

# Chapter 11: THERMAL PROPERTIES OF MATTER

# Examples

Example 11.1 Show that the coefficient of area expansion.  $\{\Delta A / A\} / \Delta T$ , of a rectangular sheet of the solid is twice its linear expansivity,  $\alpha$ .

## Answer:



Let, a rectangular sheet of the solid material of length *a* and breadth *b* (*Fig.* 11.8). When the temperature increases by  $\Delta T$ , *a* increases by  $\Delta a = \alpha a \Delta T$  and *b* increases by  $\Delta b = \alpha_1 b \Delta T$ . From *Fig.* 11.8, the increase in area

$$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3 \Delta A$$
  
=  $a\Delta b + b\Delta a + (\Delta a)(\Delta b)$   
=  $a\alpha b\Delta T + b\alpha a\Delta T + (\alpha_1)^2 ab(\Delta T)$   
=  $\alpha ab\Delta T (2 + \alpha_1 \Delta T)$   
=  $\alpha_1 A\Delta (2 + \alpha_1 \Delta T)$ 

Since  $\alpha_1 \simeq 10^{-5} \text{ K}^{-1}$ , from Table 11.1, the product  $\alpha_1 \Delta T$  for fractional temperature is small in comparison with 2 and may be neglected. Hence,

$$\left(\frac{\Delta A}{A}\right)\frac{1}{\Delta T} = 2\alpha_1$$

Example 11.2 A blacksmith fixes iron ring on the rim of the wooden wheel of a horse cart. The diameter of the rim and the iron ring are 5.243m and 5.231m, respectively at  $27^{\circ}C$ . To what temperature should the ring be heated so as to fit the rim of the wheel?

Answer: From the question, we get,



 $T_{u} = 27^{\circ} C$   $L_{r1} = 5.231 m$   $L_{r2} = 5.243 m$ So,  $L_{Tr} = L_{T} \left[ 1 + \alpha_{1} \left[ T_{2} - T_{d} \right] \right]$   $5.243 m = 5.231 m \left[ 1 + 1.20 \times 10^{-5} \text{ K}^{-1} \left( T_{2} - 27^{\circ} \text{ C} \right) \right]$   $orT_{2} = 218^{\circ} \text{ C}.$ 

Example 11.3 A sphere of 0.047 kg aluminium is placed for sufficient time in a vessel containing boiling water, so that the sphere is at  $100^{\circ}\text{C}$ . It is then immediately transferred to 0.14 kg copper calorimeter containing 0.25 kg water at  $20^{\circ}\text{C}$ . The temperature of water rises and attains a steady state at  $23^{\circ}\text{C}$ . Calculate the specific heat capacity of aluminium.

**Answer:** Mass of aluminium sphere  $(m_1) = 0.047 \text{ kg}$ 

Initial temperature of aluminium sphere  $= 100^{\circ}$ C

Final temperature  $= 23^{\circ}C$ 

Change in temperature  $(\Delta T) = (100^{\circ} \text{C} - 23^{\circ} \text{C}) = 77^{\circ} \text{C}$ 

Allow aluminum's specific heat capacity to be  $s_{AI}$ 

The amount of heat that the aluminium sphere loses  $= m_1 s_{Al} \Delta T = 0.047 \text{ kg} \times s_{Al} \times 77^{\circ} \text{ C}$ 

Mass of water  $(m_2) = 0.25 \text{ kg}$ 

Mass of calorimeter  $(m_3) = 0.14 \text{ kg}$ 

Initial temperature of water and calorimeter  $= 20^{\circ}C$ 

Final temperature of the mixture  $= 23^{\circ}C$ 

Change in temperature  $(\Delta T_2) = 23^{\circ} \text{C} - 20^{\circ} \text{C} = 3^{\circ} \text{C}$ 

Specific heat capacity of water  $(s_w) = 4.18 \times 10^3 \,\mathrm{J \, kg^{-1} \, K^{-1}}$ 

Specific heat capacity of copper calorimeter =  $0.386 \times 10^3 \, \text{J kg}^{-1} \, \text{K}^{-1}$ 

The amount of heat gained by water and caloriemeter,



 $= m_2 s_w \Delta T_2 + m_3 s_{cu} \Delta T_2$ =  $(m_2 s_w + m_3 s_{cu}) (\Delta T_2)$ =  $(0.25 \text{ kg} \times 4.18 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} + 0.14 \text{ kg} \times 0.386 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}) (23^\circ \text{C} - 20^\circ \text{C})$ 

Heat lost by the aluminium sphere = heat obtained by water + heat gained by calorimeter in the steady state. So,  $0.047 \text{ kg} \times \text{s}_{\text{A}} \times 77^{\circ} \text{C}$ 

 $= (0.25 \text{ kg} \times 4.18 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} + 0.14 \text{ kg} \times 0.386 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}) (3^{\circ} \text{ C}) \text{s}_{Al}$ = 0.911 kJ kg<sup>-1</sup> K<sup>-1</sup>

Example 11.4 When 0.15 kg of ice at 0°C is mixed with 0.30 kg of water at 50°C in a container, the resulting temperature is 6.7°C. Calculate the heat of fusion of ice.  $(s_{water} = 4186 \text{ J kg}^{-1} \text{ K}^{-1})$ 

Answer: Heat lost by water,

$$= ms_{w} (\theta_{f} - \theta_{1})_{w}$$
  
= (0.30 kg)(4186 J kg<sup>-1</sup> K<sup>-1</sup>)(50.0° C - 6.7° C)  
= 54376.14 J

Heat required to melt ice =  $m_2 L_r = (0.15 \text{ kg}) L_r$ 

Heat is necessary to bring ice water to its final temperature =  $m_1 s_w \left( \theta_f - \theta_1 \right)_{i}$ 

 $= (0.15 \text{ kg}) (4186 \text{ J kg}^{-1} \text{ K}^{-1}) (6.7^{\circ} \text{ C} - 0^{\circ} \text{ C})$ = 4206.93 J Heat lost = heat gained 54376.14 J = (0.15 kg)L<sub>f</sub> + 4206.93 J L<sub>f</sub> = 3.34×10<sup>5</sup> J kg<sup>-1</sup>.

Example 11.5 Calculate the heat required to convert 3 kg of ice at  $-12^{\circ}\text{C}$  kept in a calorimeter to steam at  $100^{\circ}\text{C}$  at atmospheric pressure. Given specific heat capacity of ice  $= 2100 \text{ J kg}^{-1} \text{ K}^{-1}$ , specific heat capacity of water  $= 4186 \text{ J kg}^{-1} \text{ K}^{-1}$ . latent heat of fusion of ice  $= 3.35 \times 10^5 \text{ J kg}^{-1}$  and latent heat of steam  $= 2.256 \times 10^6 \text{ J kg}^{-1}$ .

**Answer:** Mass of the ice, m = 3kg

Specific heat capacity of ice,  $s_{ice} = 2100 \,\mathrm{J \, kg^{-1} \, K^{-1}}$ 



Specific heat capacity of water,  $s_{water} = 4186 J kg^{-1} K^{-1}$ 

Latent heat of fusion of ice,  $L_{\text{fice}} = 3.35 \times 10^5 \,\text{J kg}^{-1}$ 

Latent heat of steam,  $L_{\text{steam}} = 2.256 \times 10^6 \,\text{J kg}^{-1}$ 

Here,

Q = heat required to convert 3kg of ice at  $-12^{\circ}$ C to steam at  $100^{\circ}$ C,

 $Q_1$  = heat required to convert ice at  $-12^{\circ}$ C to ice at  $0^{\circ}$ C.

 $= m s_{ice} \Delta T_1 = (3 \text{ kg}) (2100 \text{ J kg}^{-1} \text{ K}^{-1}) [0 - (-12)]^{\circ} \text{ C} = 75600 \text{ J}$ 

 $Q_2$  = heat required to melt ice at  $0^{\circ}$ C to water at  $0^{\circ}$ C

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= mL_{\text{itce}} = (3\,\text{kg}) (3.35 \times 10^5\,\text{J}\,\text{kg}^{-1})= 1005000\,\text{J}
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 $Q_3$  = heat required to convert water at 0°C to water at 100°C. =  $ms_w \Delta T_2$ =  $(3\text{kg})(4186\text{J}\text{kg}^{-1}\text{K}^{-1})$ 

 $(100^{\circ}C) = 1255800J$ 

 $Q_4$  = heat required to convert water at 100°C to steam at 100°C.  $mL_{\text{steam}} = (3\text{kg})(2.256 \times 10^6)\text{Jkg}^{-1}) = 6768000\text{J}$ 

After adding, we get,

 $Q = Q_1 + Q_2 + Q_3 + Q_4$ = 75600 J + 1005000 J + 1255800 J + 6768000 J = 9.1×10<sup>6</sup> J

Example 11.6 What is the temperature of the steel-copper junction in the steady state of the system shown in Fig. 11.15. Length of the steel rod = 15.0 cm, length of the copper rod

= 10.0 cm, temperature of the furnace =  $300^{\circ} \text{ C}$ , temperature of the other end =  $0^{\circ} \text{ C}$ . The area of cross section of the steel rod is twice that of the copper rod. (Thermal conductivity of steel = 50.2J } s<sup>-1</sup> m<sup>-1</sup> K<sup>-1</sup>; and of copper =  $385 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$ )





#### Answer:

Heat leakage from the sides of the rods is reduced by the insulating material wrapped around them. As a result, heat only moves along the length of the rods. Take a look at any cross section of the rod. Hea t coming into the element must equal heat flowing out of it in the steady state; otherwise, the element would experience a net gain or loss of heat, and its temperature would not be stable. As a result, at any pointalong the length of the combined steelcopper rod, the rate of heat flowing through a cross sectio n of the rod is the same in the steady state.

Let T be the temperature of the steel-copper junction in the steady state. Then,

$$\frac{K_1 A_1 (300 - T)}{L_1} = \frac{K_2 A_2 (T - 0)}{L_2}$$

here 1 = steel rod,

2 = copper rod

$$A_1 = 2A_2,$$
  
 $L_1 = 15.0 \text{ cm},$   
 $L_2 = 10.0 \text{ cm},$   
 $K_1 = 50.2 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$   
 $K_2 = 385 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$ 

we have,

$$\frac{50.2 \times 2(300 - T)}{15} = \frac{385T}{10}$$

 $T = 44.4^{\circ} C$ 

Example 11.7 An iron bar  $(L_1 = 0.1 \text{ m}, A_3 = 0.02 \text{ m}^2, K = 79 \text{ W m}^{-1} \text{ K}^{-1})$  and a brass bar  $(L_2 = 0.1 \text{ m}, A_2 = 0.02 \text{ m}^2 \cdot K_2 = 109 \text{ W m}^{-1} \text{ K}^{-1})$  are soldered end to end as shown in Fig. 11.16. The free ends of the iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expressions for and hence compute (i) the temperature of the junction of the two bars, (ii) the equivalent thermal conductivity of the compound bar, and (iii) the heat current through the compound bar.





Answer: Given,

$$L_{1} = L_{2} = L = 0.1 \text{ m},$$
  

$$A_{1} = A_{2} = A = 0.02 \text{ m}^{2}$$
  

$$K_{1} = 79 \text{ W m}^{-1} \text{ K}^{-1},$$
  

$$K_{2} = 109 \text{ W m}^{-1} \text{ K}^{-1}$$
  

$$T_{1} = 373 \text{ K}, and$$
  

$$T_{2} = 273 \text{ K}.$$

Under steady state condition, the heat current  $(H_1)$  through iron bar is equal to the heat current

 $\left(H_{2}\right)$  through brass bar.

So, 
$$H = H_1 = H_2$$
  
=  $\frac{K_1 A_1 (T_1 - T_0)}{L_1} = \frac{K_2 A_2 (T_0 - T_2)}{L_2}$ 

For

$$A_{1} = A_{2} = A$$
  

$$L_{1} = L_{2} = L$$
, this equation leads to  

$$K_{1}(T_{1} - T_{0}) = K_{2}(T_{0} - T_{2})$$

Thus, the junction temperature  $T_0$  of the two bars is

$$T_{0} = \frac{\left(K_{1}T_{1} + K_{2}T_{2}\right)}{\left(K_{1} + K_{2}\right)}$$

The heat current H via either bar is calculated using this equation.

$$H = \frac{K_1 A (T_1 - T_0)}{L} = \frac{K_2 A (T_0 - T_2)}{L}$$
$$= \left(\frac{K_1 K_2}{K_1 + K_2}\right) \frac{A (T_1 - T_0)}{L}$$
$$= \frac{A (T_1 - T_2)}{L \left(\frac{1}{K_1} + \frac{1}{K_2}\right)}$$



Applying above equations, the heat current H' through the compound bar of length  $L_1 + L_2 = 2L$  and the equivalent thermal conductivity K'', of the compound bar will be,

$$H' = \frac{K'A(T_1 - T_2)}{2L} = H$$
$$K' = \frac{2K_1K_2}{K_1 + K_2}$$

(i) } 
$$T_0 = \frac{\left(K_1 T_1 + K_2 T_2\right)}{\left(K_1 + K_2\right)}$$

 $=\frac{\left(79W\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}\right)(373\,\mathrm{K})+\left(109\,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}\right)(273\,\mathrm{K})}{79\,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}+109\,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}}=315\,\mathrm{K}$ 

(ii) 
$$K' = \frac{2K_1K_2}{K_1 + K_2}$$
  

$$= \frac{2 \times (79 \,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}) \times (109 \,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1})}{79 \,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1} + 109 \,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}} = 91.6 \,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}$$
(iii)  $H' = H = \frac{K' A (T_1 - T_2)}{2L}$ 

$$= \frac{(91.6 \,\mathrm{W}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}) \times (0.02 \,\mathrm{m}^2) \times (373 \,\mathrm{K} - 273 \,\mathrm{K})}{2 \times (0.1 \,\mathrm{m})} = 916.1 \,\mathrm{W}$$

Example 11.8 A pan filled with hot food cools from  $94^{\circ}Cto86^{\circ}C$  in 2 minutes when the room temperature is at 20°C. How long will it take to cool from  $71^{\circ}Cto69^{\circ}C$ ?

**Answer:** The average temperature of  $94^{\circ}C$  and  $86^{\circ}C$  is  $90^{\circ}C$ , which is  $70^{\circ}C$  above the room temperature. Under these conditions the pan cools  $8^{\circ}C$  in 2 *minutes*.

Applying Eq. (11.21), we get

 $\frac{\text{Change in temperature}}{\text{Time}} = K$  $\Delta T \frac{8^{\circ} \text{C}}{2 \min} = K (70^{\circ} \text{C})$ 

The average of  $69^{\circ}C$  and  $71^{\circ}C$  is  $70^{\circ}C$ , which is  $50^{\circ}C$  above room temperature.

K is the same in this case as it was in the first.



When we divide the two equations above, we get

$$\frac{8^{\circ} \mathrm{C} / 2 \min}{2^{\circ} \mathrm{C} / \mathrm{time}} = \frac{K(70^{\circ} \mathrm{C})}{K(50^{\circ} \mathrm{C})}$$

Time  $= 0.7 \min = 42 \mathrm{s}$ 

# (Chapter 11) (THERMAL PROPERTIES OF MATTER)

### XI

#### Exercises

**11.1** The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Answer: The Kelvin and Celsius scales are connected in the following way:

$$T_{\rm C} = T_{\rm K} - 273.15...$$
 (i)

The Fahrenheit and Celsius scales are connected in the following way:

$$T_F = \frac{9}{5}T_C + 32$$

For neon: 
$$T_{\rm K} = 24.57 \,\rm K$$

$$T_{\rm C} = 24.57 - 273.15$$
  
= -248.58°CT<sub>E</sub>  
=  $\frac{9}{5}T_{\rm C} + 32$   
=  $\frac{9}{5}(-248.58) + 32$   
= 415.44°F

<u>For carbon dioxide:</u>  $T_{\rm K} = 216.55 \,{\rm K}$ ∴  $T_{\rm C} = 216.55 - 273.15 = -56.60^{\circ} {\rm C}$ 

$$T_F = \frac{9}{5} (T_C) + 32$$
$$= \frac{9}{5} (-56.60) + 32$$
$$= -69.88^{\circ} C$$



**11.2** Two absolute scales A and B have triple points of water defined to be 200A and 350B. What is the relation between  $T_A and T_B$ ?

**Answer:** Triple point of water on absolute scale A,  $T_1 = 200$  A

Triple point of water on absolute scale B,  $T_2 = 350 \text{ B}$ 

Triple point of water on Kelvin scale,  $T_{\rm K} = 273.15 \,\rm K$ 

The temperature 273.15K on Kelvin scale is equivalent to 200A on absolute scale A.

 $T_1 = T_K$ 200 A = 273.15 K  $\therefore$  A =  $\frac{273.15}{200}$ 

The temperature 273.15 K on Kelvin scale is equivalent to 350 B on absolute scale B.

$$T_2 = T_K$$
  
350 B = 273.15

$$\therefore \mathbf{B} = \frac{273.15}{350}$$

 $T_{\rm A}$  is triple point of water on scale A.

 $T_{\rm B}$  is triple point of water on scale B.

$$\therefore \frac{273.15}{200} \times T_{\rm A} = \frac{273.15}{350} \times T_{\rm A}$$

$$T_{\rm A} = \frac{200}{350} T_{\rm B}$$

Therefore, the ratio  $T_{\rm A}$ :  $T_{\rm B}$  is given as 4: 7.

**11.3** The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:

$$R = R_{\circ} \left[ 1 + \alpha \left( T - T_{\circ} \right) \right]$$

The resistance is  $101.6\Omega$  at the triple-point of water 273.16K, and  $165.5\Omega$  at the normal melting point of lead (600.5K). What is the temperature when the resistance is  $123.4\Omega$ ?

Answer: Here,

$$R = R_0 \left[ 1 + \alpha \left( T - T_0 \right) \right] \dots (i)$$

Where,



 $R_0$  and  $T_0$  are the resistance and temperature at the start of the experiment, respectively.

R and T are the resistance and temperature of the finished product, respectively.

 $\alpha$  is a constant

At the triple point of water,  $T_0 = 273.15 \text{ K}$ 

Resistance of lead,  $R_0 = 101.6\Omega$ 

At normal melting point of lead, T = 600.5 K

Resistance of lead,  $R = 165.5\Omega$ 

Putting these values in equation (*i*), we receive:  $R = R_0 \left[ 1 + \alpha \left( T - T_0 \right) \right]$ 

165.5 = 101.6[1 + α(600.5 − 273.15)]  
1.629 = 1 + α(327.35)  
∴ α = 
$$\frac{0.629}{327.35}$$
  
= 1.92×10<sup>-3</sup> K<sup>-1</sup>

For resistance,  $R_1 = 123.4\Omega$ 

$$\mathbf{R}_{1} = \mathbf{R}_{0} \left[ 1 + \alpha \left( \mathbf{T} - \mathbf{T}_{0} \right) \right]$$

Where, T is the temperature when the resistance of lead is  $123.4\Omega$ 

$$123.4 = 101.6 \left[ 1 + 1.92 \times 10^{-3} (T - 273.15) \right]$$
  
$$1.214 = 1 + 1.92 \times 10^{-3} (T - 273.15) \frac{0.214}{1.92 \times 10^{-7}}$$
  
$$= T - 273.15$$
  
$$\therefore T = 384.61 K$$

# **11.4 Answer the following:**

The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

There were two fixed points in the original Celsius scale as mentioned above which were assigned the number  $0^{\circ}C$  and  $100^{\circ}C$  respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16K. What is the other fixed point on this (Kelvin) scale?

The absolute temperature (Kelvin scale) T is related to the temperature  $t_c$  on the Celsius scale by  $t_c = T - 273.15$ 



# Why do we have 273.15 in this relation, and not 273.16?

What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Answer: Water's triple point has a one-of-a-kind value of 273.16K.

The triple point of water is always present at specific volume and pressure values 273.16 K. Because the melting point of ice and the boiling point of water are dependent on pressure and tempera ture, these points do not have specific values. The absolute zero or 0 K is the other fixed point on the Kelvin absolute scale.

The temperature 273.16K is the triple point of water. It is not the melting point of ice. The melting point of ice is measured in degrees Celsius. Its corresponding value, on Kelvin scale is 273.15K.

Hence, absolute temperature (Kelvin scale) T, is related to temperature  $t_c$ , on Celsius scale as:

 $t_{\rm c} = T - 273.15$ 

Assume  $T_{\rm F}$  is temp. on Fahrenheit scale

 $T_{\rm K}$  is temp. on absolute scale.

$$\frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

Let  $T_{\rm F}$  is temperature on Fahrenheit scale and  $T_{\rm K}$  be the temperature on absolute scale. Both temperatures can be correlated as follows:

$$\frac{T_{\rm F} - 32}{180} = \frac{T_{\rm K} - 273.15}{100}$$

It is given that:

$$T_{K1} - T_{K} = 1K$$

$$\frac{T_{FI} - T_{F}}{180} = \frac{T_{KI} - T_{K}}{100}$$

$$= \frac{1}{100}T_{FH} - T_{F}$$

$$= \frac{1 \times 180}{100}$$

$$= \frac{9}{5}$$

Triple point of water = 273.16 K

So, Triple point of water on absolute scale =  $273.16 \times \frac{9}{5} = 491.69$ 



11.5 Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:

Temperature	Pressure Thermometer A	<b>Pressure Thermometer</b> <i>B</i>
Triple point of water	1.250×10 <sup>5</sup> Pa	0.200×10 <sup>5</sup> Pa
Normal melting point of sulphur	$1.797 \times 10^5$ Pa	$0.287 \times 10^5$ Pa

What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?

What do you think is the reason behind the slight difference in answers of themometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?

**Answer:** Triple point of water, T = 273.16 K.

At this temperature, pressure in the mometer  $A, P_A = 1.250 \times 10^5 \text{ Pa}$ 

Let  $T_1$  be the sulphur's typical melting point.

At this temperature, pressure in thermometer A,  $P_1 = 1.797 \times 10^5$  Pa

According to Charles' law, we get:

$$\frac{P_{A}}{T} = \frac{P_{1}}{T_{1}}$$
$$\therefore T_{1} = \frac{PT}{P_{A}}$$
$$= \frac{1.797 \times 10^{5} \times 273.16}{1.250 \times 10^{5}}$$
$$= 392.69 \,\mathrm{K}$$

As a result, the absolute temperature of sulphur's typical melting point, as measured using a thermome ter A is 392.69 K.

At triple point 273.16K, the pressure in thermometer B,  $P_{\rm B} = 0.200 \times 10^5$  Pa

At temperature  $T_1$ , the pressure in thermometer B,  $P_2 = 0.287 \times 10^5$  Pa

We may write the relationship as follows, according to Charles' law:

Infinite Sri Chaita  $\frac{P_{\rm B}}{T} = \frac{P_{\rm 1}}{T_{\rm 1}}$   $\frac{0.200 \times 10^5}{273.16}$   $= \frac{0.287 \times 10^5}{T_{\rm 1}}$   $\therefore T_{\rm 1} = \frac{0.287 \times 10^9}{0.200 \times 10^5} \times 273.16$   $= 391.98 \, \rm K$ 

As a result, the absolute temperature of sulphur's typical melting point, as measured using a thermome ter B is 391.98 K.

The oxygen and hydrogen gas present in thermometers A and B respectively are not perfect ideal gases. As a result, there is a little discrepancy in thermometer readings A and B

The experiment should be carried out at low pressure settings to minimise the disparity between the t wo values. These gases act like perfect ideal gases at low pressure.

11.6 A steel tape 1m long is correctly calibrated for a temperature of  $27.0^{\circ}$  C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is  $45.0^{\circ}$  C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is  $27.0^{\circ}$  C? Coefficient of linear expansion of steel =  $1.20 \times 10^{-5}$  K<sup>-1</sup>. Answer: Length of the steel tape at temperature,

 $T = 27^{\circ}$ C, l = 1 m = 100 cm

At temperature  $T_1 = 45^{\circ}$ C, the length of the steel rod,  $l_1 = 63$  cm

Coefficient of linear expansion of steel,  $\alpha = 1.20 \times 10^{-5} \text{ K}^{-1}$ 

Let  $l_2$  be the steel rod's true length and  $l_2$  be the steel tape's length at  $45^{\circ}$ C.

 $l' = l + \alpha l \left( T_1 - T \right)$ 

 $\therefore l' = 100 + 1.20 \times 10^{-5} \times 100(45 - 27)$ = 100.0216 cm

As a result, the steel rod's actual length as measured by the steel tape at  $45^{\circ}C$ :

$$l_2 = \frac{100.0216}{100} \times 63 = 63.0136 \,\mathrm{cm}$$

Therefore, the actual length of the rod at  $45.0^{\circ}$ C is 63.0136 cm. Its length at  $27.0^{\circ}$ C is 63.0 cm.



11.7 A large steel wheel is to be fitted on to a shaft of the same material. At 27°C, the outer diameter of the shaft is 8.70cm and the diameter of the central hole in the wheel is 8.69cm. The shaft is cooled using 'dry ice', At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range:  $\alpha_{steel} = 1.20 \times 10^{-3} \text{ K}^{-1}$ .

#### Answer:

The given temperature,  $T = 27^{\circ}C$  can be written in Kelvin as:

$$27 + 273 = 300 \,\mathrm{K}$$

Outer diameter of the steel shaft at  $T, d_1 = 8.70 \text{ cm}$ 

The diameter of the wheel's centre hole at  $T, d_2 = 8.69 \text{ cm}$ 

Coefficient of linear expansion of steel,  $a_{\text{steel}} = 1.20 \times 10^{-5} \text{ K}^{-1}$ 

After the shaft is cooled using 'dry ice', its temperature becomes  $T_1$ 

If the diameter of the shaft changes, the wheel will slip,  $\Delta d = 8.69 - 8.70 = -0.01$  cm

Temperature  $T_1$ ,:

$$\Delta d = d_1 a_{\text{steel}} (T_1 - T)$$
  
= 8.70×1.20×10<sup>-5</sup> (T<sub>1</sub> - 30  
(T<sub>1</sub> - 300) = 95.78  
 $\therefore$  T<sub>1</sub> = 204.21K  
= 204.21 - 273.16  
= -68.95° C

As a result, when the shaft temperature is  $-69^{\circ}$ C., the wheel will slip on it.

11.8 A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at  $27.0^{\circ}\text{C}$ . What is the change in the diameter of the hole when the sheet is heated to  $227^{\circ}\text{C}$ ? Coefficient of linear expansion of copper =  $1.70 \times 10^{-5} \text{ K}^{-1}$ .

**Answer:** Initial temperature,  $T_1 = 27.0^{\circ}$ C

Diameter of the hole at  $T_1, d_1 = 4.24 \text{ cm}$ 

Final temperature,  $T_2 = 227^{\circ} C$ 

Diameter of the hole at  $T_2 = d_2$ 



Co-efficient of linear expansion of copper,  $a_{Cu} = 1.70 \times 10^{-5} \text{ K}^{-1}$ 

For co-efficient of superficial expansion  $\beta$ , and change in temperature  $\Delta T$ ,

$$\frac{\text{Change in area } (\Delta A)}{\text{Original area } (A)} = \beta \Delta T$$

$$\frac{\left(\pi \frac{d_2^2}{4} - \pi \frac{d_1^2}{4}\right)}{\left(\pi \frac{d_1^2}{4}\right)} = \frac{\Delta A}{A}$$

$$\therefore \frac{\Delta A}{A} = \frac{d_2^2 - d_1^2}{d_1^2}$$
But  $\beta = 2\alpha$ 

$$\therefore \frac{d_2^2 - d_1^2}{d_1^2} = 2\alpha \Delta T$$

$$\frac{d_2^2}{d_1^2} - 1 = 2\alpha \left(T_2 - T_1\right)$$

$$\frac{d_2^2}{(4.24)^2} = 2 \times 1.7 \times 10^{-5} (227 - 27) + 12$$

$$d_2^2 = 17.98 \times 1.0068 = 18.1$$

$$\therefore d_2 = 4.2544 \text{ cm}$$

Change in diameter  $= d_2 - d_1 = 4.2544 - 4.24 = 0.0144$  cm

Hence, the diameter increases by  $1.44 \times 10^{-2}$  cm.

11.9 A brass wire 1.8 m long at  $27^{\circ}\text{C}$  is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of  $-39^{\circ}\text{C}$ , what is the tension developed in the wire, if its diameter is 2.0 mm? Co-efficient of linear expansion of brass =  $2.0 \times 10^{-5}$ ; Young's modulus of brass =  $0.91 \times 10^{11}$  Pa.

**Answer:** Initial temperature,  $T_1 = 27^{\circ} C$ 

Length of the brass wire at  $T_1$ , l = 1.8 m

Final temperature,  $T_2 = -39^{\circ}$ C

Diameter of the wire,  $d = 2.0 \text{ mm} = 2 \times 10^{-3} \text{ m}$ 

Tension developed in the wire = F

Coefficient of linear expansion of brass,  $\alpha = 2.0 \times 10^{-5} \text{ K}^{-1}$ 



Young's modulus of brass,  $Y = 0.91 \times 10^{11}$  Pa

Young's modulus is given by the relation:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$
$$\Delta L = \frac{F \times L}{A \times Y} \dots (i)$$

Where,

F = Tension developed in the wire

A = Area of cross-section of the wire.

 $\Delta L$  = Change in length,

$$\Delta L = aL(T_2 - T_1)\dots(ii)$$

Equating equations (i) and (ii), we get:

$$\alpha L(T_2 - T_1) = \frac{FL}{\pi \left(\frac{d}{2}\right)^2 \times Y}$$
  

$$F = \alpha \left(T_2 - T_1\right) \pi Y \left(\frac{d}{2}\right)^2$$
  

$$F = 2 \times 10^{-3} \times (-39 - 27) \times 3.14 \times 0.91 \times 10^{11} \times \left(\frac{2 \times 10^{-3}}{2}\right)^2$$
  

$$= -3.8 \times 10^2 \text{ N}$$

(The negative sign denotes that the strain is internal.)

As a result, the wire's tension is  $3.8 \times 10^2$  N.

11.10 A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C, if the original lengths are at 40.0°C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Co-efficient of linear expansion of brass =  $2.0 \times 10^{-5} \text{ K}^{-1}$ , steel =  $1.2 \times 10^{-5} \text{ K}^{-1}$ ).

**Answer:** Initial temperature,  $T_1 = 40^{\circ}$ C

Final temperature,  $T_2 = 250^{\circ} \text{C}$ 

Change in temperature,  $\Delta T = T_2 - T_1 = 210^{\circ} \text{C}$ 



Length of the brass rod at  $T_1, l_1 = 50 \text{ cm}$ Diameter of the brass rod at  $T_1, d_1 = 3.0 \text{ mm}$ Length of the steel rod at  $T_2, l_2 = 50 \text{ cm}$ Diameter of the steel rod at  $T_2, d_2 = 3.0 \text{ mm}$ Coefficient of linear expansion of brass,  $\alpha_1 = 2.0 \times 10^{-5} \text{ K}^{-1}$ Coefficient of linear expansion of steel,  $\alpha_2 = 1.2 \times 10^{-5} \text{ K}^{-1}$ Expansion in brass rod,:  $\frac{\text{Change in length } (\Delta_1)}{\text{Original length } (l_1)} = \alpha_1 \Delta T$ 

$$\therefore \Delta U_1 = 50 \times (2.1 \times 10^{-5}) \times 210$$
$$= 0.2205 \text{ cm}$$

For the expansion in the steel rod, we have:

Change in length 
$$(\Delta_2)$$
  
Original length  $(l_2)$  =  $\alpha_2 \Delta T$ 

$$\therefore \Delta U_2 = 50 \times (1.2 \times 10^{-5}) \times 210$$

$$= 0.126 \,\mathrm{cm}$$

Total change in the lengths of brass and steel,

$$\Delta l = \Delta l_1 + \Delta l_2$$
$$= 0.2205 + 0.126$$

= 0.346 cm

Total change in the length of the combined rod = 0.346 cm

There is no heat tension at the connection since the rod extends freely from both ends.

11.11 The coefficient of volume expansion of glycerin is  $49 \times 10^{-5} \text{ K}^{-1}$ . What is the fractional change in its density for a  $30^{\circ}$ C rise in temperature?

Answer: Coefficient of volume expansion of glycerin,  $\alpha_V = 49 \times 10^{-5} \text{ K}^{-1}$ 

Rise in temperature,  $\Delta T = 30^{\circ} \text{C}$ 

Fractional change in its volume  $=\frac{\Delta V}{V}$ 



This shift is linked to temperature shifts as follows:

$$\frac{\Delta V}{V} = \alpha_{\eta} \Delta T$$
$$V_{T_2} - V_{T_1} = V_{T_1} \alpha_V \Delta T$$

$$\frac{m}{\rho_{T_2}} - \frac{m}{\rho_{T_1}} = \frac{m}{\rho_{T_1}} \alpha_V \Delta T$$

Where, m = Mass of glycerine

$$\rho_{T_1}$$
 = initial density at  $T_1 \frac{\rho_{T_1} - \rho_{T_3}}{\rho_{T_2}} = \alpha_V \Delta T$ 

Where,

$$\frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} =$$
Fractional change in density

Fractional change in the density of glycerin =  $49 \times 10^{-5} \times 30 = 1.47 \times 10^{-2}$ 

11.12 A 10kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium =  $0.91J g^{-1} K^{-1}$ .

Answer: Power of the drilling machine,  $P = 10 \text{ kW} = 10 \times 10^3 \text{ W}$ 

Mass of the aluminum block,  $m = 8.0 \text{ kg} = 8 \times 10^3 \text{ g}$ 

Time for which the machine is used,  $t = 2.5 \text{ min} = 2.5 \times 60 = 150 \text{ s}$ 

Specific heat of aluminium,  $c = 0.91 \text{Jg}^{-1} \text{K}^{-1}$ 

After drilling, the temperature of the block rises =  $\delta T$ 

Total energy of the drilling machine = Pt

$$=10\times10^{3}\times150=1.5\times10^{6}$$
 J

It is given that only 50% of the power is useful.

Useful energy, 
$$\Delta Q = \frac{50}{100} \times 1.5 \times 10^6 = 7.5 \times 10^5 \text{ J}$$

But  $\Delta Q = mc\Delta T$ 



So, in 2.5 temperature of the stone rises after minutes of drilling is 103°C.

11.13 A copper block of mass 2.5 kg is heated in a furnace to a temperature of  $500^{\circ}\text{C}$  and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper =  $0.39 \text{ Jg}^{-1} \text{ K}^{-1}$ , heat of fusion of water =  $335 \text{ Jg}^{-1}$ ).

Answer: Mass of the copper block, m = 2.5 kg = 2500 g

The copper block's temperature has increased  $\Delta \theta = 500^{\circ} \text{C}$ 

Specific heat of copper,  $C = 0.39 \,\mathrm{Jg^{-1^{\circ}}C^{-1}}$ 

Heat of fusion of water,  $L = 335 \,\mathrm{Jg}^{-1}$ 

The greatest amount of heat that a copper block may lose in a given amount of time,  $Q = mC\Delta\theta = 2500 \times 0.39 \times 500$ 

Let  $m_1$  g when the copper block is placed on the ice block, the amount of ice that melts.

The heat gained by the melted ice,  $Q = m_1 L$ 

$$\therefore m_1 = \frac{Q}{L} = \frac{487500}{335} = 1455.22 \,\mathrm{g}$$

As a result, the total quantity of ice that may melt is 1.45 kg.

11.14 In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at  $150^{\circ}\text{C}$  is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing  $150 \text{ cm}^3$  of water at  $27^{\circ}\text{C}$ . The final temperature is  $40^{\circ}\text{C}$ . Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal?

Answer: Mass of the metal, m = 0.20 kg = 200 g

Initial temperature of the metal,  $T_1 = 150^{\circ}$ C

Final temperature of the metal,  $T_2 = 40^{\circ} \text{C}$ 

Calorimeter has water equivalent of mass, m' = 0.025 kg = 25 g



Volume of water,  $V = 150 \text{ cm}^3$ 

Mass (*M*) of water at temperature  $T = 27^{\circ}$ C :

 $150 \times 1 = 150 \text{ g}$ 

Fall in the temperature of the metal:

 $\Delta T = T_1 - T_2 = 150 - 40 = 110^{\circ} \text{C}$ 

Specific heat of water,  $C_{\rm w} = 4.186 \,{\rm J} \,{\rm / g \, / ^{\circ} K}$ 

Specific heat of the metal = C

Heat lost by the metal,  $\theta = mC\Delta T...(i)$ 

The temperature of the water and the calorimeter system has risen:

$$\Delta T' = 40 - 27 = 13^{\circ} C$$

The water and calorimeter system gained heat:

$$\Delta \theta^{''} = m_1 C_w \Delta T$$
$$= (M + m') C_w \Delta T'_{\dots} \dots (ii)$$

The heat lost by the metal is replaced by the heat absorbed by the water and the colorimeter device.

$$\Delta \theta^{''} = m_1 C_w \Delta T' = (M + m') C_w \Delta T' \dots (ii)$$

The metal loses the same amount of heat as the water and colorimeter system gain.

 $mC\Delta T = (M+m)C_{w}\Delta T$  $200 \times C \times 110 = (150+25) \times 4.186 \times 13$ 

$$\therefore C = \frac{175 \times 4.186 \times 13}{110 \times 200} = 0.43 \,\mathrm{J}\,\mathrm{g}^{-1}\,\mathrm{K}^{-1}$$

If some heat is lost to the surroundings, then the value of C will be smaller than the actual value.

# 11.15 Given below are observations on molar specific heats at room temperature of some common gases.

gas	molar specific heat $(Cv)$	
	$(calmol^{-1} K^{-1})$	
Hydrogen	4.87	
Nitrogen	4.97	
Oxygen	5.02	
Nitric Oxide	4.99	
Carbon Monoxide	5.01	
Chlorine	6.17	



The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92cal / molK . Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

Answer: The gases indicated in the table are diatomic in nature.

They have extra degrees of flexibility in addition to translational freedom (modes of motion). To raise the temperature of these gases, heat must be given. The average energy of all types of motion increases as a result of this. As a result, diatomic gases have a higher molar specific heat than monatomic gases. Only the rotating mode of motion is taken into account when calculating the molar specific heat of a d iatomic gas.

$$=\frac{5}{2}R = \frac{5}{2} \times 1.98 = 4.95 \text{ calmol}^{-1} \text{ K}^{-1}$$

With the exception of chlorine, all the observations in the given table agree with  $\int \frac{3}{2}R$ 

This is because, in addition to rotational and translational modes of motion, chlorine exhibits vibration al modes of motion at ambient temperature.

11.16 Answer the following questions based on the P-T phase diagram of carbon dioxide:

At what temperature and pressure can the solid, liquid and vapour phases of  $CO_2$  coexist in equilibrium? What is the effect of decrease of pressure on the fusion and boiling point of  $CO_2$ ? What are the critical temperature and pressure for  $CO_2$ ? What is their significance? Is  $CO_2$  solid, liquid or gas at (a)  $-70^{\circ}$ C under 1 atm, (b)  $-60^{\circ}$ C under 10 atm, (c) $15^{\circ}$ C under \$56 atm ?

Answer: The P-T phase diagram for CO<sub>2</sub> { is shown in the following figure. }





C is the triple point of the equatorial plane  $CO_2$  diag. This means that at the temperature and pressure corresponding to this point (i.e., at  $-56.6^{\circ}C$  and 5.11atm), the solid, liquid, and vaporous phases of  $CO_2$  co-exist in equilibrium.

With a drop in pressure, the fusion and boiling points of  $CO_2$  decrease.

The critical temperature and critical pressure of  $CO_2$  are 31.1°C and 73 atm respectively.

Even if it is compressed to a pressure greater than 73 atm,  $CO_2$  will not liquefy above the critical temperature.

It can be concluded from the P-T phase diagram of CO<sub>2</sub> that:

 $CO_2$  is gaseous at  $-70^{\circ}C$ , under 1 atm pressure.

 $CO_2$  is solid at  $-60^{\circ}C$ , under 10atm pressure.

 $CO_2$  is liquid at  $15^{\circ}C$ , under 56 *atm* pressure.

11.17 Answer the following questions based on the P-T phase diagram of CO<sub>2</sub>:

 $CO_2$  at 1 atm pressure and temperature  $-60^{\circ}C$  is compressed isothermally. Does it go through a liquid phase? What happens when  $CO_2$  at 4 atm pressure is cooled from room temperature at constant pressure? Describe qualitatively the changes in a given mass of solid  $CO_2$  at 10 atm pressure and temperature  $(5^{\circ}C)$  and it is based on the group temperature at constant.

10 atm pressure and temperature  $-65^{\circ}$ C as it is heated up to room temperature at constant pressure.

 $CO_2$  is heated to a temperature 70 °C and compressed isothermally. What changes in its properties do you expect to observe?

Answer: No,

It immediately condenses into a solid. The junction point where this parallel line cuts, The fusion and boiling points are determined by the fusion and vaporisation curves. As pressure rises, the behaviour of the gas deviates from that of an ideal gas.

The P-T phase diagram for CO<sub>2</sub> is shown in the following figure.





At 1 *atm* pressure and at  $-60^{\circ}$  C, CO<sub>2</sub> lies to the left of  $-56.6^{\circ}$  C (triple point C). As a result, it exists in between the vaporous and solid phases.

Thus,  $CO_2$  without passing through the liquid state, it condenses into a solid form.

At 4 *atm* pressure,  $CO_2$  lies below 5.11 *atm* (triple point C).

As a result, it exists in between the vaporous and solid phases. As a result, it condenses directly into th e solid form, by passing the liquid state.

When the temperature of a mass of solid  $CO_2$  (at 10 atm pressure and at -65°C) is increased, it

changes to the liquid phase and then to the vaporous phase. At the temperature 10 atm it produces a line parallel to it.

The junction point where this parallel line cuts the fusion and vaporisation curves determines the fusio n and boiling points.

If  $CO_2$  is heated to  $70^{\circ}C$  It will not transition to the liquid state if it is squeezed isothermally.

This is because 70 °C is higher than the critical temperature of  $CO_2$ . It will stay in the vapour state, but when pressure rises, it will deviate from its optimal behaviour.

11.18 A child running a temperature of  $101^{\circ}$  F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to  $98^{\circ}$  F in  $20 \min$ , what is the average rate of extra evaporation caused, by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about  $580 \text{ calg}^{-1}$ .

**Answer:** The child's body temperature at the start,  $T_1 = 101^{\circ}$  F

The child's body temperature at the end of the day,  $T_2 = 98^{\circ} \text{F}$ 



Change in temperature, 
$$\Delta T = \left[ (101 - 98) \times \frac{5}{9} \right]_{\circ} C$$

Time taken to reduce the temperature,  $t = 20 \min$ 

Mass of the child,  $m = 30 \text{ kg} = 30 \times 10^3 \text{ g}$ 

The human body's specific heat = Water's specific heat = c

$$= 1000 \text{ cal} / \text{ kg} / ^{\circ} \text{ C}$$

Latent heat of evaporation of water,  $L = 580 calg^{-1}$ 

The child's heat loss is calculated as follows:

$$\Delta \theta = mc \Delta T$$

 $=30\times1000\times(101-98)\times\frac{5}{9}$ 

Let  $m_1$  be the mass of the water evaporated from the child's body in 20 min.

The following factors contribute to heat loss through water:

$$\Delta \theta = m_1 L$$
  
$$\therefore m_1 = \frac{\Delta \theta}{L}$$
  
$$= \frac{50000}{580} = 86.2 \text{ g}$$

Extra evaporation rate produced by the medication on average  $=\frac{m_1}{t}=\frac{86.2}{200}=4.3 \text{ g}/\text{min}$ .

11.19 A 'thermacole' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6h. The outside temperature is  $45^{\circ}$ C, and co-efficient of thermal conductivity of thermacole is  $0.01 J s^{-1} m^{-1} K^{-1}$ . [Heat of fusion of water =  $335 \times 10^3 J k g^{-1}$ ]

**Answer:** Side of the given cubical ice box, s = 30 cm = 0.3 m

Thickness of the ice box, l = 5.0 cm = 0.05 m

In the ice box, there is a lot of ice of mass, m = 4 kg

Time gap,  $t = 6h = 6 \times 60 \times 60s$ 

Outside temperature,  $T = 45^{\circ}C$ 



Coefficient of thermal conductivity of thermacole,  $K = 0.01 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$ 

Heat of fusion of water,  $L = 335 \times 10^3 \, \mathrm{J \, kg^{-1}}$ 

Let m be the total amount of ice that melts in 6h.

The quantity of heat lost by the meal is calculated as follows:

$$\theta = \frac{KA(T-0)t}{l}$$

Where,

 $A = \text{Surface area of the box} = 6s^2 = 6 \times (0.3)^2 = 0.54 \text{ m}^3$  $\theta = \frac{0.01 \times 0.54 \times (45) \times 6 \times 60 \times 60}{0.05} = 104976 \text{ J}$ But  $\theta = m'L$  $\therefore m' = \frac{\theta}{L}$  $= \frac{104976}{335 \times 10^3}$ 

= 0.313kg

Mass of ice left = 4 - 0.313 = 3.687 kg

Hence, the amount of ice remaining after 6 his 3.687 kg.

11.20 A brass boiler has a base area of  $0.15 \text{ m}^2$  and thickness 1.0 cm. It boils water at the rate of 6.0 kg / min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass =  $109 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$ ; Heat of vaporisation of water =  $2256 \times 10^3 \text{ J kg}^{-1}$ .

**Answer:** Base area of the boiler,  $A = 0.15 \text{ m}^2$ 

Thickness of the boiler, l = 1.0 cm = 0.01 m

Boiling rate of water, R = 6.0 kg / min

Mass, m = 6kg

Time,  $t = 1 \min = 60 \mathrm{s}$ 

Thermal conductivity of brass,  $K = 109 \,\mathrm{J \, s^{-1} \, m^{-1} \, K^{-1}}$ 



Heat of vaporisation,  $L = 2256 \times 10^3 \, \mathrm{J \, kg^{-1}}$ 

The quantity of heat that flows into the water via the boiler's brass base is determined by:

$$\theta = \frac{KA(T_1 - T_2)t}{l}$$

Where,

 $T_1$  = The temperature of the flame that comes into contact with the boiler.

 $T_2$  = Boiling point of water = 100° C

Heat required for boiling the water:

 $\theta = mL...(ii)$ 

Equating equations (i) and (ii), we get:

$$\therefore mL = \frac{KA(T_1 - T_2)t}{l}$$

 $T_{1} - T_{2} = \frac{mLl}{KAt}$  $= \frac{6 \times 2256 \times 10^{3} \times 0.01}{109 \times 0.15 \times 60}$  $= 137.98^{\circ} C$ 

As a result, the temperature of the portion of the flame that comes into contact with the boiler is  $237.98^{\circ}$ C.

11.21 Explain why: a body with large reflectivity is a poor emitter a brass tumbler feels much colder than a wooden tray on a chilly day an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace the earth without its atmosphere would be inhospitably cold heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

**Answer:** A body with a high reflectivity absorbs light radiations poorly. A poor radiation absorber will also be a poor radiation emitter. As a result, a body with a high reflectivity is an inefficient emitter. Brass is an excellent heat conductor.

When a person touches a brass tumbler, heat is easily transferred from the body to the tumbler. As a result, the body's temperature drops to a lower level, and one feels colder. Wood is a poor heat conductor.

When one touches a wooden tray, just a small amount of heat is transferred from the body to the tray. As a result, there is only a minor reduction in body temperature, and one does not feel cool. On a cool day, a metal tumbler seems colder than a wooden tray.

A temperature measurement for a red hot iron object left out in the open from an optical pyrometer set for optimal black body radiation is considerably too low.

Black body radiation equation is given by:



Where,

E = Energy radiation

T = Temperature of optical

 $pyrometerT_{o} = Temperature$ 

# $\sigma = Constant$

As a result, increasing the temperature of open space lowers the amount of energy radiated.

When the same piece of iron is heated in a furnace, the amount of radiation energy produced is the sa me,

 $E = \sigma T^4$ 

Earth would be inhospitably cold without its atmosphere. There will be no additional heat trapped in t he absence of air gases. The heat would be transmitted back to the surface of the earth. In terms of warming a structure, a heating system based on the circulation of steam is more efficient t han one based on the circulation of hot water.

This is due to the fact that steam contains excess heat in the form of latent heat (540 cal/g).

# **11.22** A body cools from $80^{\circ}$ C to $50^{\circ}$ C in 5 minutes. Calculate the time it takes to cool from $60^{\circ}$ C to $30^{\circ}$ C. The temperature of the surroundings is $20^{\circ}$ C.

Answer: Applying Newton's law of cooling, we have:

$$-\frac{dT}{dt} = K\left(T - T_0\right)$$

$$\frac{dT}{K(T-T_0)} = -Kd$$

Where,

Temperature of the body = T

Temperature of the surroundings  $= T_0 = 20^{\circ} C$ 

K is a constant

Temperature of the body falls from 80°C to 50°C in time,  $t = 5 \min = 300$ s

Integrating equation (i), we get:



$$\int_{50}^{80} \frac{K(T - T_0)}{K(T - T_0)} = -\int_0^{80} K dt$$

$$\left[ \log_e (T - T_0) \right]_{50}^{80} = -K[t]_0^{300}$$

$$\frac{2.3026}{K} \log_{10} \frac{80 - 20}{50 - 20}$$

$$= -30 \frac{2.3026}{K} \log_{10} 2$$

$$= -300 \frac{-2.3026}{300} \log_{10} 2 = K$$

The temperature of the body falls from  $60^{\circ}$ C to  $30^{\circ}$ C in time = t

The temperature of the body falls from  $60^{\circ}$ C to  $30^{\circ}$ C in time = *i* 

Hence, we get:

$$\frac{2.3026}{K}\log_{10}\frac{60-20}{30-20} = -k$$
$$\frac{-2.3026}{t}\log_{10}4 = K$$

Equating equations (ii) and (iii), we get:

$$\frac{-2.3026}{t}\log_{10}4 = \frac{-2.3026}{300}\log_{10}2$$

$$\therefore i = 300 \times 2 = 600 \text{ s} = 10 \text{ min}$$

So, the time taken to cool the body from  $60^{\circ}$ C to  $30^{\circ}$ C is 10 *minutes*.