Chapter – 10: Mechanical Properties of Fluids

Example:

10.1 The two thigh bones (femurs), each of cross-sectional area $10\ cm^2$ support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs.

Solution:

Total cross-sectional area of the femurs is $A = 2\times 10\ cm^2$

$= 20\times 10^{-4} m^2$.

The force acting on them is $F = 40\ kg$

$wt = 400\ N$ (taking $g = 10\ m\ s^{-2}$).

This force is acting vertically down and hence, normally on the femurs. Thus, the average pressure is

$$p_{av} = \frac{F}{A}$$

$= 2\times 10^5\ N\ m^{-2}$

10.2 What is the pressure on a swimmer 10 m below the surface of a lake?

Solution:

Here $h = 10\ m$ and $\rho = 1000\ kg\ m^{-3}$.

Take $g = 10\ m\ s^{-2}$.

$P = P_a + \rho gh$

$= 1.01\times 10^5 Pa + 1000\ kg\ m^{-3} \times 10\ m\ s^{-2} \times 10\ m$

$= 2.01\times 10^5 Pa$

$\approx 2\ atm$

This is a 100% increase in pressure from surface level. At a depth of $1\ km$, the increase in pressure is $100\ atm$! Submarines are designed to withstand such enormous pressures.

10.3 The density of the atmosphere at sea level is $1.29\ kg/m^3$. Assume that it does not change with altitude. Then how high would the atmosphere extend?

Solution:

$\rho gh = 1.29\ kg\ m^{-3} \times 9.8\ m\ s^2 \times hm$

$= 1.01\times 10^5 Pa$

$\therefore h = 7989\ m \approx 8\ km$

In reality the density of air decreases with height. So does the value of g. The atmospheric cover extends with decreasing pressure over $100\ km$. We should also note that the sea level atmospheric pressure is not always $760\ mm\ of\ Hg$. A drop in the Hg level by $10\ mm$ or more is a sign of an approaching storm.

10.4 At a depth of $1000\ m$ in an ocean (a) what is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area $20\ cm\times 20\ cm$ of a submarine at this depth, the interior of which is maintained at sea- level atmospheric pressure. (The density of sea water is $1.03\times 10^3 kgm^{-3}, g = 10\ m\ s^{-2}$.)

Solution:

Here $h = 1000\ m$ and $\rho = 1.03\times10^3 kg\ m^{-3}$.

(a) Absolute pressure

$P = P_a + \rho gh$

$= 1.01\times10^5 Pa + 1.03\times10^3 kg\ m^{-3}\times10\ m\ s^{-2}\times1000\ m$

$= 104.01\times10^5 Pa$

$\approx 104\ atm$

(b) Gauge pressure

$P - P_a = \rho gh = P_g$

$Pg = 1.03\times10^3 kg\ m^{-3}\times10\ ms^2\times1000\ m$

$= 103\times10^5 Pa$

$\approx 103\ atm$

(c) The pressure outside the submarine

$P = P_a + \rho gh$ and the pressure inside it is P_a. Hence, the net pressure acting on the window is gauge pressure, $P_g = \rho gh$. Since the area of the window is $A = 0.04\ m^2$, the force acting on it is

$F = P_g A = 10^3\times10^5 Pa\times0.04\ m^2$

$= 4.12\times10^5 N$

10.5 Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are $1.0 cm$ and 3.0 cm respectively.

(a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston.

(b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

Solution:

(a) Since pressure is transmitted undiminished throughout the fluid,

$$F_2 = \frac{A_2}{A_1}F_1$$

$$= \frac{\pi(3/2\times10^{-2}m)^2}{\pi(1/2\times10^{-2}m)^2}\times10\,N$$

$$= 90\ N$$

(b) Water is considered to be perfectly incompressible. Volume covered by the movement of smaller piston inwards is equal to volume moved outwards due to the larger piston.

$$L_1A_1 = L_2A_2$$

$$L_2 = \frac{A_1}{A_2}L_1$$

$$= \frac{\pi(1/2\times10^{-2}m)^2}{\pi(3/2\times10^{-2}m)^2}\times6\times10^{-2}m$$
$$\simeq 0.67\times10^{-2}m$$
$$= 0.67\ cm$$

Note, atmospheric pressure is common to both pistons and has been ignored.

10.6 In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5.0 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, calculate F_1. What is the pressure necessary to accomplish this task? $(g = 9.8\ ms^{-2})$.

Solution:

Since pressure is transmitted undiminished throughout the fluid,

$$F_1 = \frac{A_1}{A_2}F_2$$
$$= \frac{\pi(5\times10^{-2}m)^2}{\pi(15\times10^{-2}m)^2}\times(1350\,kg\times9.8\,ms^{-2})$$
$$= 1470\ N$$
$$\approx 1.5\times103N$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1}$$
$$= \frac{1.5\times10^3N}{\pi(5\times10^{-2})^2m}$$
$$= 1.9\times10^5 Pa$$

10.7 Blood velocity: The flow of blood in a large artery of an anesthetized dog is diverted through a Venturi meter. The wider part of the meter has a cross- sectional area equal to that of the artery. $A = 8\ mm^2$. The narrower part has an area $a = 4\ mm^2$. The pressure drop in the artery is $24\ Pa$. What is the speed of the blood in the artery?

Solution:

We take the density of blood from Table 10.1 to be $1.06\times10^3 kg\ m^{-3}$. The ratio of the areas is

$$\left(\frac{A}{a}\right) = 2.$$
$$\upsilon = \sqrt{\frac{2\times24\,Pa}{1060\,kg\,m^{-3}\times(2^2-1)}}$$
$$= 0.123\,m\,s^{-1}$$

10.8 A fully loaded Boeing aircraft has a mass of 3.3×10^5kg. Its total wing area is $500\ m^2$. It is in level flight with a speed of 960 km/h.

(a) Estimate the pressure difference between the lower and upper surfaces of the wings

(b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. [The density of air is $\rho = 1.2 \text{ kg m}^{-3}$].

Solution:

(a) The weight of the Boeing aircraft is balanced by the upward force due to the pressure difference

$$\Delta P \times A = 3.3 \times 10^5 kg \times 9.8$$

$$\Delta P = (3.3 \times 10^5 kg \times 9.8 \, m \, s^{-2}) \, / \, 500 \, m^2$$

$$= 6.5 \times 10^3 Nm^{-2}$$

(b) We ignore the small height difference between the top and bottom sides. The pressure difference between them is then

$$\Delta p = \frac{\rho}{2}(\upsilon_2^2 - \upsilon_1^2)$$

where v_2 is the speed of air over the upper surface and v_1 is the speed under the bottom surface.

$$(\upsilon_2 - \upsilon_1) = \frac{2\Delta P}{\rho(\upsilon_2 + \upsilon_1)}$$

Taking the average speed

$$v_{av} = \frac{(v_2 + v_1)}{2}$$

$$= 960 \, km / h$$

$$= 267 \, m \, s^{-1}$$

we have

$$\frac{(\upsilon_2 - \upsilon_1)}{\upsilon_{av}} = \frac{\Delta P}{\rho \upsilon_{av}^2}$$

$$\approx 0.08$$

The speed above the wing needs to be only 8% higher than that below.

10.9 A metal block of area $0.10 \, m^2$ is connected to a 0.010 kg mass via a string that passes over an ideal pulley (considered massless and frictionless). A liquid with a film thickness of $0.30 \, mm$ is placed between the block and the table. When released the block moves to the right with a constant speed of $0.085 \, m \, s^{-1}$. Find the coefficient of viscosity of the liquid.

Solution:

The metal block moves to the right because of the tension in the string. The tension T is equal in magnitude to the weight of the suspended mass m. Thus, the shear force F is

$$F = T = mg = 0.010 \, kg \times 9.8 \, m \, s^{-2}$$

$$= 9.8 \times 10^{-2} N$$

Shear stress on the fluid $= \dfrac{F}{A}$

$$= \frac{9.8 \times 10^{-2}}{0.10} N / m^2$$

Strain rate $= \dfrac{\upsilon}{t}$

$= \dfrac{0.085}{0.30\times10^{-3}}$

$\eta = \dfrac{stress}{strain\ rate} s^{-1}$

$= \dfrac{(9.8\times10^{-2}\,N)(0.30\times10^{-3}\,m)}{(0.085\,m\,s^{-1})(0.10\,m^2)}$

$= 3.46\times10^{-3}\,Pa\ s$

10.10 The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20^oC is $6.5\ cm\ s^{-1}$. Compute the viscosity of the oil at 20^oC. Density of oil is $1.5\times10^3 kg\ m^{-3}$, density of copper is $8.9\times10^3 kg\ m^{-3}$.

Solution:

We have $v_t = 6.5\times10^{-2} ms^{-1}$, $a = 2\times10^{-3} m$,

$g = 9.8\ ms^{-2}$, $\rho = 8.9\times10^3 kg\ m^{-3}$,

$\sigma = 1.5\times10^3 kg\ m^{-3}$.

$\eta = \dfrac{2}{9}\times\dfrac{(2\times10^{-3})^2\,m^2\times9.8\,m\,s^{-2}}{6.5\times10^2\,m\,s^{-1}}\times7.4\times10^3\,kg\,m^{-3}$

$= 9.9\times10^{-1} kg\ m^{-1}s^{-1}$

10.11 The lower end of a capillary tube of diameter 2.00 mm is dipped $8.00\,cm$ below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at temperature of the experiments is $7.30\times10^{-2}\,Nm^{-1}$. 1 atmospheric pressure $= 1.01\times10^5\,Pa$, density of water $= 1000\ kg/m^3$, $g = 9.80\ m\ s^{-2}$. Also calculate the excess pressure.

Solution:

$P_o = (1.01\times10^5\,Pa + 0.08\,m\times1000\,kg\ m^{-3}\times9.80\,m\,s^{-2})$

$= 1.01784\times10^5\,Pa$

Therefore, the pressure inside the bubble is $P_i = P_o + 2S/r$

$= 1.01784\times10^5\,Pa + (2\times7.3\times10^{-2}\,Pa\ m/10^{-3}m)$

$= (1.01784 + 0.00146)\times10^5\,Pa$

$= 1.02\times10^5\,Pa$

Exercise:

10.1 Explain why

(a) **The blood pressure in humans is greater at the feet than at the brain.**

(b) **Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km.**

(c) **Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.**

Solution:

The pressure of a liquid is given by the relation:

$P = h\rho g$

Where,

P = Pressure

h = Height of the liquid column

ρ = Density of the liquid

g = Acceleration due to the gravity

(a) It can be inferred that pressure is directly proportional to height. Hence, the blood pressure in human vessels depends on the height of the blood column in the body. The height of the blood column is more at the feet than it is at the brain. Hence, the blood pressure at the feet is more than it is at the brain.

(b) Density of air is the maximum near the sea level. Density of air decreases with increase in height from the surface. At a height of about 6 km, density decreases to nearly half of its value at the sea level. Atmospheric pressure is proportional to density. Hence, at a height of 6 km from the surface, it decreases to nearly half of its value at the sea level.

(c) When force is applied on a liquid, the pressure in the liquid is transmitted in all directions. Hence, hydrostatic pressure does not have a fixed direction and it is a scalar physical quantity.

10.2 Explain why

(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)

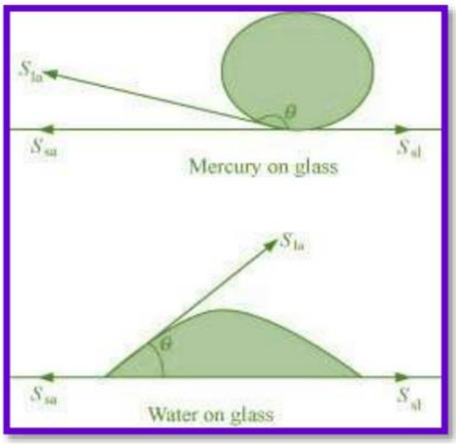
(c) Surface tension of a liquid is independent of the area of the surface.

(d) Water with detergent dissolved in it should have small angles of contact.

(e) A drop of liquid under no external forces is always spherical in shape.

Solution:

The angle between the tangent to the liquid surface at the point of contact and the surface inside the liquid is called the angle of contact (θ), as shown in the given figure.



S_{la}, S_{sa}, and S_{sl} are the respective interfacial tensions between the liquid-air, solid-air, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium, i.e.,

$$\cos\theta = \frac{S_{la} - S_{sa}}{S_{la}}$$

The angle of contact θ, is obtuse if $S_{sa} < S_{la}$ (as in the case of mercury on glass). This angle is acute if $S_{sl} < S_{la}$ (as in the case of water on glass).

a) Mercury molecules have a high affinity between themselves and a weak attraction toward solids (due to their obtuse angle with glass). As a result, they are prone to forming droplets.
b) Water molecules, on the other hand, form acute angles with glass. They have a mild affinity to one other and a high attraction to solids. As a result, they tend to disperse.
c) The force operating per unit length at the contact between the plane of a liquid and any other surface is known as surface tension. The area of the liquid surface has no bearing on this force. As a result, surface tension is unaffected by the size of the liquid surface.
d) The angles of contact in water with detergent dissolved in it are minimal (θ). This is due to the detergent's rapid capillary increase in the cloth for a tiny amount. The cosine of the angle of contact (θ) is exactly related to the capillary rise of a liquid. If is tiny, cos (θ) will be large, and the detergent water in the cloth will rise quickly.
e) Due to the presence of surface tension, a liquid seeks to obtain the smallest possible surface area. A sphere's surface area is the smallest for a given volume. As a result, liquid drops always take on a spherical shape in the absence of external forces.

10.3 Fill in the blanks using the word(s) from the list appended with each statement:

a) Surface tension of liquids generally . . . with temperatures (increases / decreases).
b) Viscosity of gases. .. with temperature, whereas viscosity of liquids . . . with temperature (increases / decreases).
c) For solids with elastic modulus of rigidity, the shearing force is proportional to . . . , while for fluids it is proportional to. .. (shear strain / rate of shear strain).
d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle).
e) For the model of a plane in a wind tunnel, turbulence occurs at a ... speed for turbulence for an actual plane (greater / smaller).

Solution:

a) Decreases

A liquid's surface tension is inversely proportional to its temperature.

b) Increases; decreases

The mobility of most fluids is hampered by resistance. Internal mechanical friction, often known as viscosity, is similar to this. The viscosity of gases increases as the temperature rises, but the viscosity of liquids falls as the temperature rises.

c) Shear strain; Rate of shear strain

The shearing force is proportional to the shear strain when the elastic modulus of rigidity for solids is used. The shearing force is proportional to the rate of shear strain when considering the elastic modulus of rigidity for fluids.

d) Conservation of mass/Bernoulli's principle

The conservation of mass/principle Bernoulli's governs the rise in flow speed of a steady-flowing fluid at a constriction.

e) **Greater**

Turbulence happens at a faster rate in a wind tunnel model of a plane than it does in a real plane. This is due to Bernoulli's principle, and the motions of the two planes are connected with distinct Reynolds' numbers.

10.4 Explain why

a) **To keep a piece of paper horizontal, you should blow over, not under, it.**
b) **When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.**
c) **The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.**
d) **A fluid flowing out of a small hole in a vessel results in a backward thrust on the Vessel.**
e) **A spinning cricket ball in air does not follow a parabolic trajectory.**

Solution:

a) When air is blasted beneath a piece of paper, the velocity of the air is greater than it is above it. The atmospheric pressure decreases under the paper, according to Bernoulli's principle. As a result, the paper begins to fall. Blowing over a piece of paper will keep it horizontal. The air velocity above the paper is increased as a result of this. The atmospheric pressure decreases above the paper, and the paper remains horizontal, according to Bernoulli's principle.
According to the continuity equation: $Area \times Velocity = Constant$

b) When a fluid flows through a smaller aperture, its velocity is higher than when the opening is larger. Fast jets of water gush through the gaps between our fingers as we try to close a tap with our fingertips. This is due to the fact that the water is allowed to flow out of the pipe through very small gaps. As a result, area and velocity are inversely proportional.

c) The velocity of fluid flow is greater for a smaller opening than it is for a larger opening. Fast jets of water squirt through the gaps between our fingers when we try to close a water tap with our fingertips.This is due to the fact that the water flow out of the pipe through very small gaps. As a result, the relationship between area and velocity is inverse.

d) The vessel experiences a rearward thrust as fluid runs out of a small hole in it. The velocity of a fluid pouring out of a small hole is high. Because there are no external forces acting on the system, the vessel achieves a backward velocity according to the equation of continuity: $Area \times Velocity = Constant$. Because there are no external forces acting on the system, the vessel attains a backward velocity according to the law of conservation of momentum.

e) The rotatory and linear motions of a spinning cricket ball are both active at the same time. The effects of these two forms of motion are diametrically opposed. The air velocity below the ball is reduced as a result of this. As a result, the upper side of the ball experiences less pressure than the lower side. The ball is subjected to an upward push. As a result, the ball follows a bent path. It does not take a path that is parabolic.

10.5 A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

Solution:

Mass of the girl, $m = 50\ kg$

Diameter of the heel, $d = 1\ cm = 0.01\ m$

Radius of the heel, $r = \frac{d}{2} = 0.005m$

Area of the heel $= \pi r^2$

$= \pi\ (0.005)^2$

$= 7.85 \times 10^{-5}\ m^2$

Force exerted by the heel on the floor:

$F = mg$

$= 50 \times 9.8$

$= 490\ N$

Pressure exerted by the heel on the floor:

$P = \frac{Force}{Area}$

$= \frac{490}{7.85 \times 10^{-5}}$

$= 6.24 \times 10^6\ N\ m^{-2}$

Therefore, the pressure exerted by the heel on the horizontal floor is $6.24 \times 106\ Nm^{-2}$.

10.6 Toricelli's barometer used mercury. Pascal duplicated it using French wine of density $984\ \text{kg m}^{-3}$. Determine the height of the wine column for normal atmospheric pressure.

Solution:

Density of mercury, $\rho_1 = 13.6 \times 10^3\ kg/m^3$

Height of the mercury column, $h_1 = 0.76\ m$

Density of French wine, $\rho_2 = 984\ kg/m^3$

Height of the French wine column $= h_2$

Acceleration due to gravity, $g = 9.8\ m/s^2$

The pressure in both the columns is equal, i.e.,

Pressure in the mercury column = Pressure in the French wine column

$\rho_1 h_1 g = \rho_2 h_2 g$

$h_2 = \frac{\rho_1 h_1}{\rho_2}$

$= \frac{13.6 \times 10^3 \times 0.76}{984}$

$= 10.5\ m$

Hence, the height of the French wine column for normal atmospheric pressure is $10.5\ m$.

10.7 A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

Solution:

Yes.

The maximum allowable stress for the structure, $P = 10^9 \ Pa$

Depth of the ocean, $d = 3 \ km = 3 \times 10^3 \ m$

Density of water, $\rho = 10^3 \ kg / m^3$

Acceleration due to gravity, $g = 9.8 \ m / s^2$

The pressure exerted because of the sea water at depth,

$d = \rho dg$

$= 3 \times 10^3 \times 10^3 \times 9.8$

$= 2.94 \times 10^7 \ Pa$

The maximum allowable stress for the structure $\left(10^9 \ Pa\right)$ is greater than the pressure of the sea water $\left(2.94 \times 10^7 \ Pa\right)$. The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

10.8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

Solution:

The maximum mass of a car that can be lifted, $m = 3000 \ kg$

Area of cross-section of the load-carrying piston,

$A = 425 \ cm^2$

$= 425 \times 10^{-4} \ m^2$

The maximum force exerted by the load,

$F = mg$

$= 3000 \times 9.8$

$= 29400 \ N$

The maximum pressure exerted on the load-carrying piston,

$P = \dfrac{F}{A}$

$= \dfrac{29400}{425 \times 10^{-4}}$

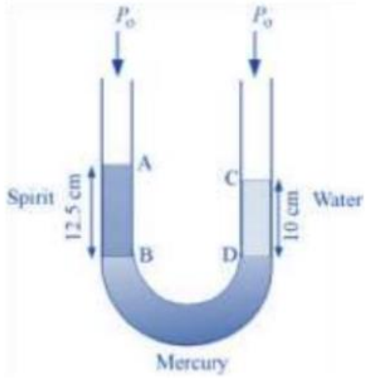
$= 6.917 \times 10^5 \ Pa$

Pressure is transmitted equally in all directions in a liquid. Therefore, the maximum pressure that the smaller piston would have to bear is $6.917 \times 10^5 \ Pa$.

10.9 A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

Solution:

The given system of water, mercury, and methylated spirit is shown as follows:



Height of the spirit column, $h_1 = 12.5\ cm = 0.125\ m$

Height of the water column, $h_2 = 10\ cm = 0.1\ m$

$P_0 =$ Atmospheric pressure

$\rho_1 =$ Density of spirit

$\rho_2 =$ Density of water

Pressure at point $B = P_0 + h_1\rho_1 g$

Pressure at point $D = P_0 + h_2\rho_2 g$

Pressure at points B and D is the same.

$P_0 + h_1\rho_1 g = h_2\rho_2 g$

$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$

$= \frac{10}{12.5}$

$= 0.8$

Therefore, the specific gravity of spirit is 0.8.

10.10 In problem 10.9, if 15.0 cm of water and spirit each are further poured into the respective
arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury $= 13.6$)

Solution:

Height of the water column, $h_1 = 10 + 15 = 25\ cm$

Height of the spirit column, $h_2 = 12.5 + 15 = 27.5\ cm$

Density of water, $\rho_1 = 1\ g\ cm^{-3}$

Density of spirit, $\rho_2 = 0.8\ g\ cm^{-3}$

Density of mercury $= 13.6\ g\ cm^{-3}$

Let h be the difference between the levels of mercury in the two arms.

Pressure exerted by height h, of the mercury column:

$= h\rho g = h \times 13.6g$ .... (i)

Difference between the pressures exerted by water and spirit:

$= h_1\rho_1 g - h_1\rho_1 g$

$= g(25\times 1 - 27.5\times 0.8)$

$= 3g$ ... (ii)

Equating equations (i) and (ii), we get:

$13.6\ hg = 3g\ h$

$= 0.220588 \approx 0.221\ cm$

Hence, the difference between the levels of mercury in the two arms is $0.221\ cm$.

10.11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river?Explain.

Solution:

No.

Because of the turbulent flow of water, Bernoulli's equation cannot be utilised to explain the flow of water through a rapid in a river. Only a streamline flow can be used to apply this idea.

10.12 Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.

Solution:

No.

When implementing Bernoulli's equation, it makes no difference whether you use gauge pressure or absolute pressure. The air pressures should be significantly different at the two sites when Bernoulli's equation is used.

10.13 Glycerine flows steadily through a horizontal tube of length $1.5\ \mathrm{m}$ and radius $1.0\ \mathrm{cm}$. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3}\ \mathrm{kg\ s^{-1}}$, what is the pressure difference between the two ends of the tube? (Density of glycerine $= 1.3 \times 10^{3}\ \mathrm{kg\ m^{-3}}$ and viscosity of glycerine $= 0.83\ \mathrm{Pa\ s}$). [You may also like to check if the assumption of laminar flow in the tube is correct].

Solution:

Length of the horizontal tube, $l = 1.5\ m$

Radius of the tube, $r = 1\ cm = 0.01\ m$

Diameter of the tube, $d = 2r = 0.02\ m$

Glycerine is flowing at a rate of $4.0 \times 10^{-3}\ kg\ s^{-1}$.

$M = 4.0 \times 10^{-3}\ kg\ s^{-1}$

Density of glycerine, $\rho = 1.3 \times 10^{3}\ kg\ m^{-3}$

Viscosity of glycerine, $\eta = 0.83\ Pa\ s$

Volume of glycerine flowing per sec:

$$V = \frac{M}{\rho}$$

$$= \frac{4.0\times 10^{-3}}{1.3\times 10^{3}}$$

$$= 3.08 \times 10^{-6}\ m^3\ s^{-1}$$

According to Poiseville's formula, we have the relation for the rate of flow:

$$V = \frac{\pi p r^4}{8\eta l}$$

Where, p is the pressure difference between the two ends of the tube

$$\therefore p = \frac{V8\eta l}{\pi r^4}$$

$$= \frac{3.08\times10^{-4}\times8\times0.83\times1.5}{\pi\times(0.01)^4}$$

$$= 9.8 \times 10^2 \ Pa$$

Reynolds' number is given by the relation:

$$R = \frac{4\rho V}{\pi d\eta}$$

$$= \frac{4\times1.3\times10^3\times3.08\times10^{-6}}{\pi\times(0.02)\times0.83}$$

$$= 0.3$$

Reynolds' number is about 0.3.

Hence, the flow is laminar.

10.14 In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.

Solution:

Speed of wind on the upper surface of the wing, $V_1 = 70\ m/s$

Speed of wind on the lower surface of the wing, $V_2 = 63\ m/s$

Area of the wing, $A = 2.5\ m^2$

Density of air, $\rho = 1.3\ kg\ m^{-3}$

According to Bernoulli's theorem, we have the relation:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

Where,

$P_1 =$ Pressure on the upper surface of the wing

$P_2 =$ Pressure on the lower surface of the wing

The pressure difference between the upper and lower surfaces of the wing provides lift to the aeroplane.

Lift on the wing $= (P_2 - P_1)A$

$$= \frac{1}{2}\rho(V_1^2 - V_2^2)A$$

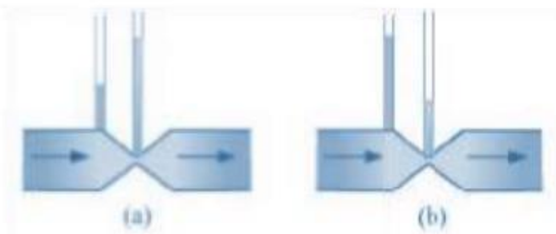
$$= \frac{1}{2}1.3((70)^2 - (63)^2)\times2.5$$

$$= 1512.87$$

$$= 151\times10^3\ N$$

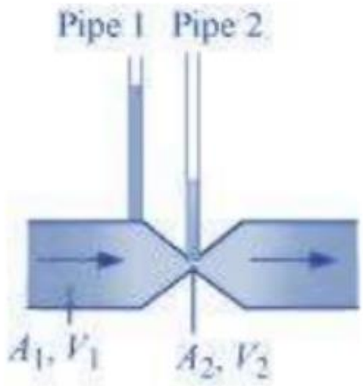
Therefore, the lift on the wing of the aeroplane is $1.51 \times 10^3\ N$.

10.15 Figures 10.23 (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect? Why?



Solution:

(a) Take the case given in figure (b).



Where,

A_1 = Area of pipe1

A_2 = Area of pipe 2

V_1 = Speed of the fluid in pipe1

V_2 = Speed of the fluid in pipe 2

From the law of continuity, we have:

$A_1V_1 = A_2V_2$

When the area of cross-section in the middle of the venturimeter is small, the speed of the flow of liquid through this part is more. According to Bernoulli's principle, if speed is more, then pressure is less.

Pressure is directly proportional to height. Hence, the level of water in pipe 2 is less. Therefore, figure (a) is not possible.

10.16 The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes?

Solution:

Area of cross-section of the spray pump, $A_1 = 8 \, cm^2 = 8 \times 10^{-4} \, m^2$

Number of holes, $n = 40$

Diameter of each hole, $d = 1 \, mm = 1 \times 10^{-3} \, m$

Radius of each hole, $r = \frac{d}{2} = 0.5 \times 10^{-3}\ m$

Area of cross-section of each hole, $a = \pi r^2$

$= \pi\left(0.5 \times 10^{-3}\right)^2\ m^2$

Total area of 40 holes, $A_2 = n \times a$

$= 40 \times \pi\left(0.5 \times 10^{-3}\right)^2\ m^2$

$= 31.41 \times 10^{-6}\ m^2$

Speed of flow of liquid inside the tube, $V_1 = 1.5\ m/min = 0.025\ m/s$

Speed of ejection of liquid through the holes $= V_2$

According to the law of continuity, we have:

$A_1V_1 = A_2V_2$

$V_2 = \frac{A_1V_1}{A_2}$

$= \frac{8\times10^{-4}\times0.025}{31.61\times10^{-6}}$

$= 0.633\ m/s$

Therefore, the speed of ejection of the liquid through the holes is $0.633\ m/s$.

10.17 A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

Solution:

The weight that the soap film supports, $W = 1.5 \times 10^{-2}\ N$

Length of the slider, $l = 30\ cm = 0.3\ m$

A soap film has two free surfaces.

$\therefore$ Total length $= 2l = 2 \times 0.3 = 0.6\ m$

Surface tension,

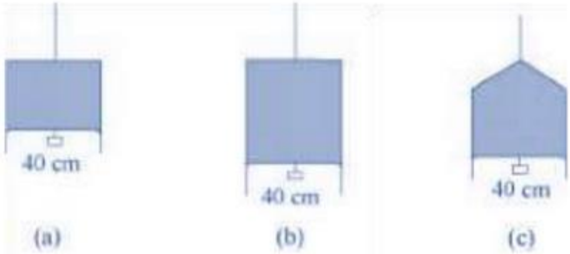
$S = \frac{Force\ or\ Weight}{2l}$

$= \frac{1.5\times10^{-2}}{0.6}$

$= 2.5\times10^{-2}\ N/m$

Therefore, the surface tension of the film is $2.5 \times 10^{-2}\ N\ m^{-1}$.

10.18 Figure 10.24 (a) shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2}$ N. What
is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.



Solution:

Take case (a):

The length of the liquid film supported by the weight, $l = 40\ cm = 0.4\ cm$

The weight supported by the film, $W = 4.5 \times 10^{-2}\ N$

A liquid film has two free surfaces.

$\therefore$ Surface tension $= \dfrac{W}{2l}$

$= \dfrac{4.5\times10^{-2}}{2\times0.4}$

$= 5.625\times10^{-2}\ N\,m^{-1}$

In all the three figures, the liquid is the same. Temperature is also the same for each case. Hence, the surface tension in figure (b) and figure (c) is the same as in figure (a), i.e., $5.625\times10^{-2}\ N\,m^{-1}$.

Since the length of the film in all the cases is 40 cm, the weight supported in each case i $4.5 \times 10^{-2}\ N$.

10.19 What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature?Surface tension of mercury at that temperature ($20°C$) is 4.65×10^{-1} N m^{-1}. The atmospheric pressure is 1.01×10^{5} Pa. Also give the excess pressure inside the drop.

Solution:

Radius of the mercury drop, $r = 3.00\ mm = 3 \times 10^{-3}\ m$

Surface tension of mercury, $S = 4.65 \times 10^{-1}\ N\ m^{-1}$

Atmospheric pressure, , $P_0 = 1.01 \times 10^{5}\ Pa$

Total pressure inside the mercury drop

$= Excess\ pressure\ inside\ mercury + Atmospheric\ pressure$

$= \dfrac{2S}{r} + P_0$

$= \dfrac{2\times4.65\times10^{-1}}{3\times10^{-3}} + 1.01\times10^{5}$

$= 1.0131 \times 10^{5}$

$= 1.01 \times10^{5}\ Pa$

Excess pressure $= \dfrac{2S}{r}$

$= \dfrac{2\times4.65\times10^{-1}}{3\times10^{-3}}$

$= 310\ Pa$

10.20 What is the excess pressure inside a bubble of soap solution of radius $5.00\ \text{mm}$, given that the surface tension of soap solution at the temperature ($20°\text{C}$) is $2.50 \times 10^{-2}\ \text{N m}^{-1}$? If an air bubble of the same dimension were formed at depth of $40.0\ \text{cm}$ inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is $1.01 \times 10^5\ \text{Pa}$).

Soluton:

Excess pressure inside the soap bubble is $20\ Pa$;

Pressure inside the air bubble is $1.06 \times 10^5\ Pa$

Soap bubble is of radius, $r = 5.00\ mm = 5 \times 10^{-3}\ m$

Surface tension of the soap solution, $S = 2.50 \times 10^{-2}\ Nm^{-1}$

Relative density of the soap solution $= 1.20$

Density of the soap solution, $\rho = 1.2 \times 10^3\ kg/m^3$

Air bubble formed at a depth, $h = 40\ cm = 0.4\ m$

Radius of the air bubble, $r = 5\ mm = 5 \times 10^{-3}\ m$

1 atmospheric pressure $= 1.01 \times 10^5\ Pa$

Acceleration due to gravity, $g = 9.8\ m/s^2$

Hence, the excess pressure inside the soap bubble is given by the relation:

$$P = \frac{4S}{r}$$

$$= \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}}$$

$$= 20\,Pa$$

Therefore, the excess pressure inside the soap bubble is $20\ Pa$.

The excess pressure inside the air bubble is given by the relation:

$$P' = \frac{4S}{r}$$

$$= \frac{2 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}}$$

$$= 10\,Pa$$

Therefore, the excess pressure inside the air bubble is $10\ Pa$.

At a depth of $0.4\ m$, the total pressure inside the air bubble

$= Atmospheric\ pressure + h\rho g + P'$

$= 1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + 10$

$= 1.057 \times 10^5\ Pa$

$= 1.06 \times 10^5\ Pa$

Therefore, the pressure inside the air bubble is $1.06 \times 10^5\ Pa$.

10.21 A tank with a square base of area $1.0\ m^2$ is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area $20\ cm^2$. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of $4.0\ m$. compute the force necessary to keep the door close.

Solution:

Base area of the given tank, $A = 1.0\ m^2$

Area of the hinged door, $a = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

Density of water, $\rho_1 = 10^3\ kg/m^3$

Density of acid, $\rho_2 = 1.7 \times 10^3\ kg/m^3$

Height of the water column, $h_1 = 4\ m$

Height of the acid column, $h_2 = 4\ m$

Acceleration due to gravity, $g = 9.8$ Pressure due to water is given as:

$P_1 = h_1\rho_1 g$

$= 4 \times 10^3 \times 9.8$

$= 3.92 \times 10^4\ Pa$

Pressure due to acid is given as:

$P_2 = h_2\rho_2 g$

$= 4 \times 1.7 \times 10^3 \times 9.8$

$= 6.664 \times 10^4\ Pa$

Pressure difference between the water and acid columns:

$\Delta P = P_2 - P_1$

$= 6.664 \times 10^4 - 3.92 \times 10^4$

$= 2.744 \times 10^4\ Pa$

Hence, the force exerted on the door $= \Delta P \times a$

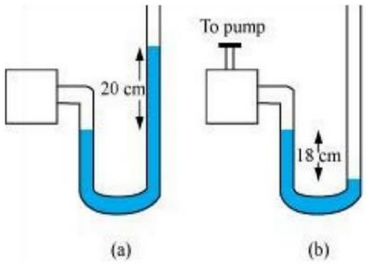
$= 2.744 \times 10^4 \times 20 \times 10^{-4}$

$= 54.88\ N$

Therefore, the force necessary to keep the door closed is $54.88\ N$.

10.22 A manometer reads the pressure of a gas in an enclosure as shown in Fig. 10.25 (a) When a pump removes some of the gas, the manometer reads as in Fig. 10.25 (b) The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

- **f) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.**
- **g) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in the volume of the gas).**



Solution:

(a) $96\ cm$ of Hg & $20\ cm$ of Hg; $58\ cm$ of Hg & $-18\ cm$ of Hg $19\ cm$

For figure (a)

Atmospheric pressure, $P_0 = 76\ cm$ of Hg

Difference between the levels of mercury in the two limbs gives gauge pressure

Hence, gauge pressure is $20\ cm$ of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure

$= 76 + 20 = 96\ cm$ of Hg

For figure (b)

Difference between the levels of mercury in the two limbs $= -18\ cm$

Hence, gauge pressure is $-18\ cm$ of Hg.

$Absolute\ pressure = Atmospheric\ pressure + Gauge\ pressure$

$= 76\ cm - 18\ cm$

$= 58\ cm$

$13.6\ cm$ of water is poured into the right limb of figure (b).

Relative density of mercury $= 13.6$

Hence, a column of $13.6\ cm$ of water is equivalent to $1\ cm$ of mercury.

(b) Let h be the difference between the levels of mercury in the two limbs.

The pressure in the right limb is given as:

$P_R =$ Atmospheric pressure $+ 1\ cm$ of Hg

$= 76 + 1 = 77\ cm\ of\ Hg$ ..... (i)

The mercury column will rise in the left limb.

Hence, pressure in the left limb, $P_L = 58 + h$ …... (ii)

Equating equations (i) and (ii), we get:

$77 = 58 + h$

$\therefore h = 19\ cm$ Hence, the difference between the levels of mercury in the two limbs will be $19\ cm$.

10.23 Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

Solution:

Yes

The force and pressure are the same on the shared base area of two vessels having the same base area. Due to the different geometries of the two vessels, the force exerted on the sides of the vessels has non-zero vertical components. When the total force on one vessel is added to the total force on the other vessel, the total force on one vessel is greater. As a result, when both jars are filled to the same height with water, the weighing scale readings are different.

10.24 During blood transfusion the needle is inserted in a vein where the gauge pressure is $2000\ \text{Pa}$. At what height must the blood container be placed so that blood may just enter the vein?

Solution:

Gauge pressure, $P = 2000\ Pa$

Density of whole blood, $\rho = 1.06 \times 10^3\ kg\ m^{-3}$

Acceleration due to gravity, $g = 9.8\ m/s^2$

Height of the blood container $= h$

Pressure of the blood container, $P = h\rho g$

$$\therefore h = \frac{P}{\rho g}$$

$$= \frac{2000}{1.06\times10^3\times9.8}$$

$$= 0.1925\,m$$

The blood may enter the vein if the blood container is kept at a height greater than $0.1925\ m$, i.e., about $0.2\ m$.

10.25 In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar? (b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.

Solution:

(a) $1.966\ m/s$

(b) Yes

Diameter of the artery, $d = 2 \times 10^{-3}\ m$

Viscosity of blood, $\eta = 2.084\times10^{-3}\ Pa\,s$

Density of blood, $\rho = 1.06 \times 10^3\ kg/m^3$

Reynolds' number for laminar flow, $N_R = 2000$

The largest average velocity of blood is given as:

$$V_{\text{arg}} = \frac{N_R \eta}{\rho d}$$

$$= \frac{2000\times2.084\times10^{-3}}{1.06\times10^3\times2\times10^{-3}}$$

$$= 1.966\,m/s$$

Therefore, the largest average velocity of blood is $1.966\ m/s$.

As the fluid velocity increases, the dissipative forces become more important. This is because of the rise of turbulence. Turbulent flow causes dissipative loss in a fluid.

10.26 What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar? (b) What is the corresponding flow rate? (Take viscosity of blood to be 2.084×10^{-3} Pa s).

Solution:

Radius of the artery, $r = 2 \times 10^{-3}\ m$

Diameter of the artery, $d = 2 \times 2 \times 10^{-3}\ m$

$= 4 \times 10^{-3}\ m$

Viscosity of blood, $\eta = 2.084 \times 10^{-3}\ Pa\,s$

Density of blood, $\rho = 1.06 \times 10^{3}\ kg/m^{3}$

Reynolds' number for laminar flow, $N_R = 2000$

The largest average velocity of blood is given by the relation:

$$V_{\arg} = \frac{N_R \eta}{\rho d}$$

$$= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^{3} \times 4 \times 10^{-3}}$$

$$= 0.983\, m/s$$

Therefore, the largest average velocity of blood is $0.983\ m/s$.

Flow rate is given by the relation:

$$R = \pi r^2 V_{\arg}$$

$$= 3.14 \times (2 \times 10^{-3})^2 \times 0.983$$

$$= 1.235 \times 10^{-5}\ m^3 s^{-1}$$

Therefore, the corresponding flow rate is $1.235 \times 10^{-5}\ m^3 s^{-1}$.

10.27 A plane is in level flight at constant speed and each of its two wings has an area of $25\ \mathrm{m}^2$. If the speed of the air is $180\ \mathrm{km/h}$ over the lower wing and $234\ \mathrm{km/h}$ over the upper wing surface, determine the plane's mass. (Take air density to be $1\ \mathrm{kg\ m^{-3}}$).

Solution:

The area of the wings of the plane, $A = 2 \times 25 = 50\ m^2$

Speed of air over the lower wing, $V_1 = 180\ km/h = 50\ m/s$

Speed of air over the upper wing, $V_2 = 234\ km/h = 65\ m/s$

Density of air, $\rho = 1\ kg\ m^{-3}$

Pressure of air over the lower wing $= P_1$

Pressure of air over the upper wing $= P_2$

The upward force on the plane can be obtained using Bernoulli's equation as:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho (V_2^2 - V_1^2) \qquad ..(i)$$

The upward force (F) on the plane can be calculated as:

$$(P_1 - P_2)A$$

$$= \frac{1}{2}\rho (V_2^2 - V_1^2)A \qquad \text{Using equation (i)}$$

$$= \frac{1}{2} \times 1 \times \left((65)^2 - (50)^2\right) \times 50$$

$$= 43125\, N$$

Using Newton's force equation, we can obtain the mass (m) of the plane as:

$F = mg$

$\therefore m = \frac{43125}{9.8}$

$= 4400.51 kg$

$\sim 4400 kg$

Hence, the mass of the plane is about $4400\ kg$.

10.28 In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m – 3 ? Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Solution:

Terminal speed = $5.8\ cm/s$;

Viscous force = $3.9 \times 10^{-10}\ N$

Radius of the given uncharged drop, $r = 2.0 \times 10^{-5}\ m$

Density of the uncharged drop, $\rho = 1.2 \times 10^3\ kg\ m^{-3}$

Viscosity of air, $\eta = 1.8 \times 10^{-5}\ Pa\,s$

Density of air (ρ_o) can be taken as zero in order to neglect buoyancy of air.

Acceleration due to gravity, $g = 9.8\ m/s^2$

Terminal velocity (v) is given by the relation:

$v = \frac{2r^2 \times (\rho - \rho_0) g}{9\eta}$

$= \frac{2 \times (2.0 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8}{9 \times 1.8 \times 10^{-5}}$

$= 5.807 \times 10^{-2}\ m\,s^{-1}$

$= 5.8\,cm\,s^{-1}$

Hence, the terminal speed of the drop is $5.8\,cm\,s^{-1}$.

The viscous force on the drop is given by:

$F = 6\pi\eta r v$

$\therefore F = 6 \times 3.14 \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.8 \times 10^{-2}$

$= 3.9 \times 10^{-10} N$

Hence, the viscous force on the drop is $3.9 \times 10^{-10}\ N$.

10.29 Mercury has an angle of contact equal to $140°$ with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 N m^{-1}. Density of mercury $= 13.6 \times 10^3$ kg m^{-3}.

Solution:

Angle of contact between mercury and soda lime glass, $\theta = 140°$

Radius of the narrow tube, $r = 1\,mm = 1 \times 10^{-3}\ m$

Surface tension of mercury at the given temperature, $s = 0.465\ N\ m^{-1}$

Density of mercury, $\rho = 13.6 \times 10^{3}\ kg/m^{3}$

Dip in the height of mercury $= h$

Acceleration due to gravity, $g = 9.8\ m/s^{2}$

Surface tension is related with the angle of contact and the dip in the height as:

$$s = \frac{h\rho g r}{2\cos\theta}$$

$$\therefore h = \frac{2s\cos\theta}{r\rho g}$$

$$= \frac{2\times 0.465\times \cos 140}{1\times 10^{-3}\times 13.6\times 10^{3}\times 9.8}$$

$$= -0.00534\,m$$

$$= -5.34\,mm$$

Here, the negative sign shows the decreasing level of mercury. Hence, the mercury level dips by $5.34\ mm$.

10.30 Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} N m^{-1}. Take the angle of contact to be zero and density of water to be 1.0×103 kg m^{-3} $(g = 9.8\ m\ s^{-2})$.

Solution:

Diameter of the first bore, $d_1 = 3.0\ mm = 3 \times 10^{-3}\ m$

Hence, the radius of the first bore, $r_1 = \frac{d_1}{2} = 1.5\times 10^{-3}\,m$

Diameter of the second bore, $d_2 = 6.0\ mm$

Hence, the radius of the second bore, $r_2 = \frac{d_2}{2} = 3\times 10^{-3}\,m$

Surface tension of water, $s = 7.3 \times 10^{-2}\ N\ m^{-1}$

Angle of contact between the bore surface and water, $\theta = 0$

Density of water, $\rho = 1.0 \times 10^{3}\ kg/m^{-3}$

Acceleration due to gravity, $g = 9.8\ m/s^{2}$

Let h_1 and h_2 be the heights to which water rises in the first and second tubes respectively.

These heights are given by the relations:

$$h_1 = \frac{2s\cos\theta}{r_1\rho g} \qquad \text{.... (i)}$$

$$h_2 = \frac{2s\cos\theta}{r_2\rho g} \qquad \text{.... (ii)}$$

The difference between the levels of water in the two limbs of the tube can be calculated as:

$$= \frac{2s\cos\theta}{r_1\rho g} - \frac{2s\cos\theta}{r_2\rho g}$$

$$= \frac{2s\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2\times 7.3\times 10^{-2}\times 1}{1\times 10^{3}\times 9.8}\left[\frac{1}{1.5\times 10^{-3}} - \frac{1}{3\times 10^{-3}}\right]$$

$$= 4.966\times 10^{-3}\, m$$

$$= 4.97\ mm$$

Hence, the difference between levels of water in the two bores is $4.97\ mm$.

Calculator/Computer – Based Problem

10.31

(a) It is known that density ρ of air decreases with height y as $_0e^{\frac{v}{y_0}}$ **Where** $\rho_0 =$ **1.25 kg m**^{3} **is the density at sea level, and** y_0 **is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.**

(b) A large He balloon of volume $1425\ m^3$ **is used to lift a payload of** $400\ kg$**. Assume that the balloon maintains constant radius as it rises. How high does it rise? [Take y0= 8000 m and = 0.18 kg m–3].**

Solution:

Volume of the balloon, $V = 1425\ m^3$

Mass of the payload, $m = 400\ kg$

Acceleration due to gravity, $g = 9.8\ m/s^2$

$y_0 = 8000\, m$

$\rho_{He} = 0.18\, kg\, m^{-3}$

Density of the balloon $= \rho$

Height to which the balloon rises $= y$

Density (ρ) of air decreases with height (y) as:

$$\rho = \rho_0 e^{-y/y_0}$$

$$\frac{\rho}{\rho_0} = e^{-y/y_0} \qquad \text{.... (i)}$$

This density variation is called the law of atmospherics.

It can be inferred from equation (i) that the rate of decrease of density with height is directly proportional to **ρ**, i.e.,

$$-\frac{d\rho}{dy} \propto \rho$$

$$\frac{d\rho}{dy} = -k\rho$$

$$\frac{d\rho}{\rho} = -kdy$$

Where, k is the constant of proportionality

Height changes from 0 to y, while density changes from ρ_0 to $\boldsymbol{\rho}$.

Integrating the sides between these limits, we get:

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\upsilon} = -\int_0^y kdy$$

$$[\log_e \rho]_{\rho_0}^{\rho} = -ky$$

$$\log_e \frac{\rho}{\rho_0} = -ky$$

$$\frac{\rho}{\rho_0} = e^{-ky} \quad \text{.... (i)}$$

Comparing equations (i) and (ii), we get:

$$y_0 = \frac{1}{k}$$

$$k = \frac{1}{y_0} \quad \text{.... (ii)}$$

From equations (i) and (ii), we get:

$$\rho = \rho_0 e^{-y/y_0}$$

$$Density\ \rho = \frac{Mass}{Volume}$$

$$= \frac{Mass\ of\ the\ payload + Mass\ of\ helium}{Volume}$$

$$= \frac{m + V\rho_{He}}{V}$$

$$= \frac{400 + 1425 \times 0.18}{1425}$$

$$= 0.46\,kg/m^3$$

From equations (ii) and (iii) , we can obtain y as:

$$\rho = \rho_0 e^{-y/y_0}$$

$$\log_e \frac{\rho}{\rho_0} = \frac{y}{y_o}$$

$$\therefore y = -8000 \times \log_e \frac{0.46}{1.25}$$

$$= -8000 \times -1$$

$$= 8000\,m$$

$$= 8\,km$$

Hence, the balloon will rise to a height of $8\ km$.