbf{F}\cdot\mathbf{d}=F_x d_x+F_y d_y+F_z d_z$$
$$=3(5)+4(4)+(-5)(3)$$
$$=16 \text{ unit}$$

As a result, $\mathbf{F}\cdot\mathbf{d}=Fd\cos\theta=16$ unit

$$\mathbf{F}\cdot\mathbf{F}=F^2$$
$$=F_x^2+F_y^2+F_z^2$$
$$=9+16+25$$
$$=50 \text{ unit}$$
$$=d^2=d_x^2+d_y^2+d_z^2$$
$$=25+16+9$$
$$=50 \text{ unit}$$
$$\therefore \cos\theta=\frac{16}{\sqrt{50}\sqrt{50}}=\frac{16}{50}=0.32,$$
$$\theta=\cos^{-1}0.32$$

6.2 It is well known that a raindrop falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass $1.00\,\text{g}$ falling from a height $1.00\,\text{km}$. It hits the ground with a speed of $50.0\,\text{m s}^{-1}$.

(a) What is the work done by the gravitational force?

Solution: The drop's decrease in kinetic energy is

$$\Delta K=\frac{1}{2}mv^2-0$$
$$=\frac{1}{2}\times10^{-3}\times50\times50$$
$$=1.25\,\text{J}$$

Where we've assumed the descent is at rest at first.

Let g is a constant having value $10\,\text{m}/\text{s}^2$,

The gravitational pull does the following work:

$$W_g=mgh$$
$$=10^{-3}\times10\times10^3$$
$$=10.0\,\text{J}$$

(b) What is the work done by the unknown resistive force?

Solution: According to the work-energy theorem,

$\Delta K = W_g + W_r$

Here, W_r is the work done on the raindrop by the resistive force.

$W_r = \Delta K - W_g$

$= 1.25 - 10$

$= -8.75\,\text{J}$

6.3 A cyclist comes to a skidding stop in $10\,\text{m}$. During this process, the force on the cycle due to the road is $200\,\text{N}$ and is directly opposed to the motion.

(a) How much work does the road do on the cycle?

Solution: The work done on the bike by the road is the work done on the cycle by the road's stopping (frictional) force. The stopping force and the displacement intersect at an angle of $180^{\circ}(\pi rad)$.

So, work done by the road,

$W_r = Fd\cos\theta$

$= 200 \times 10 \times \cos\pi$

$= -2000\,\text{J}$

According to the WE theorem, it is this negative work that draws the cycle to a halt.

(b) How much work does the cycle do on the road?

Solution:

The cycle, according to Newton's Third Law, exerts an equal and opposite force on the road.

It has a magnitude of $200\,\text{N}$. The road, on the other hand, does not move.

As a result, the work done by cycling on the road is nil.

6.4 In a ballistics demonstration a police officer fires a bullet of mass $50.0\,\text{g}$ with speed $200\,\text{m}\,\text{s}^{-1}$ on soft plywood of thickness $2.00\,\text{cm}$. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Solution: The initial kinetic energy of the bullet, $mv^2/2 = 1000\,\text{J}$.

It has a final kinetic energy of $0.1 \times 1000 = 100\,\text{J}$.

If the emergent speed of the bullet is v_f

$\frac{1}{2}mv_f^2 = 100\,\text{J}$

$v_f = \sqrt{\frac{2 \times 100\,\text{J}}{0.05\,\text{kg}}}$

$= 63.2\,\text{m}\,\text{s}^{-1}$

The speed is reduced by approx. 68%.

6.5 A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of $100\,\text{N}$ over a distance of $10\,\text{m}$. Thereafter, she gets progressively tired and her applied

force reduces linearly with distance to $50\,\text{N}$. The total distance through which the trunk has been moved is $20\,\text{m}$. Plot the force applied by the woman and the frictional force, which is $50\,\text{N}$ versus displacement. Calculate the work done by the two forces over $20\,\text{m}$.

Solution:

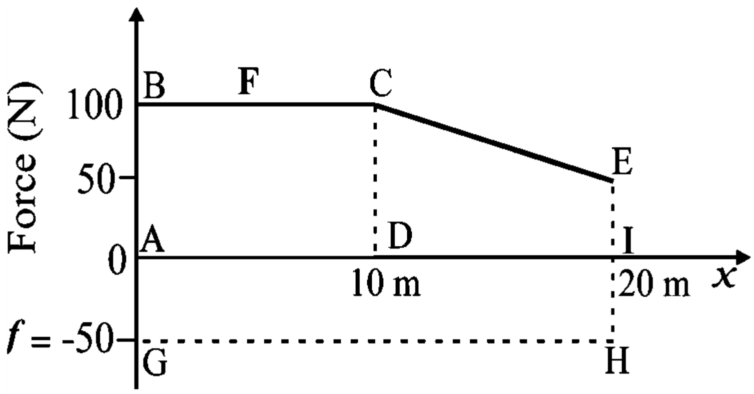


Figure shows a plot of the applied force.

At $x = 20\,\text{m}$, $F = 50\,\text{N}(\neq 0)$.

The frictional force f is $|\mathbf{f}| = 50\,\text{N}$.

It opposes motion and acts in the opposite direction of $\mathbf{F}$.

As a result, it is depicted on the minus side of the force axis.

The work done by the woman is

$W_F \rightarrow$ Area of the rectangle $\text{ABCD} +$ area of the trapezium CEID

$$W_F = 100 \times 10 + \frac{1}{2}(100 + 50) \times 10$$

$$= 1000 + 750$$

$$= 1750\,\text{J}$$

The frictional force's work is

$W_f \rightarrow$ Area of the rectangle $AGHI$,

$$W_f = (-50) \times 20 = -1000\,\text{J}$$

The region on the force axis's negative side has a negative sign.

6.6 A block of mass $m = 1\,\text{kg}$, moving on a horizontal surface with speed $v_1 = 2\,\text{ms}^{-1}$ enters a rough patch ranging from $x = 0.10\,\text{m}$ to $x = 2.01\,\text{m}$. The retarding force F_r on the block in this range is inversely proportional to x over this range,

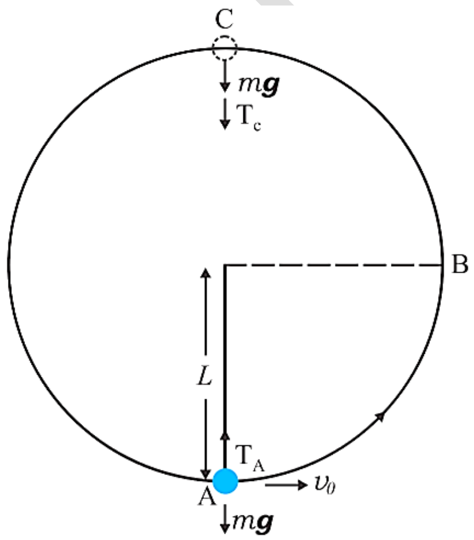
$F_r = \dfrac{-k}{x}$ for $0.1 < x < 2.01\,\text{m} = 0$ for $x < 0.1\,\text{m}$ and $x > 2.01\,\text{m}$, where $k = 0.5\,\text{J}$. What is the final kinetic energy and speed v_r of the block as it crosses this patch?

Solution:

$$K_f = K_i + \int_{0.1}^{2.01} \frac{(-k)}{x} dx$$
$$= \frac{1}{2} m v_i^2 - k \ln(x)\Big|_{0.1}^{2.01}$$
$$= \frac{1}{2} m v_i^2 - k \ln(2.01/0.1)$$
$$= 2 - 0.5 \ln(20.1)$$
$$= 2 - 1.5$$
$$= 0.5 \text{ J}$$
$$v_f = \sqrt{2K_f / m} = 1 \text{ m s}^{-1}$$

Note that the notation $\ln$ refers to the natural logarithm to the base e, not the logarithm to the base $10 \left[\ln X = \log_2 X = 2.303 \log_{10} X\right]$.

6.7 A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. This is shown in figure.



Obtain an expression for

(i) v_i **;**

Solution: The bob is subjected to two external forces: gravity and string tension (T).

The latter is useless since the bob's displacement is always normal to the string. The bob's potential en ergy is thus only related to the gravitational pull. The system's entire mechanical energy E is preserved.

At the lowest point A, we assume the system's potential energy is zero.

As a result, at A:

$$E = \frac{1}{2} m v_0^2$$

$$T_A - mg = \frac{m v_0^2}{L} \text{ [Newton's Second Law]}$$

Here T_A is the tension in the string at A. At the highest point C, the string slackens, as the tension in the string (T_c) becomes 0.

At C

$$E=\frac{1}{2}mv_c^2+2mgL$$

$$mg=\frac{mv_c^2}{L}$$ {Newton's Second Law}

Here v_C is the speed at C.

$$E=\frac{5}{2}mgL$$

Equating this to the energy at A

$$\frac{5}{2}mgL=\frac{m}{2}v_o^2$$

or, $v_0=\sqrt{5gL}$

(ii) The speeds at points B and C;

Solution:

$$v_C=\sqrt{gL}$$

At B, the energy is

$$E=\frac{1}{2}mv_B^2+mgL$$

Equating this to the energy at A and employing the result from (i), namely $v_o^2=5\text{gL}$,

$$\frac{1}{2}mv_B^2+mgL=\frac{1}{2}mv_o^2$$

$$=\frac{5}{2}mgL$$

$$\therefore v_B=\sqrt{3gL}$$

(iii) The ratio of the kinetic energies (K_B/K_0) at B and C. Comment on the nature of the trajectory of the bob after it reaches the point C.

Solution: The ratio of the kinetic energies at B and C is:

$$\frac{K_B}{K_C}=\frac{\frac{1}{2}mv_B^2}{\frac{1}{2}mv_C^2}=\frac{3}{1}$$

At point C, The bob's motion is horizontal and to the left as the string grows slack.
If the connecting thread is severed at this point, the bob will perform a projectile motion with horizont al projection, similar to a rock kicked horizontally from a cliff's edge.
Otherwise, the bob will complete the rotation by continuing on its circular route.

6.8 To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass $1000\,\mathrm{kg}$ moving with a speed $18.0\,\mathrm{km/h}$ on a smooth road and colliding with a horizontally mounted spring of spring constant $6.25\times10^{3}\,\mathrm{N\,m^{-1}}$. What is the maximum compression of the spring?

Solution:

At maximal compression, the car's kinetic energy is completely transferred into the spring's potential e nergy.

A moving car's kinetic energy is

$$K=\frac{1}{2}mv^{2}$$

$$K=\frac{1}{2}\times10^{3}\times5\times5$$

$$K=1.25\times10^{4}\,\mathrm{J}$$

Here, we changed $18\,\mathrm{km\,h^{-1}}$ to $5\,\mathrm{m\,s^{-1}}$ $[36\,\mathrm{km\,h^{-1}}=10\,\mathrm{m\,s^{-1}}$. At maximum compression x_m, the potential energy V of the spring $=$ the kinetic energy K of the moving car from the principle of conservation of mechanical energy.

$$V=\frac{1}{2}kx_m^{2}$$

$$x_m=2.00\,\mathrm{m}$$

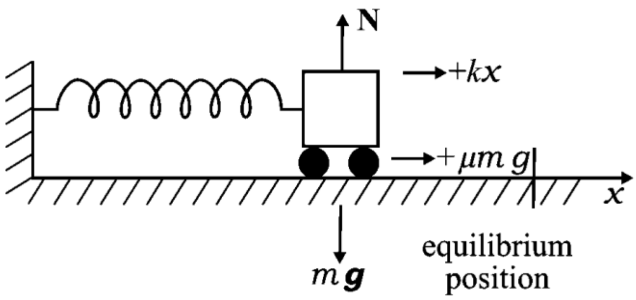
We acknowledge that we have exaggerated the problem. The spring is thought to have no bulk. The friction on the surface has been estimated to be minimal.

6.9 Consider Example 6.8 taking the coefficient of friction, μ, to be 0.5 and calculate the maximum compression of the spring.

Solution:

Both the spring force and the frictional force operate to counteract the compression of the spring in th e presence of friction, as shown in figure.

Rather of the conservation of mechanical energy, we use the work-energy theorem.



The change in kinetic energy is

$$\Delta K=K_f-K_i=0-\frac{1}{2}mv^{2}$$

The work done by the net force,

$$W=-\frac{1}{2}kx_m^{2}-\mu mgx_m$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 + \mu mgx_m$$

Now $\mu \text{mg} = 0.5\times10^3 \times 10 = 5\times10^3\ \text{N}$ ($g = 10.0\,\text{m s}^{-2}$).

We get the following quadratic equation in the unknown x_m by rearranging the preceding equation.

$$kx_m^2 + 2\mu mgx_m - mv^2 = 0$$

$$x_m = \frac{-\mu mg + \left[\mu^2 m^2 g^2 + mkv^2\right]^{1/2}}{k}$$

Since x_m is positive, we take the positive square root.

We get what we want by plugging in numerical values.

$x_m = 1.35\,\text{m}$

The conservation of mechanical energy formula will have to be changed if the two forces on the body are a conservative force F_c and a non-conservative force F_{nc}.

According to the WE theorem,

$$\left(F_c + F_{nc}\right)\Delta x = \Delta K$$

$$F_c \Delta x = -\Delta V$$

$$\Delta(K+V) = F_{nc}\Delta x$$

$$\Delta E = F_{nc}\Delta x$$

where E is the total mechanical energy.

$$E_f - E_i = W_{nc}$$

here W_{nc} is the total work done by the non-conservative forces over the path.

6.10 Examine Tables $6.1 - 6.3$ and express

(a) The energy required to break one bond in DNA in eV:

Solution: The amount of energy required to break one DNA link is

$$\frac{10^{-20}\ \text{J}}{1.6\times10^{-19}\ \text{J/eV}} \simeq 0.06\text{eV}$$

$0.1\text{eV} = 100\text{meV}.$

(b) The kinetic energy of an air molecule $\left(10^{-21} J\right)$ in eV ;

Solution: An air molecule's kinetic energy equals

$$\frac{10^{-21}\ \text{J}}{1.6\times10^{-19}\ \text{J/eV}} \simeq 0.0062\text{eV}$$

(c) The daily intake of a human adult in kilocalories.

Solution: A day's average human consumption is

$$\frac{10^{7}\ \text{J}}{4.2\times10^{3}\ \text{J/kcal}} \simeq 2400\text{kcal}$$

6.11 An elevator can carry a maximum load of $1800\,\mathrm{kg}$ (elevator + passengers) is moving up with a constant speed of $2\,\mathrm{m\,s^{-1}}$. The frictional force opposing the motion is $4000\,\mathrm{N}$. Determine the minimum power delivered by the motor to the elevator in watts as well as in horsepower.

Solution: The elevator's descending force is $F = mg + F_f = (1800 \times 10) + 4000 = 22000\,\mathrm{N}$

The motor must be powerful enough to counteract this force. As a result,

$P = \mathbf{F} \cdot \mathbf{v} = 22000 \times 2 = 44000\,\mathrm{W} = 59\mathrm{hp}$

6.12 Slowing down of neutrons: In a nuclear reactor a neutron of high speed (typically $10^7\,\mathrm{m\,s^{-1}}$) must be slowed to $10^3\,\mathrm{m\,s^{-1}}$ so that it can have a high probability of interacting with isotope ${}^{235}_{92}\mathrm{U}$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nuclei like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water $\left(\mathrm{D_2O}\right)$ or graphite, is called a moderator.

Solution: The neutron's initial kinetic energy is

$$K_{1i} = \frac{1}{2} m_1 v_{1i}^2$$

$$K_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_1 \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 v_{1i}^2$$

The kinetic energy lost as a proportion of total kinetic energy is

$$f_1 = \frac{K_{1f}}{K_{1i}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

whereas the moderating nuclei's fractional kinetic energy gain K_{2f} / K_{1i} is

$$f_2 = 1 - f_1 \text{(elastic collision) }\}$$

$$= \frac{4 m_1 m_2}{\left(m_1 + m_2\right)^2}$$

For deuterium $m_2 = 2m_1$ and we get $f_1 = 1/9$ whereas $f_2 = 8/9$. Almost 90% of the neutron's energy is travelled to deuterium. For carbon $f_1 = 71.6$ \% and $f_2 = 28.4\%$.

6.13 Consider the collision depicted in Fig. 6.10 to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to 'sink' the target ball in a corner pocket, which is at an angle $\theta_2 = 37^\circ$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1.

Solution: Because the masses are equivalent, momentum conservation applies.

$$\mathbf{v}_{1i} = \mathbf{v}_{1f} + \mathbf{v}_{2f}$$

$$v_{1i}{}^2 = \left(\mathbf{v}_{1f} + \mathbf{v}_{2f}\right) \cdot \left(\mathbf{v}_{1f} + \mathbf{v}_{2f}\right)$$

$$= v_{1f}{}^2 + v_{2f}{}^2 + 2\mathbf{v}_{1f} \cdot \mathbf{v}_{2f}$$

$$= \left\{v_{1f}^2 + v_{2f}^2 + 2v_{1f} v_{2f} \cos\left(\theta_1 + 37^\circ\right)\right\}$$

Because the collision is elastomeric and $m_1 = m_2$, The conservation of kinetic energy dictates that

$v_{1i}^{\ 2} = v_{1f}^{\ 2} + v_{2f}^{\ 2}$

Now, $\cos\left(\theta_1 + 37^\circ\right) = 0$

$\theta_1 + 37^\circ = 90^\circ$

So, $\theta_1 = 53^\circ$

This shows the following: when two equal masses collide in a glancing elastic collision with one of them at rest, they will move at right angles to each other after the impact.

Exercises

6.1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

Solution: Positive. Force and displacement are in the same direction in this example.

As a result, the indicator of completed job is good. The job is done on the bucket in this example.

(b) Work done by gravitational force in the above case,

Solution: Negative.

The direction of force (vertically downward) and displacement (vertically upward) are the polar opposites in this example. As a result, the indication of completed job is negative.

(c) Work done by friction on a body sliding down an inclined plane,

Solution: Negative

The work done by frictional force is negative in this scenario because the direction of frictional force is opposite the direction of motion.

(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

Solution: Positive. The body is travelling on a rough horizontal plane in this image.

The frictional force resists the body's motion.

As a result, a uniform force must be given to the body in order to maintain a consistent velocity.

The work done is positive because the applied force operates in the direction of the body's motion.

(e) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Solution: Negative.

Air has a resistive force that operates in the opposite direction of the pendulum's motion. As a result, the work completed in this example is negative.

6.2 A body of mass $2\,\text{kg}$ initially at rest moves under the action of an applied horizontal force of $7\,\text{N}$ on a table with coefficient of kinetic friction $= 0.1$. Compute the

(a) Work done by the applied force in $10\,\text{s}$,

Solution: Mass of the body, $m = 2\,\text{kg}$

Applied force, $F = 7\,\text{N}$

Coefficient of kinetic friction, $\mu = 0.1$

Initial velocity, $u = 0$

Time, $t = 10\,\text{s}$

Newton's second law of motion states that the applied force causes the body to accelerate:

$$a' = \frac{F}{m} = \frac{7}{2} = 3.5\,\text{m/s}^2$$

Frictional force is:

$$f = \mu m g$$

$$= 0.1 \times 2 \times 9.8 = -1.96\,\text{N}$$

The frictional force produces the following acceleration:

$$a'' = -\frac{1.96}{2} = -0.98\,\text{m/s}^2$$

Total acceleration of the body:

$$a = a' + a''$$

$$= 3.5 + (-0.98) = 2.52\,\text{m/s}^2$$

The equation of motion gives the distance travelled by the body:

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 2.52 \times (10)^2 = 126\,\text{m}$$

Work done by the applied force, $W_a = F \times s = 7 \times 126 = 882\,\text{J}$

(b) Work done by friction in $10\,\text{s}$,

Solution: Work done by the friction, $W_f - F \times s = -1.96 \times 126 = -247\,\text{J}$

(c) Work done by the net force on the body in $10\,\text{s}$,

Solution: Net force $= 7 + (-1.96) = 5.04\,\text{N}$

Work done by the net force, $W_{net} = 5.04 \times 126 = 635\,\text{J}$

(d) Change in kinetic energy of the body in $10\,\text{s}$, and interpret your results.

Solution: The final velocity may be determined using the first equation of motion:

$$v = u + at$$

$$= 0 + 2.52 \times 10 = 25.2\,\text{m/s}$$

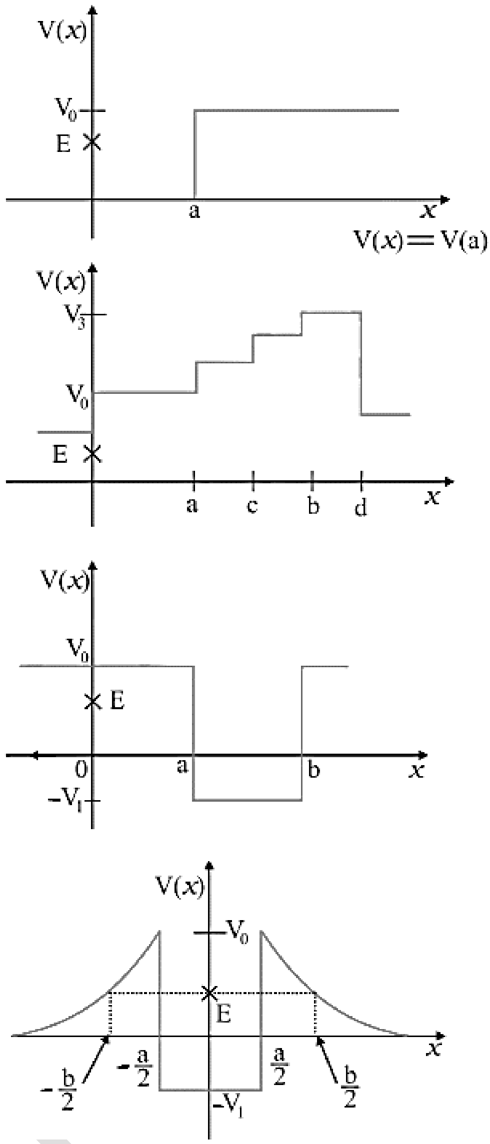
Change in kinetic energy,

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2} \times 2\left(v^2 - u^2\right) = (25.2)^2 - 0^2 = 635\,\text{J}$$

6.3 Given in figure are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the

minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.



Solution: $x > a$; 0

The relationship gives the total energy of a system:

$E = \text{P.E.} + \text{K.E}$

$\therefore \text{K.E.} = E - \text{P.E.}$

A body's kinetic energy is a positive number.
It can't possibly be negative. As a result, at a region where K.E. turns negative, the particle will not ex ist.

For $x > a$, the particle's potential energy (V_0) exceeds its total energy (E) in the given situation.

As a result, kinetic energy in this area is negative. As a result, the particle will not be found in this are a.

The particle's minimal total energy is zero.

All regions

In the given case, the potential energy (V_0) is more than total energy (E) in all regions.

As a result, the particle will not be found in this area.

$x > a$ and $x < b; -V_1$

In the given case, the condition regarding the positivity of K.E , is o.k. only in the region between $x > a$ and $x < b$.

In this example, the minimal potential energy is

$-V_1$.

$$\text{K.E.} = E - (-V_1) = E + V_1$$

As a result, for the kinetic energy to be positive, the particle's total energy must be larger than $-V_1$.

As a result, the particle's minimal total energy is

$$-V_1 . -\frac{b}{2} < x < \frac{a}{2}; \quad \frac{a}{2} < x < \frac{b}{2}; -V_1$$

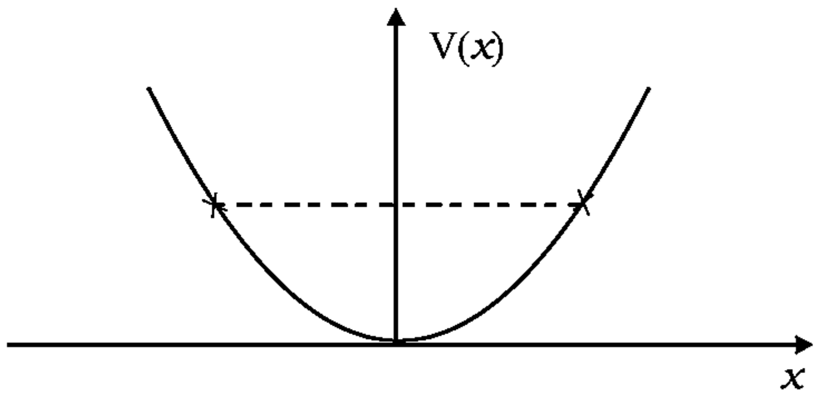
In the given case, the potential energy (V_0) of the particle becomes greater than the total energy (E)

for $-\frac{b}{2} < x < \frac{b}{2}$

$-\frac{a}{2} < x < \frac{a}{2}$

As a result, the particle will not be found in these areas.

6.4 The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5\,\text{N m}^{-1}$. The graph of $V(x)$ versus x is shown in Fig. 6.12. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2\,\text{m}$.



Solution: Total energy of the particle, $E = 1\text{J}$

Force constant, $k = 0.5\,\text{N m}^{-1}$

Kinetic energy of the particle, $K = \frac{1}{2}mv^2$

Using conservation law:

$E = V + K$

$$1 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

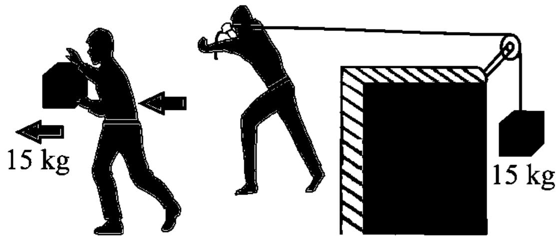
When you 'turn back,' your velocity (and so your K) becomes zero.

$\therefore \quad 1=\frac{1}{2}kx^2$

$\frac{1}{2}\times 0.5.x^2 = 1nx^2 = 4nx = \pm 2$

As a result, the particle turns back when $x=\pm 2\,\text{m}$.

6.5 Solution the following:



(i) (ii)

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?
Solution:
The burning of a rocket's casing in flight (due to friction) results in the rocket's mass being reduced.
By using conservation of energy:
Total Energy (T.E.) = Potential energy (P.E.) + Kinetic energy (K.E.)

$= mgh + \frac{1}{2}mv^2$

The overall energy decreases as the mass of the rocket decreases.
As a result, the rocket provides the heat energy necessary for the burning.

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?
Solution: The force of gravity is a conservative force.
The work done by the gravitational force across every full orbit of a comet is zero because the work d one by a conservative force over a closed route is zero.

(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed mere's progressively as it comes closer and closer to the earth?
Solution:
Because of the decrease in height, the potential energy of an artificial satellite circling the earth reduce s as it gets closer to it. The decrease in P.E. leads in an increase in K.E. since the total energy of the sy stem stays constant. As a result, the satellite's velocity rises. However, owing to air friction, the satelli te's overall energy reduces somewhat.

(d) In figure (i) the man walks $2\,\text{m}$ carrying a mass of $15\,\text{kg}$ on his hands. In figure (ii). He walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of $15\,\text{kg}$ hangs at its other end. In which case is the work done greater?

Solution: Case (i)

Mass, $m = 15\,\text{kg}$

Displacement, $s = 2\,\text{m}$

Work done, $W = Fs\cos\theta$

Here, $\theta =$ Angle between force and displacement

$= mgs\cos\theta = 15 \times 2 \times 9.8\cos 90^{\circ}$

$= 0$

Case $(\mathbf{ii})$

Mass, $m = 15$ kg

Displacement, $s = 2\,\text{m}$

The force applied to the rope and the direction of the rope's displacement are the same in this case.

Therefore, the angle between them, $\theta = 0^{\circ}$

Since $\cos 0^{\circ} = 1$

Work done, $W = Fs\cos\theta = mgsn = 15 \times 9.8 \times 2 = 294\,\text{J}$

As a result, in the second situation, more work is done.

6.6 Underline the correct alternative:

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

Solution: Decreases.

When a conservative force displaces a body in the direction of force, it puts a positive work on it.

As a result, the body moves closer to the centre of gravity. It reduces the distance between the two, th erefore lowering the body's potential energy.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

Solution: Kinetic energy.

The velocity of a body is reduced when work is done against the direction of friction.

As a result, the body's kinetic energy is depleted.

(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.

Solution: External force.

Internal forces, regardless of their direction, cannot cause a change in a body's overall momentum. As a result, a many particle system's overall momentum is proportional to the external forces operating on it.

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Solution: Total linear momentum.

Whether an elastic or inelastic collision occurs, the total linear momentum is always preserved.

6.7 State if each of the following statements is true or false. Give reasons for your Solution.

(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

Solution: False,

In an elastic collision, the entire energy and momentum of both bodies is preserved, not the energy an d momentum of each individual body.

(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

Solution: False, Internal forces are balanced, yet they do not cause any labour on the body.
External forces are the ones who can make things happen.
As a result, external influences have the ability to affect a system's energy.

(c) Work done in the motion of a body over a closed loop is zero for every force in nature.

Solution: False,
For a conservation force alone, the work done in the motion of a body across a closed loop is zero.

(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Solution: True,
The ultimate kinetic energy of an inelastic collision is always smaller than the system's initial kinetic e nergy. This is because there is always a loss of energy in the form of heat, sound, and other forms of e nergy in such collisions.

6.8 Solution carefully, with reasons:

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)?

Solution: No,
The entire initial kinetic energy of the balls will be equal to the total final kinetic energy of the balls in an elastic collision.
The kinetic energy of the balls is not preserved when they come into touch with each other.
In reality, the kinetic energy of the balls will be transformed into potential energy during follision.

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?

Solution: Yes, in an elastic collision, the system's entire linear momentum is always preserved.

(c) What are the Solutions to (a) and (b) for an inelastic collision?

Solution: No,
There is always a loss of kinetic energy in an inelastic collision, i.e. the total kinetic energy of the billi ard balls before impact is always larger than the total kinetic energy after collision.

(d) If the potential energy of two billiard balls depends only on the separation distance between their centers, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Solution: Yes, conservation factors are at work in this situation.
This is because they rely on the distance between the billiard balls' centres.
As a result, the collision is pliable.Even in the event of an inelastic collision, the overall linear momen tum of the billiards balls system will be preserved.

6.9 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{1/2}$

(ii) t

(iii) $t^{3/2}$

(iv) t^2

Solution: (ii) t

Mass of the body $= m$

Acceleration of the body $= a$

The force experienced by the body is provided by the equation: Using Newton's second rule of motion , the force experienced by the body is given by the equation:

$$F = ma$$

Both m and a are constants. As a result, force F will also be a constant.

$$F = ma = Constant...(i)$$

For velocity v, acceleration, $a = \frac{dv}{dt} =$ Constant

$$dv = \text{ Constant } \times dt$$

$$v = \alpha t$$

Where, α is another constant

$$v \propto t...(iii)$$

Power, $P = F \cdot v$

Using equations (i) and (iii) :

$$P \propto t$$

As a result, $P \propto t$

6.10 A body is moving unidirectional under the influence of a source of constant power. Its displacement in time t is proportional to

(i) $t^{1/2}$

(ii) t

(iii) $t^{3/2}$

(iv) t^2

Solution: (iii) $t^{\frac{3}{2}}$

Power, $\&P = F\ v$

$$= mav = mv\frac{dv}{dt} = \text{ Constant } (k)$$

$$\therefore vdv = \frac{k}{m}dt$$

Integrating both sides:

$$\frac{v^2}{2} = \frac{k}{m}tv = \sqrt{\frac{2kt}{m}}$$

For displacement x of the body:

$$v = \frac{dx}{dt} = \sqrt{\frac{2k}{m}}t^{\frac{1}{2}}dx = k^{'}t^{\frac{1}{2}}dt$$

Here $k^{'} = \sqrt{\frac{2k}{3}} =$ New constant

On integrating both sides:

$x=\frac{2}{3}k't^{\frac{3}{2}}$

$\therefore x \propto t^{\frac{3}{2}}$

6.11 A body constrained to move along the $z-$axis of a coordinate system is subject to a constant force F given by

$\mathbf{F}=-\bar{\mathbf{i}}+2\bar{\mathbf{j}}+3\bar{\mathbf{k}}\text{N}$

Where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\mathbf{k}$ are unit vectors along the $x-y$ $-$and $z-$axis of the system respectively. What is the work done by this force in moving the body a distance of 4m along the $z-$axis ?

Solution: Force, $\mathbf{F}=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\mathbf{kN}$

Displacement, $s=4\mathbf{k}\text{m}$

Work done, $W-F.s$

$=(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\mathbf{k})\cdot(4\mathbf{k})$

$=0+0-3\times4$

$=12\,\text{J}$

As a result, work done is $12\,\text{J}$.

6.12 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10keV, and the second with 100keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (Electron mass $=9.11\times10^{-31}$ kg, proton mass $=1.67\times10^{-27}$ kg, $1\text{eV}=1.60\times10^{-19}$ J

Solution: The Electron is faster; Ratio of their speeds is $13.54:1$

Mass of the electron, $m_e=9.11\times10^{-31}$ kg

Mass of the proton, $m_p=1.67\times10^{-27}$ kg

$K.E.$ of the electron, $E_{Ke}=10\text{keV}=10^4\text{eV}$

$=10^4\times1.60\times10^{-19}$

$=1.60\times10^{-15}$ J

$K.E.$ of the proton, $E_K=100\text{keV}=10^5\text{eV}=1.60\times10^{-14}$ J

The kinetic energy of an electron ve is determined by the following relationship:

$E_{Ke}=\frac{1}{2}mv_c^2$

$\therefore v_e=\sqrt{\frac{2\times E_{Ke}}{m}}$

$=\sqrt{\frac{2\times1.60\times10^{-15}}{9.11\times10^{-31}}}=5.93\times10^7\ \text{m/s}$

$E_{\mathrm{Kp}} = \frac{1}{2} m v_{\mathrm{p}}^2 v_p$

$= \sqrt{\frac{2 \times E_{Kp}}{m}}$

$\therefore v_p = \sqrt{\frac{2 \times 1.6 \times 10^{-14}}{1.67 \times 10^{-27}}} = 4.38 \times 10^6 \mathrm{~m/s}$

As a result, the electron travels quicker than the proton.

The following is the ratio of their speeds:

$\frac{v_{\mathrm{c}}}{v_{\mathrm{p}}} = \frac{5.93 \times 10^7}{4.38 \times 10^6} = 13.54 : 1$

6. 13 A rain drop of radius $2\,\mathrm{mm}$ falls from a height of $500\,\mathrm{m}$ above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is $10\,\mathrm{m\,s^{-1}}$?

Solution: Rain drop's radius is , $r = 2\,\mathrm{mm} = 2 \times 10^{-3}\,\mathrm{m}$

Rain drop's Volume,

$V = \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \mathrm{m}^{-3}$

Density of water, $\rho = 10^3\,\mathrm{kg\,m}^{-3}$

Rain drop's Mass, $m = \rho V$

$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3\,\mathrm{kg}$

Gravitational force, $F = m\mathrm{g}$

$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8\,\mathrm{N}$

In the first half of its travel, the gravitational force did the following work on the drop:

$W_1 = Fs$

$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8 \times 250$

$= 0.082\,\mathrm{J}$

This quantity of effort is the same as the work done on the drop by gravity in the second half of its jou rney, i.e.

$W_{\mathrm{I},} = 0.082\,\mathrm{J}$

The total energy of the rain drop will remain the same if there is no resistive force present, according t o the law of conservation of energy.

Total energy at the top,

$E_{\mathrm{T}} = mgh + 0$

$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8 \times 500 \times 10^{-5}$

$= 0.164\,\mathrm{J}$

$10\,\mathrm{m/s}.$

The drop hits the ground at a velocity of due to the presence of a resistive force.

Total energy at the ground:

$E_{\mathrm{G}} = \frac{1}{2} mv^2 + 0$

$= \frac{1}{2} \times \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8 \times (10)^2$

$= 1.675 \times 10^{-3}\,\mathrm{J}$

Resistive force $= E_{\mathrm{G}} - E_{\mathrm{T}} = -0.162\,\mathrm{J}$

6.14 A molecule in a gas container hits a horizontal wall with speed $200\,\mathrm{m\,s^{-1}}$ and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Solution: Collision is elastic, therefore yes.

Whether the collision is elastic or inelastic, the gas molecule's momentum is preserved.

The gas molecule has a velocity of $200\,\mathrm{m/s}$ and collides with the container's stationary wall, rebounding at the same speed.

It demonstrates that the wall's rebound velocity stays zero.

As a result, the molecule's entire kinetic energy is preserved during the impact.

An elastic collision is exemplified by the above collision.

6.15 A pump on the ground floor of a building can pump up water to fill a tank of volume $30\,\mathrm{m^3}$ in $15\,\mathrm{min}$. If the tank is $40\,\mathrm{m}$ above the ground, and the efficiency of the pump is 30%. How much electric power is consumed by the pump?

Solution: Volume of the tank, $V = 30\,\mathrm{m^3}$

Time of operation, $t = 15\,\mathrm{min} = 15 \times 60 = 900\,\mathrm{s}$

Height of the tank, $h = 40\,\mathrm{m}$

Efficiency of the pump, $\eta = 30\%$

Density of water, $\rho = 10^3\,\mathrm{kg/m^3}$

Mass of water, $m = \rho V = 30 \times 10^3\,\mathrm{kg}$

Output power:

$P_0 = \frac{\text{Work done}}{\text{Time}} = \frac{mgh}{t}$

$= \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 13.067 \times 10^3\,\mathrm{W}$

For input power P_i, efficiency η,

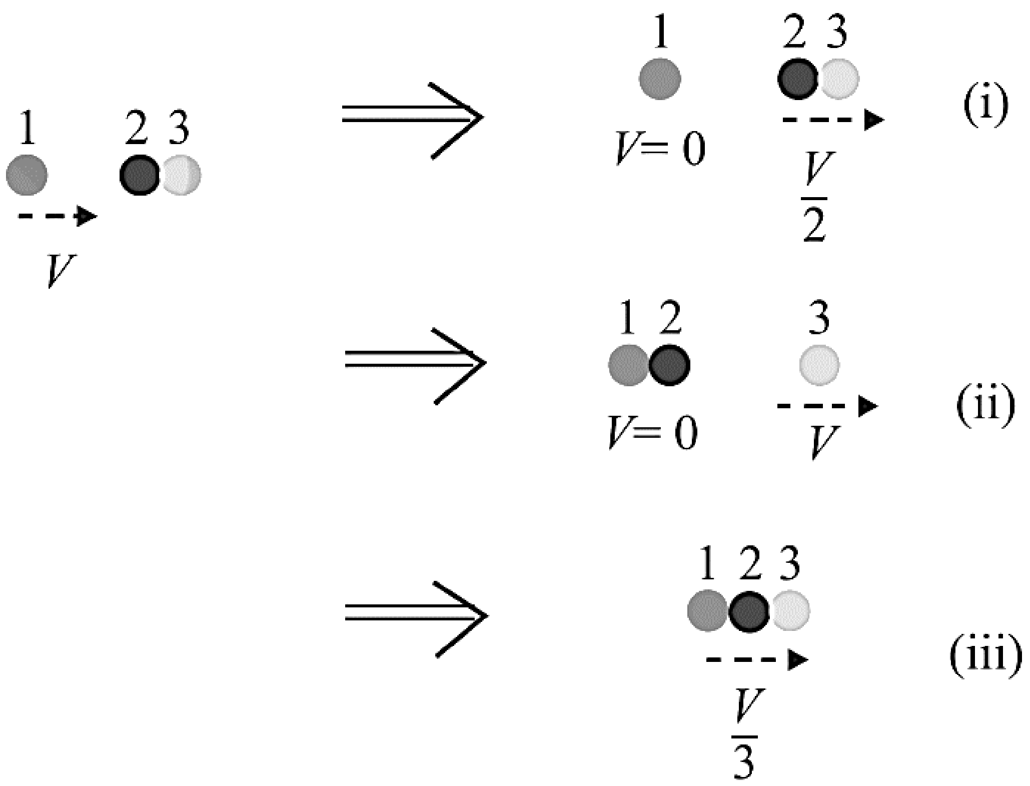
$\eta = \frac{P_0}{P_i} = 30\%$

$$P_i = \frac{13.067}{30} \times 100 \times 10^3$$

$$= 0.436 \times 10^5 \text{ W}$$

$$= 43.6 \text{ kW}$$

6.16 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following is a possible result after collision?



Solution: Case (ii)

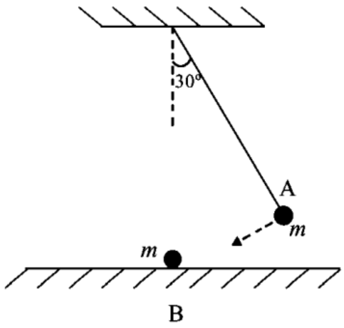
It is clear that the overall momentum before and after impact is constant in each situation.

The total kinetic energy of a system remains constant before and after an elastic collision.Before the i mpact, the system's total kinetic energy was:

$$= \frac{1}{2} mV^2 + \frac{1}{2}(2m)0$$

$$= \frac{1}{2} mV^2$$

6.17 The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in figure. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.



Solution: Bob A will not rise after the collision.
After an elastic collision between two equal masses, one of which is stationary and the other of which is travelling with a velocity, the stationary mass obtains the same velocity as the moving body, while t he moving mass comes to a complete stop.
A full transfer of momentum from the advancing mass to the stationary mass occurs in this situation.
As a result, bob A of mass m, Will come to a halt after colliding with a bob
B of same mass, whereas bob B will travel at the same velocity as bob A at the time of contact.

6.18 The bob of a pendulum is released from a horizontal position. If the length of the pendulum Is 1.5m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?

Solution: Length of the pendulum, $l = 1.5\text{m}$
Mass of the bob $= m$
Energy dissipated $= 5\%$
The overall energy of the system remains constant according to the rule of conservation of energy.
Positioned horizontally:
Potential energy of the bob, $E_P = mgl$
Kinetic energy of the bob, $E_K = 0$
Total energy $= mgl \ldots (i)$
At the very bottom (mean position):
Potential energy of the bob, $E_P = 0$
Kinetic energy of the bob, $E_K = \frac{1}{2}mv^2$
Total energy $E_x = \frac{1}{2}mv^2 ...(ii)$
5% of the bob's energy is wasted as it goes from the horizontal position to the lowest point.
The entire energy at the bottom is 95% of the total energy at the top, i.e.,

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgl$$

$$\therefore v = \sqrt{\frac{2 \times 95 \times 1.5 \times 9.8}{100}} = 5.28\text{m/s}$$

6.19 A trolley of mass 300kg carrying a sandbag of 25kg is moving uniformly with a speed of 27km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of

the trolley at the rate of $0.05\,\text{kg}\,\text{s}^{-1}$. What is the speed of the trolley after the entire sand bag is empty?

Solution: The sand bag is loaded onto a trolley that moves at a constant pace of $27\,\text{km/h}$.
The external forces operating on the sandbag and trolley system are nil.
There will be no change in the trolley's velocity when the sand begins to flow from the bag.
This is due to the fact that the leaking activity has no external force on the system.
According to Newton's first law of motion, this is the case.
As a result, the trolley's speed will remain $27\,\text{km/h}$.

6.20 A body of mass $0.5\,\text{kg}$ travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\,\text{m}^{-1/2}\,\text{s}^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2\,\text{m}$?

Solution: Mass of the body, $m = 0.5\,\text{kg}$

Velocity of the body, $v = ax^{\frac{3}{2}}$ with $a = 5\,\text{m}^{\frac{-1}{2}}\,\text{s}^{-1}$

Initial velocity, $u(x=0) = 0$

Final velocity $v($ at $x = 2\,\text{m}) = 10\sqrt{2}\,\text{m/s}$

Work done, $W =$ Change in kinetic energy

$= \frac{1}{2}m\left(v^2 - u^2\right)$

$= \frac{1}{2}\times 0.5\left[(10\sqrt{2})^2 - (0)^2\right]$

$= \frac{1}{2}\times 0.5\times 10\times 10\times 2$

$= 50\,\text{J}$

6.21 The blades of a windmill sweep out a circle of area A.

(a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t?

Solution: Area of the circle swept by the windmill $= A$
Velocity of the wind $= v$
Density of air $= \rho$

Volume of the wind flowing through the windmill / sec $= Av$

Mass of the wind flowing through the windmill / sec $= \rho Av$

Mass m, of the wind flowing through the windmill in time $t = \rho Avt$

(b) What is the kinetic energy of the air?

Solution: Kinetic energy of air,

$= \frac{1}{2}mv^2$

$= \frac{1}{2}(\rho Avt)v^2$

$= \frac{1}{2}\rho Av^3 t$

(c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30\,\text{m}^2, v = 36\text{km/h}$ and the density of air is $1.2\,\text{kg}\,\text{m}^{-3}$. What is the electrical power produced?

Solution: Area of the circle swept by the windmill $= A = 30\,\text{m}^2$

Velocity of the wind $= v = 36\,\text{km/h}$

Density of air, $\rho = 1.2\,\text{kg}\,\text{m}^{-3}$

Electric energy produced $= 25\%$ of the wind energy $= \frac{25}{100} \times K.E.$ of air

$= \frac{1}{8}\rho A v^3 t$

Electrical power $= \frac{\text{Electrical energy}}{\text{Time}}$

$= \frac{1}{8}\frac{\rho A v^3 t}{t}$

$= \frac{1}{8}\rho A v^3 = \frac{1}{8} \times 1.2 \times 30 \times (10)^3$

$= 4.5 \times 10^3\,\text{W} = 4.5\,\text{kW}$

6.22 A person trying to lose weight (dieter) lifts a $10\,\text{kg}$ mass, one thousand times, to a height of $0.5\,\text{m}$ each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force?

Solution: Mass of the weight, $m = 10\,\text{kg}$

The person's height at which the weight is lifted, $h = 0.5\,\text{m}$

Number of times the weight is lifted, $n = 1000$

Work done against gravitational force:

$= n(mgh)$

$= 1000 \times 10 \times 9.8 \times 0.5$

$= 49 \times 10^3\,J = 49\,\text{kJ}$

(b) Fat supplies $3.8 \times 10^7\,\text{J}$ of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

Solution: Energy equivalent of 1kg of fat $= 3.8 \times 10^7\,\text{J}$

Efficiency rate $= 20\%$

The person's body provides mechanical energy in the form of:

$= \frac{20}{100} \times 3.8 \times 10^7\,\text{J} = \frac{1}{5} \times 3.8 \times 10^7\,\text{J}$

Dieter's equivalent mass of fat lost:

$$= \frac{1}{\frac{1}{5}\times 3.8\times 10^{7}}\times 49\times 10^{3}$$

$$= \frac{245}{3.8}\times 10^{-4}$$

$$= 6.45\times 10^{-3}\,\text{kg}$$

6.23 A family uses $8\,\text{kW}$ of power.

(a) Direct solar energy is incident on the horizontal surface at an average rate of $200\,\text{W}$ per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply $8\,\text{kW}$?

Solution: $200\,\text{m}^2$

Power used by the family, $P = 8\,\text{kW} = 8\times 10^3\,\text{W}$

Solar energy received per square meter $= 200\,\text{W}$

Conversion efficiency from solar to electricity energy $= 20\%$

Area required to generate the desired electricity $= A$

Now,

$$8\times 10^3 = 20\%\times (A\times 200)$$

$$= \frac{20}{100}\times A\times 200$$

$$\therefore A = \frac{8\times 10^3}{40} = 200\,\text{m}^2$$

(b) Compare this area to that of the roof of a typical house.

Solution: The size of a solar plate required to create $8\,\text{kW}$ of power is about equal to the roof area of a structure with dimensions of $14\,\text{m}\times 14\,\text{m}$.

Additional Exercises

6.24 A bullet of mass $0.012\,\text{kg}$ and horizontal speed $70\,\text{m s}^{-1}$ strikes a block of wood of mass $0.4\,\text{kg}$ and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Solution: Mass of the bullet, $m = 0.012\,\text{kg}$

Initial speed of the bullet, $u_b = 70\,\text{m/s}$

Mass of the wooden block, $M = 0.4\,\text{kg}$

Initial speed of the wooden block, $u_B = 0$

The final speed of the bullet and block system $= v$

Using the law of conservation of momentum:

$$mu_b + Mu_B = (m+M)v$$

$$0.012\times 70 + 0.4\times 0 = (0.012+0.4)v$$

$\therefore v=\frac{0.84}{0.412}=2.04\,\text{m/s}$

The bullet and wooden block system is as follows:

Mass of the system, $m'=0.412\,\text{kg}$

Velocity of the system $=2.04\,\text{m/s}$

Height up to which the system rises $=h$

Using the law of conservation of energy:

At the greatest point, potential energy = kinetic energy at the lowest position

$$m'gh=\frac{1}{2}m'v^2$$

.

$$\therefore h=\frac{1}{2}\left(\frac{v^2}{g}\right)$$

$$=\frac{1}{2}\times\frac{(2.04)^2}{9.8}$$

$$=0.2123\,\text{m}$$

The wooden block will reach a height of $0.2123\,\text{m}$.

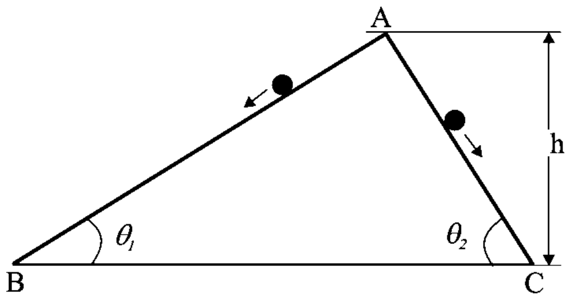
Heat produced $=$ Kinetic energy of the bullet $-$ Kinetic energy of the system

$$=\frac{1}{2}mu^2-\frac{1}{2}m'v^2$$

$$=\frac{1}{2}\times0.012\times(70)^2-\frac{1}{2}\times0.412\times(2.04)^2$$

$$=29.4-0.857=28.54\,\text{J}$$

6.25 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig. 6.16). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1=30^\circ,\theta_2=60^\circ$, and $h=10\,\text{m}$, what are the speeds and times taken by the two stones?



Solution: No, the stone that is travelling down the steep plane will be the first to reach the bottom. Yes, the stones will all arrive at the same time at the bottom.

$v_B=v_C=14\,\text{m/s}$

$t_1=2.86\,\text{s};$

$t_2=1.65\,\text{s}$

The kinetic energy of the stones at positions B and C will likewise be the same according to the rule o f conservation of energy, i.e.

$\frac{1}{2}mv_1^2=\frac{1}{2}mv_2^2$

$v_1=v_2=v$,

Where, $m=$ Mass of each stone

$v=$ Speed of each stone at points B and C

As a result, both stones will arrive at the bottom at the same time, v.

For stone I:

Net force:

$F_{\text{net}}=ma_1=mg\sin\theta_1$

$a_1=g\sin\theta_1$

For stone II:

$a_2=g\sin\theta_2$

$\because \theta_2>\theta_1$

$\therefore \sin\theta_2>\sin\theta_1$

$\therefore a_2>a_1$

Applying the first equation of motion,

$v=u+at$

$\therefore t=\frac{v}{a}\quad (\because u=0)$

For stone I:

$t_1=\frac{v}{a_1}$

For stone II:

$t_2=\frac{v}{a_2}$

$\because a_2>a_1$

$\therefore t_2<t_1$

As a result, the stone travelling down the steep plane will be the first to reach the bottom.

$mgh=\frac{1}{2}mv^2$

$\therefore v=\sqrt{2gh}$

$=\sqrt{2\times 9.8\times 10}$

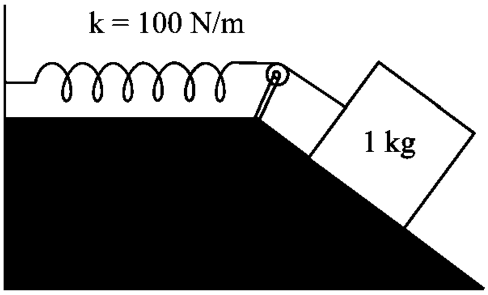
$=\sqrt{196}=14\,\text{m/s}$

The times are:

$t_1=\frac{v}{a_1}=\frac{v}{g\sin\theta_1}=\frac{14}{9.8\times\sin 30}=\frac{14}{9.8\times\frac{1}{2}}=2.86\,\text{s}$

$t_2=\frac{v}{a_2}=\frac{v}{g\sin\theta_2}=\frac{14}{9.8\times\sin 60}=\frac{14}{9.8\times\frac{\sqrt{3}}{2}}=1.65\,\text{s}$

6.26 A 1kg block situated on a rough incline is connected to a spring of spring constant 100Nm^{-1} as shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.



Solution: Mass of the block, $m=1\text{kg}$

Spring constant, $k=100\,\text{N}\,\text{m}^{-1}$

Displacement in the block, $x=10\text{cm}=0.1\text{m}$

At equilibrium:

Normal reaction, $R=mg\cos 37^{\circ}$

Frictional force, $f=\mu_R=mg\sin 37^{\circ}$

Where, coefficient of friction $=\mu$

Net force acting on the block $=mg\sin 37^{\circ}-f$

$=mg\sin 37^{\circ}-\mu mg\cos 37^{\circ}$

$=mg\left(\sin 37^{\circ}-\mu\cos 37^{\circ}\right)$

$mg\left(\sin 37^{\circ}-\mu\cos 37^{\circ}\right)x$

$=\frac{1}{2}kx^2 1\times 9.8\left(\sin 37^{\circ}-\mu\cos 37^{\circ}\right)$

$=\frac{1}{2}\times 100\times 0.10.602-\mu\times 0.799=0.510$

$\therefore \mu=\frac{0.092}{0.799}=0.115$

6.27 A bolt of mass 0.3kg falls from the ceiling of an elevator moving down with an uniform speed of 7ms^{-1}. It hits the floor of the elevator (length of the elevator $=3\text{m}$) and does not rebound. What is the heat produced by the impact? Would your Solution be different if the elevator were stationary?

Solution: Mass of the bolt, $m=0.3\text{kg}$

Speed of the elevator $=7\text{m}/\text{s}$

Height, $h=3\text{m}$

Because the bolt's relative velocity to the lift is zero, potential energy is transferred to thermal energy at the instant of impact.

Heat produced $=$ Loss of potential energy

$= mgh = 0.3 \times 9.8 \times 3$

$= 8.82\,\text{J}$

Even if the lift is motionless, the amount of heat produced remains constant.
This is due to the fact that the bolt's relative velocity in relation to the lift will stay zero.

6.28 A trolley of mass $200\,\text{kg}$ moves with a uniform speed of $36\,\text{km/h}$ on a frictionless track. A child of mass $20\,\text{kg}$ runs on the trolley from one end to the other ($10\,\text{m}$ away) with a speed of $4\,\text{ms}^{-1}$ relative to the trolley in a direction opposite to the its motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Solution: Mass of the trolley, $M = 200\,\text{kg}$

Speed of the trolley, $v = 36\,\text{km/h} = 10\,\text{m/s}$

Mass of the boy, $m = 20\,\text{kg}$

The boy's and trolley's systems' initial velocity

$= (M + m)v$

$= (200 + 20) \times 10$

$= 2200\,\text{kg m/s}$

Suppose, final velocity of the trolley $= v'$

Final velocity of the boy $= v' - 4$

Final momentum $= Mv' + m\left(v' - 4\right)$

$= 200v' + 20v' - 80$

$= 220v' - 80$

Acc. to law of conservation of momentum:

Initial momentum $=$ Final momentum

$2200 = 220v' - 80$

$\therefore v' = \frac{2280}{220} = 10.36\,\text{m/s}$

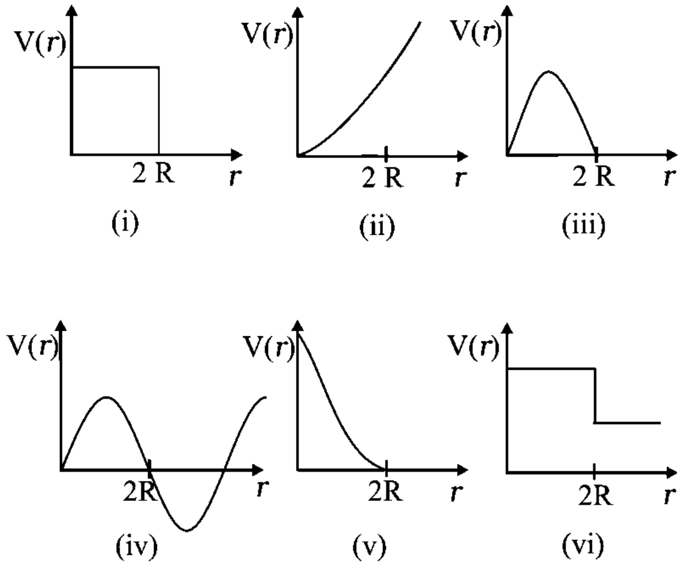
Length of the trolley, $l = 10\,\text{m}$

Speed of the boy, $v'' = 4\,\text{m/s}$

Time taken by the boy to run, $t = \frac{10}{4} = 2.5\,\text{s}$

Distance moved by the trolley $= v'' \times t = 10.36 \times 2.5 = 25.9\,\text{m}$

6.29 Which of the following potential energy curves in figure cannot possibly describe the elastic collision of two billiard balls? Here r is the distance between centers of the balls.



Solution: $(i),\ (ii),\ (iii),\ (iv),\ and\ (vi)$

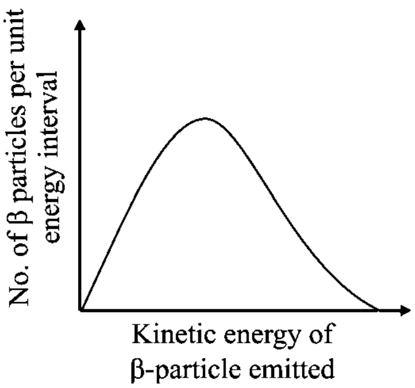
A system of two masses has potential energy that is inversely proportional to the distance between them. As the two balls go closer to each other in the aforementioned scenario, the potential energy of the system will decrease.
When the two balls touch, i.e. when $\mathrm{V}(r)=0$

, where R is the radius of each billiard ball, it becomes zero (i.e. $V(r)=0$).

Figures, $(i),\ (ii),\ (iii),\ (iv),\ and\ (vi)$

show potential energy curves that do not meet these two requirements.
As a result, they are unable to characterise the elastic collisions that occur between them.

6.30 Consider the decay of a free neutron at rest: $n \to p + e^{-}$
Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β **-decay of a neutron or a nucleus.**



Solution:
At rest, the free neutron decays as follows:

$n \to p + e^{-}$

The energy of an electron is calculated using Einstein's mass-energy relationship, Δmc^2
Where,

$\Delta m =$ Mass defect $=$ Mass of neutron $-$ (Mass of proton $+$ Mass of electron)

$c =$ Speed of light

Δm and c are constants.

As a result, the two-body decay cannot account for the energy distribution in the continuous state,

$\beta -$

decay of a neutron or a nucleus. The continuous energy distribution is appropriately explained by the existence of neutrinos from the LHS of the decay.